Exercise 15.1

Question 1:

The following graph shows the temperature of a patient in a hospital, recorded every hour.

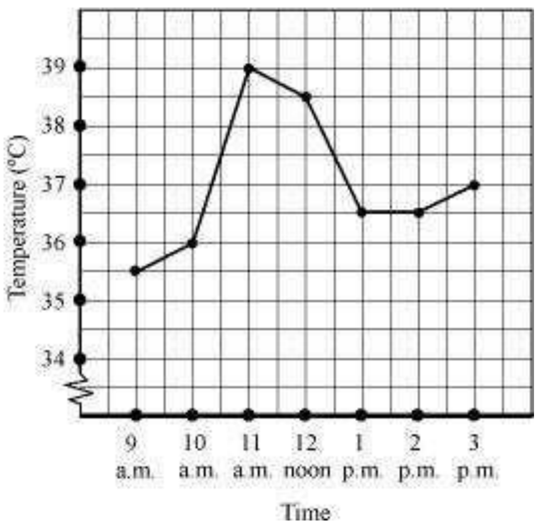
(a) What was the patient's temperature at 1 p.m.?

(b) When was the patient's temperature 38.5°C?

(c) The patient's temperature was the same two times during the period given. What were these two times?

(d) What was the temperature at 1.30 p.m? How did you arrive at your answer?

(e) During which periods did the patient's temperature show an upward trend?



Answer:

(a) At 1 p.m., the patient's temperature was 36.5°C.

(b) The patient's temperature was 38.5°C at 12 noon.

(c) The patient's temperature was same at 1 p.m. and 2 p.m.

(d) The graph between the times 1 p.m. and 2 p.m. is parallel to the x-axis. The temperature at 1 p.m. and 2 p.m. is 36.5°C. So, the temperature at 1:30 p.m. is 36.5°C.

(e) During the following periods, the patient's temperature showed an upward trend.

9 a.m. to 10 a.m., 10 a.m. to 11 a.m., 2 p.m. to 3 p.m.

Question 2:

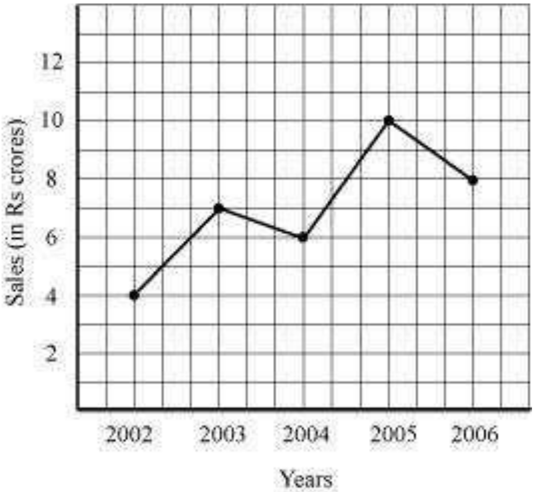
The following line graph shows the yearly sales figure for a manufacturing company.

(a) What were the sales in (i) 2002 (ii) 2006?

(b) What were the sales in (i) 2003 (ii) 2005?

(c) Compute the difference between the sales in 2002 and 2006.

(d) In which year was there the greatest difference between the sales as compared to its previous year?



Answer:

(a)

(i) In 2002, the sales were Rs 4 crores.

(ii) In 2006, the sales were Rs 8 crores.

(b)

(i) In 2003, the sales were Rs 7 crores.

(ii) In 2005, the sales were Rs 10 crores.

(c)

(i) In 2002, the sales were Rs 4 crores and in 2006, the sales were Rs 8 crores.

Difference between the sales in 2002 and 2006

= Rs (8 − 4) crores = Rs 4 crores

(d) Difference between the sales of the year 2006 and 2005

= Rs (10 − 8) crores = Rs 2 crores

Difference between the sales of the year 2005 and 2004

= Rs (10 − 6) crores = Rs 4 crores

Difference between the sales of the year 2004 and 2003

= Rs (7 − 6) crore = Rs 1 crore

Difference between the sales of the year 2003 and 2002

= Rs (7 − 4) crores = Rs 3 crores

Hence, the difference was the maximum in the year 2005 as compared to its previous year 2004.

Question 3:

For an experiment in Botany, two different plants, plant A and plant B were grown under similar laboratory conditions. Their heights were measured at the end of each week for 3 weeks. The results are shown by the following graph.

(a) How high was Plant A after (i) 2 weeks (ii) 3weeks?

(b) How high was Plant B after (i) 2 weeks (ii) 3weeks?

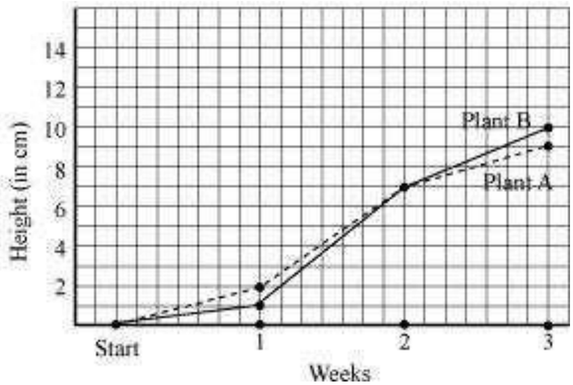
(c) How much did Plant A grow during the 3rd week?

(d) How much did Plant B grow from the end of the 2nd week to the end of the 3rd week?

(e) During which week did Plant A grow most?

(f) During which week did Plant B grow least?

(g) Were the two plants of the same height during any week shown here? Specify.



Answer:

(a)

(i) After 2 weeks, the height of plant A was 7 cm.

(ii) After 3 weeks, the height of plant A was 9 cm.

(b)

(i) After 2 weeks, the height of plant B was 7 cm.

(ii) After 3 weeks, the height of plant B was 10 cm.

(c) Growth of plant A during 3rd week = 9 cm − 7 cm = 2 cm

(d) Growth of plant B from the end of the 2nd week to the end of the 3rd week

= 10 cm − 7 cm = 3 cm

(e) Growth of plant A during 1st week = 2 cm − 0 cm = 2 cm

Growth of plant A during 2nd week = 7 cm − 2 cm = 5 cm

Growth of plant A during 3rd week = 9 cm − 7 cm = 2 cm

Therefore, plant A grew the most, i.e. 5 cm, during the 2nd week.

(f) Growth of plant B during 1st week = 1 cm − 0 cm = 1 cm

Growth of plant B during 2nd week = 7 cm − 1 cm = 6 cm

Growth of plant B during 3rd week = 10 cm − 7 cm = 3 cm

Therefore, plant B grew the least, i.e. 1 cm, during the 1st week.

(g) At the end of the 2nd week, the heights of both plants were same.

Question 4:

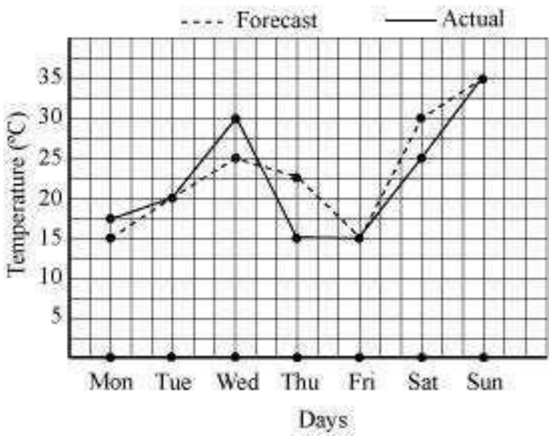
The following graph shows the temperature forecast and the actual temperature for each day of a week.

(a) On which days was the forecast temperature the same as the actual temperature?

(b) What was the maximum forecast temperature during the week?

(c) What was the minimum actual temperature during the week?

(d) On which day did the actual temperature differ the most from the forecast temperature?



Answer:

(a) The forecast temperature was same as the actual temperature on Tuesday, Friday, and Sunday.

(b) The maximum forecast temperature during the week was 35°C.

(c) The minimum actual temperature during the week was 15°C.

(d) The actual temperature differs the most from the forecast temperature on Thursday.

Question 5:

Use the tables below to draw linear graphs.

(a) The number of days a hill side city received snow in different years.

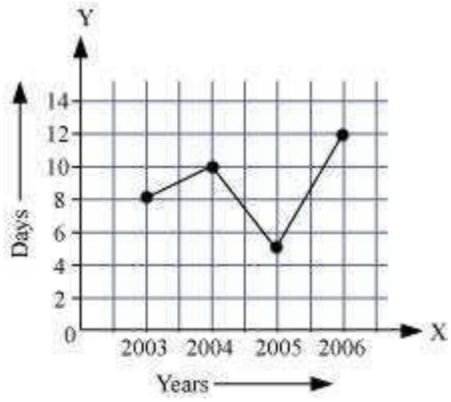
Year	2003	2004	2005	2006
Days	8	10	5	12

(b) Population (in thousands) of men and women in a village in different years.

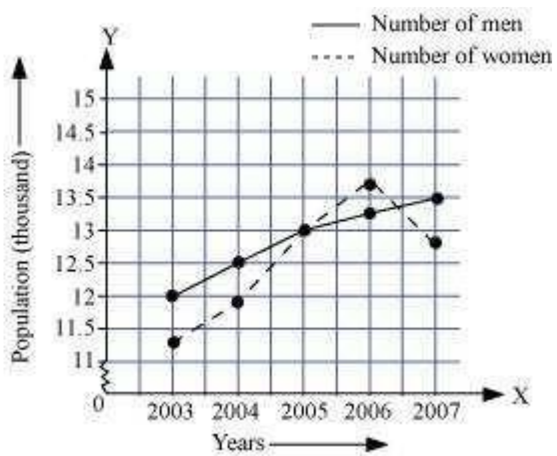
Year	2003	2004	2005	2006	2007
Number of men	12	12.5	13	13.2	13.5
Number of women	11.3	11.9	13	13.6	12.8

Answer:

(a) By taking the years on x-axis and the number of days on y-axis and taking scale as 1 unit = 2 days on y-axis and 2 unit = 1 year on x-axis, the linear graph of the given information can be drawn as follows.

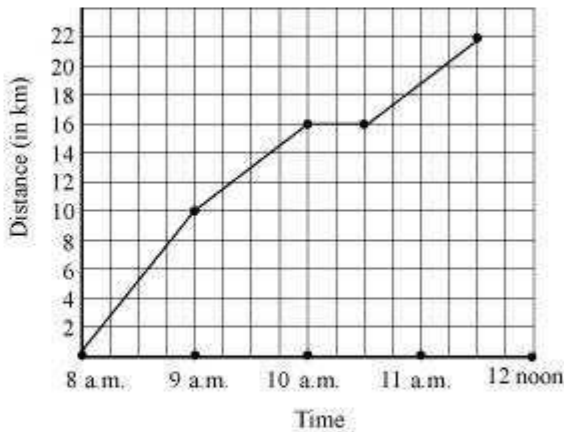


(b) By taking the years on x-axis and population on y-axis and scale as 1 unit = 0.5 thousand on y-axis and 2 unit = 1 year on x-axis, the linear graph of the given information can be drawn as follows.



Question 6:

A courier-person cycles from a town to a neighboring suburban area to deliver a parcel to a merchant. His distance from the town at different times is shown by the following graph.



(a) What is the scale taken for the time axis?

(b) How much time did the person take for the travel?

(c) How far is the place of the merchant from the town?

(d) Did the person stop on his way? Explain.

(e) During which period did he ride fastest?

Answer:

(a) Scale taken for the time axis is 4 units = 1 hour

(b) The person travelled during the time 8 a.m. − 11:30 a.m.

Therefore, the person took $3\frac{1}{2}$ hours to travel.

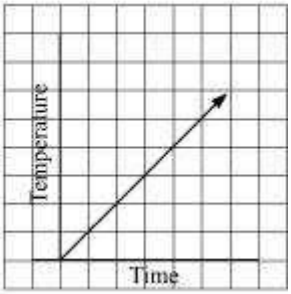
(c) The merchant is 22 km far from the town.

(d) Yes, the person stopped on his way from 10 a.m. to 10: 30 a.m. This is indicated by the horizontal part of the graph.

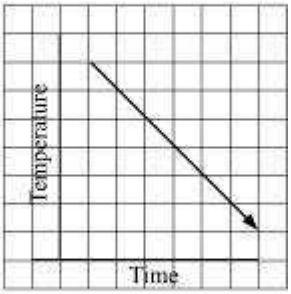
(e) From the graph, it can be observed that during 8 a.m. to 9 a.m., the person travelled the maximum distance. Thus, the person's ride was the fastest between 8 a.m. and 9 a.m.

Question 7:

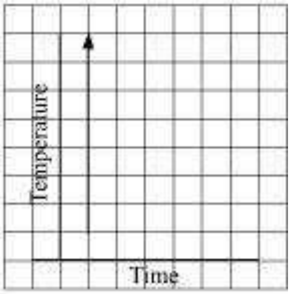
Can there be a time temperature graph as follows? Justify you're answer:



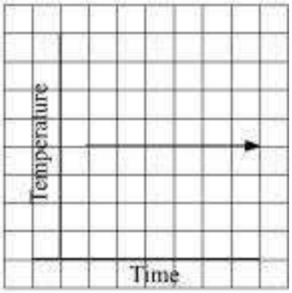
(i)



(ii)



(iii)



(iv)

Answer:

(i) This can be a time−temperature graph, as the temperature can increase with the increase in time.

(ii) This can be a time−temperature graph, as the temperature can decrease with the decrease in time.

(iii) This cannot be a time−temperature graph since different temperatures at the same time are not possible.

(iv) This can be a time−temperature graph, as same temperature at different times is possible.

Exercise 15.2

Question 1:

Plot the following points on a graph sheet. Verify if they lie on a line

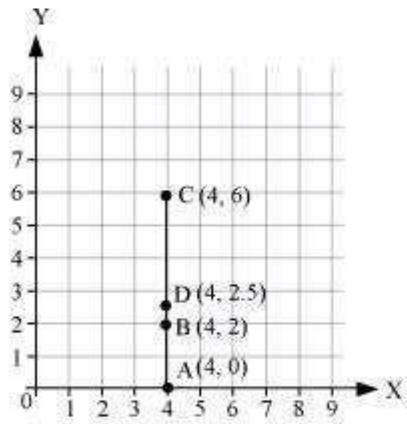
(a) A(4, 0), B(4, 2), C(4, 6), D(4, 2.5)

(b) P(1, 1), Q(2, 2), R(3, 3), S(4, 4)

(c) K(2, 3), L(5, 3), M(5, 5), N(2, 5)

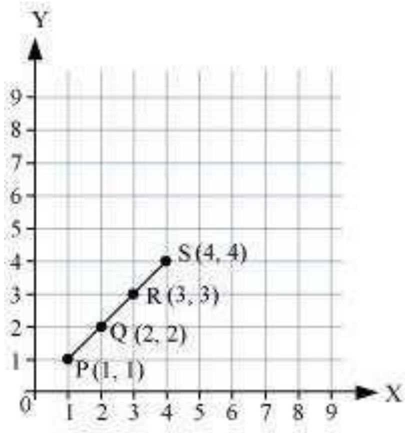
Answer:

(a) We can plot the given points and join the consecutive points on a graph paper as follows.



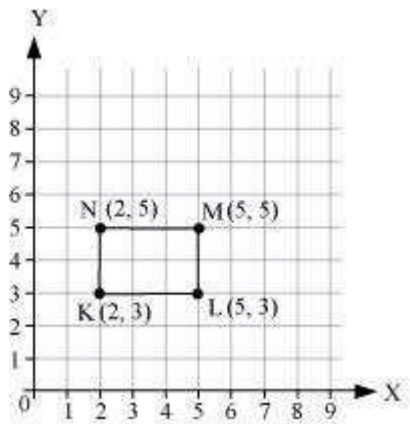
From the graph, it can be observed that the points A, B, C, and D lie on the same line.

(b) We can plot the given points and join the consecutive points on a graph paper as follows.



Hence, points P, Q, R, and S lie on the same line.

(c) We can plot the given points and join the consecutive points on a graph paper as follows.

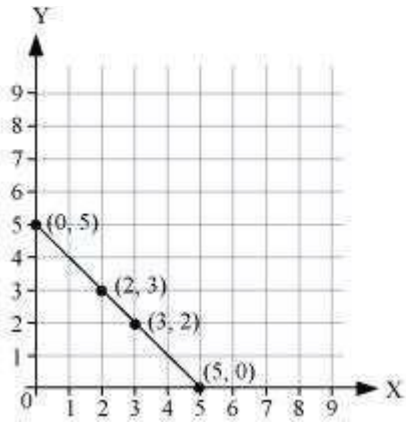


Hence, points K, L, M, and N are not lying on the same line.

Question 2:

Draw the line passing through (2, 3) and (3, 2). Find the coordinates of the points at which this line meets the x-axis and y-axis.

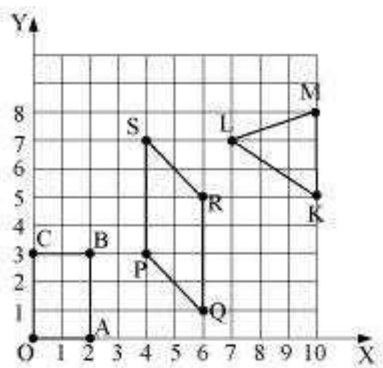
Answer:



From the graph, it can be observed that the line joining the points (2, 3) and (3, 2) meets the x-axis at the point (5, 0) and the y-axis at the point (0, 5).

Question 3:

Write the coordinates of the vertices of each of these adjoining figures.



Answer:

The coordinates of the vertices in the given figure are as follows.

O (0, 0), A (2, 0), B (2, 3), C (0, 3)

P (4, 3), Q (6, 1), R (6, 5), S (4, 7)

K (10, 5), L (7, 7), M (10, 8)

Question 4:

State whether True or False. Correct those are false.

(i) A point whose x coordinate is zero and y-coordinate is non-zero will lie on the y-axis.

(ii) A point whose y coordinate is zero and x-coordinate is 5 will lie on y-axis.

(iii) The coordinates of the origin are (0, 0).

Answer:

(i) True

(ii) False

The point whose y-coordinate is zero and x-coordinate is 5 will lie on x-axis.

(iii) True

Exercise 15.3

Question 1:

Draw the graphs for the following tables of values, with suitable scales on the axes.

(a) Cost of apples

Number of apples	1	2	3	4	5
Cost (in Rs)	5	10	15	20	25

(b) Distance travelled by a car

Time (in hours)	6 a.m.	7 a.m.	8 a.m.	9 a.m.
Distance (in km)	40	80	120	160

(i) How much distance did the car cover during the period 7.30 a.m. to 8 a.m.?

(ii) What was the time when the car had covered a distance of 100 km since its start?

(c) Interest on deposits for a year:

Deposit (in Rs)	1000	2000	3000	4000	5000
Simple interest (in Rs)	80	160	240	320	400

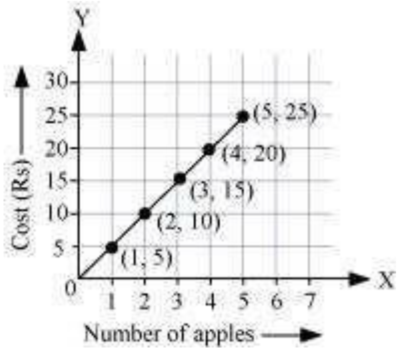
(i) Does the graph pass through the origin?

(ii) Use the graph to find the interest on Rs 2500 for a year:

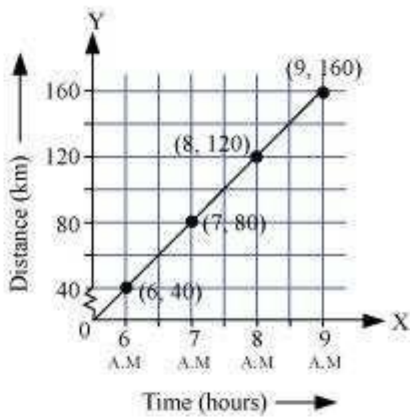
(iii) To get an interest of Rs 280 per year, how much money should be deposited?

Answer:

(a) Taking a suitable scale (for x-axis, 1 unit = 1 apple and for y-axis, 1 unit = Rs 5), we can mark the number of apples on x-axis and the cost of apples on y-axis. A graph of the given data is as follows.



(b) Taking a suitable scale (for x-axis, 2 units = 1 hour and for y-axis, 2 units = 40 km), we can represent the time on x-axis and the distance covered by the car on y-axis. A graph of the given data is as follows.



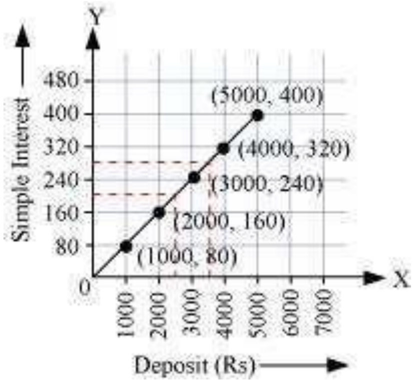
(i) During the period 7:30 a.m. to 8 a.m., the car covered a distance of 20 km.

(ii) The car covered a distance of 100 km at 7:30 a.m. since its start.

(c) Taking a suitable scale,

For x-axis, 1 unit = Rs 1000 and for y-axis, 1 unit = Rs 80

We can represent the deposit on x-axis and the interest earned on that deposit on y-axis. A graph of the given data is obtained as follows.



From the graph, the following points can be observed.

(i) Yes. The graph passes through the origin.

(ii) The interest earned in a year on a deposit of Rs 2500 is Rs 200.

(iii) To get an interest of Rs 280 per year, Rs 3500 should be deposited.

Question 2:

Draw a graph for the following.

(i)

Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24

Is it a linear graph?

(ii)

Side of square (in cm)	2	3	4	5	6
Area (in cm^2)	4	9	16	25	36

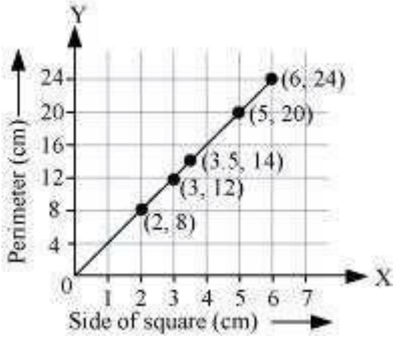
Is it a linear graph?

Answer:

(i) Choosing a suitable scale,

For x-axis, 1 unit = 1 cm and for y-axis, 1 unit = 4 cm

We can represent the side of a square on x-axis and the perimeter of that square on y-axis. A graph of the given data is drawn as follows.

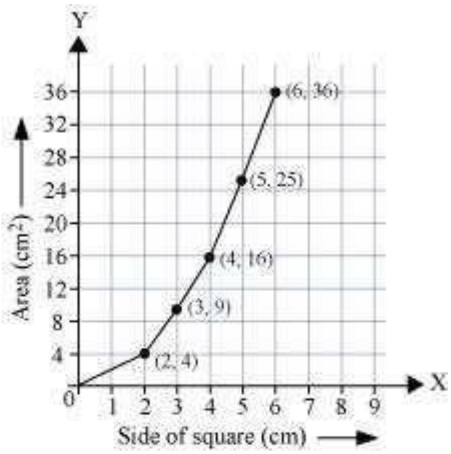


It is a linear graph.

(ii)Choosing a suitable scale,

For x-axis, 1 unit = 1 cm and for y-axis, 1 unit = 4 cm^2

We can represent the side of a square on the x-axis and the area of that square on y-axis. A graph of the given data is as follows.



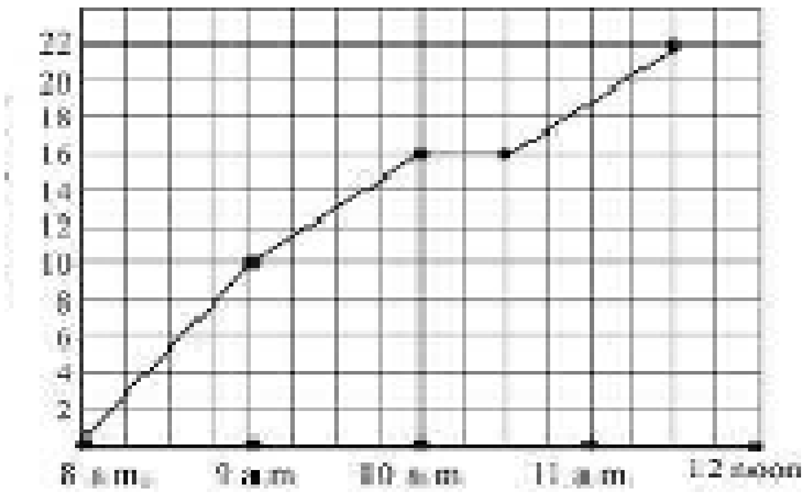
It is not a linear graph.

DELHI PUBLIC SCHOOL, GANDHINAGAR
MIND MAP
CH- 15 INTRODUCTION TO GRAPHS
SUBJECT: MATHEMATICS CLASS:VIII

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE:1 Inferences from a line graph and drawing a linear graph

- A courier person rides his bicycle from city to town to deliver parcels. His distance from the town at different times is as shown in the following graph;



(a) What is the scale taken for the time axis?
(b) How much time did the person take for travel?
(c) Did the person stop in his way? Explain.
(d) During which period did he ride fastest?

- Use the table below to draw the linear graph.

Year	2003	2004	2005	2006	2007
Number of men	12	12.5	13	13.2	13.5
Number of women	11.3	11.9	13	13.6	12.8

TYPE-II: Locating a point and it's coordinates in linear graph

- Plot the following points on graph sheet and verify if they lie on a line.
  (a) A (4,0), B (4,2), C (4,6) & D (4,2.5)
  (b) K (2,3), L (5,3), M (5,5) & N (2,5)
- Draw the line passing through (2,3) and (3,2). Find the coordinates of the points at which line meets the *x*-axis and *y*- axis.

TYPE-III: SOME APPLICATIONS

- Manoj drives a jeep constantly at a speed of 60km/hr. Draw a time-distance graph for this situation. Use it to find
  (a) The time taken by him to drive 100km.
  (b) The distance covered by him in $\frac{1}{2}$ hr.
- Draw the graph for the following data

Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24

Exercise 14.1

Question 1:

Find the common factors of the terms

(i) $12x$, 36

(ii) $2y$, $22xy$

(iii) $14pq$, $28p^2q^2$

(iv) $2x$, $3x^2$, 4

(v) $6abc$, $24ab^2$, $12a^2b$

(vi) $16x^3$, $-4x^2$, $32x$

(vii) $10pq$, $20qr$, $30rp$

(viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

Answer:

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

The common factors are 2, 2, 3.

And, $2 \times 2 \times 3 = 12$

(ii) $2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

The common factors are 2, y.

And, $2 \times y = 2y$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

The common factors are 2, 7, p, q.

And, $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

The common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

The common factors are 2, 3, a, b.

And, $2 \times 3 \times a \times b = 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$

$-4x^2 = -1 \times 2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

The common factors are 2, 2, x.

And, $2 \times 2 \times x = 4x$

(vii) $10pq = 2 \times 5 \times p \times q$

$20qr = 2 \times 2 \times 5 \times q \times r$

$30rp = 2 \times 3 \times 5 \times r \times p$

The common factors are 2, 5.

And, $2 \times 5 = 10$

(viii) $3x^2y^3 = 3 \times x \times x \times y \times y \times y$

$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$

$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$

The common factors are x, x, y, y.

And,

$x \times x \times y \times y = x^2y^2$

Question 2:

Factorise the following expressions

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$

(v) $20l^2m + 30\ alm$

(vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4\ ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

Answer:

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7.

$\therefore\ 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7\ (x - 6)$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3.

$\therefore\ 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$

$= 2 \times 3\ [p - (2 \times q)]$

$= 6\ (p - 2q)$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a.

$\therefore\ 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$

$= 7 \times a\ [a + 2] = 7a\ (a + 2)$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$\therefore\ -16z + 20z^3 = -\ (2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z)\ [-\ (2 \times 2) + (5 \times z \times z)]$

$= 4z\ (-\ 4 + 5z^2)$

(v) $20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30alm = 2 \times 3 \times 5 \times a \times l \times m$

The common factors are 2, 5, l, and m.

$\therefore\ 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$

$= (2 \times 5 \times l \times m)\ [(2 \times l) + (3 \times a)]$

$= 10lm\ (2l + 3a)$

(vi) $5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

The common factors are 5, x, and y.

$\therefore\ 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$

$= 5 \times x \times y\ [x - (3 \times y)]$

$= 5xy\ (x - 3y)$

(vii) $10a^2 = 2 \times 5 \times a \times a$

$15b^2 = 3 \times 5 \times b \times b$

$20c^2 = 2 \times 2 \times 5 \times c \times c$

The common factor is 5.

$10a^2 - 15b^2 + 20c^2 = (2 \times 5 \times a \times a) - (3 \times 5 \times b \times b) + (2 \times 2 \times 5 \times c \times c)$

$= 5\ [(2 \times a \times a) - (3 \times b \times b) + (2 \times 2 \times c \times c)]$

$= 5\ (2a^2 - 3b^2 + 4c^2)$

(viii) $4a^2 = 2 \times 2 \times a \times a$

$4ab = 2 \times 2 \times a \times b$

$4ca = 2 \times 2 \times c \times a$

The common factors are 2, 2, and a.

$\therefore\ -4a^2 + 4ab - 4ca = -\ (2 \times 2 \times a \times a) + (2 \times 2 \times a \times b) - (2 \times 2 \times c \times a)$

$= 2 \times 2 \times a\ [-\ (a) + b - c]$

$= 4a\ (-a + b - c)$

(ix) $x^2yz = x \times x \times y \times z$

$xy^2z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

The common factors are x, y, and z.

$\therefore\ x^2yz + xy^2z + xyz^2 = (x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z)$

$= x \times y \times z\ [x + y + z]$

$= xyz\ (x + y + z)$

(x) $ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

The common factors are x and y.

$ax^2y + bxy^2 + cxyz = (a \times x \times x \times y) + (b \times x \times y \times y) + (c \times x \times y \times z)$

$= (x \times y)\ [(a \times x) + (b \times y) + (c \times z)]$

$= xy\ (ax + by + cz)$

Question 3:

Factorise

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

Answer:

(i) $x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$

$= x\ (x + y) + 8\ (x + y)$

$= (x + y)\ (x + 8)$

(ii) $15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5 \times y - 2$

$= 3x\ (5y - 2) + 1\ (5y - 2)$

$= (5y - 2)\ (3x + 1)$

(iii) $ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$

$= x\ (a + b) - y\ (a + b)$

$= (a + b)\ (x - y)$

(iv) $15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$

$= 3q\ (5p + 3) + 5\ (5p + 3)$

$= (5p + 3)\ (3q + 5)$

(v) $z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$

$= z\ (1 - xy) - 7\ (1 - xy)$

$= (1 - xy)\ (z - 7)$

Exercise 14.2

Question 1:

Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Answer:

(i) $a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$

$= (a + 4)^2$ $[(x + y)^2 = x^2 + 2xy + y^2]$

(ii) $p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$

$= (p - 5)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times (7y) \times (6z) + (6z)^2$

$= (7y + 6z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(v) $4x^2 - 8x + 4 = (2x)^2 - 2\ (2x)\ (2) + (2)^2$

$= (2x - 2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= [(2)\ (x - 1)]^2 = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2 = (11b)^2 - 2\ (11b)\ (4c) + (4c)^2$

$= (11b - 4c)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(vii) $(l + m)^2 - 4lm = l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= (l - m)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(viii) $a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2\ (a^2)\ (b^2) + (b^2)^2$

$= (a^2 + b^2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

Question 2:

Factorise

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Answer:

(i) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$= (2p + 3q)\ (2p - 3q)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(ii) $63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$= 7[(3a)^2 - (4b)^2]$

$= 7(3a + 4b)\ (3a - 4b)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(iii) $49x^2 - 36 = (7x)^2 - (6)^2$

$= (7x - 6)\ (7x + 6)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(iv) $16x^5 - 144x^3 = 16x^3(x^2 - 9)$

$= 16\ x^3\ [(x)^2 - (3)^2]$

$= 16\ x^3(x - 3)\ (x + 3)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(v) $(l + m)^2 - (l - m)^2 = [(l + m) - (l - m)]\ [(l + m) + (l - m)]$

[Using identity $a^2 - b^2 = (a - b)\ (a + b)$]

$= (l + m - l + m)\ (l + m + l - m)$

$= 2m \times 2l$

$= 4ml$

$= 4lm$

(vi) $9x^2y^2 - 16 = (3xy)^2 - (4)^2$

$= (3xy - 4)\ (3xy + 4)\ [a^2 - b^2 = (a - b)\ (a + b)]$

(vii) $(x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2\ [(a - b)^2 = a^2 - 2ab + b^2]$

$= (x - y - z)(x - y + z)$ $[a^2 - b^2 = (a - b)(a + b)]$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2)$

$= (5a)^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - [(2b - 7c)^2]$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

$= [5a + (2b - 7c)]\,[5a - (2b - 7c)]$

[Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= (5a + 2b - 7c)(5a - 2b + 7c)$

Question 3:

Factorise the expressions

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Answer:

(i) $ax^2 + bx = a \times x \times x + b \times x = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7 \times p \times p + 3 \times 7 \times q \times q = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) $(lm + l) + m + 1 = lm + m + l + 1$

$= m(l + 1) + 1(l + 1)$

$= (l + 1)(m + 1)$

(vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

(vii) $5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2 = 10ab + 5b + 4a + 2$

$= 5b(2a + 1) + 2(2a + 1)$

$= (2a + 1)(5b + 2)$

(ix) $6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6$

$= 3x(2y - 3) - 2(2y - 3)$

$= (2y - 3)(3x - 2)$

Question 4:

Factorise

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Answer:

(i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2 - b^2)(a^2 + b^2)$

$= (a - b)(a + b)(a^2 + b^2)$

(ii) $p^4 - 81 = (p^2)^2 - (9)^2$

$= (p^2 - 9)(p^2 + 9)$

$= [(p)^2 - (3)^2](p^2 + 9)$

$= (p - 3)(p + 3)(p^2 + 9)$

(iii) $x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$

$= [x^2 - (y + z)^2][x^2 + (y + z)^2]$

$= [x - (y + z)][x + (y + z)][x^2 + (y + z)^2]$

$= (x - y - z)(x + y + z)[x^2 + (y + z)^2]$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - [(x - z)^2]^2$

$= [x^2 - (x - z)^2][x^2 + (x - z)^2]$

$= [x - (x - z)] [x + (x - z)] [x^2 + (x - z)^2]$

$= z(2x - z) [x^2 + x^2 - 2xz + z^2]$

$= z(2x - z) (2x^2 - 2xz + z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2 (a^2) (b^2) + (b^2)^2$

$= (a^2 - b^2)^2$

$= [(a - b) (a + b)]^2$

$= (a - b)^2 (a + b)^2$

Question 5:

Factorise the following expressions

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Answer:

(i) $p^2 + 6p + 8$

It can be observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$\therefore p^2 + 6p + 8 = p^2 + 2p + 4p + 8$

$= p(p + 2) + 4(p + 2)$

$= (p + 2) (p + 4)$

(ii) $q^2 - 10q + 21$

It can be observed that, $21 = (-7) \times (-3)$ and $(-7) + (-3) = -10$

$\therefore q^2 - 10q + 21 = q^2 - 7q - 3q + 21$

$= q(q - 7) - 3(q - 7)$

$= (q - 7) (q - 3)$

(iii) $p^2 + 6p - 16$

It can be observed that, $16 = (-2) \times 8$ and $8 + (-2) = 6$

$p^2 + 6p - 16 = p^2 + 8p - 2p - 16$

$= p(p + 8) - 2(p + 8)$

$= (p + 8) (p - 2)$

Exercise 14.3

Question 1:

Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Answer:

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii) $36y^3 = 2 \times 2 \times 3 \times 3 \times y \times y \times y$

$9y^2 = 3 \times 3 \times y \times y$

$$-36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y} = -4y$$

(iii) $66\,pq^2\,r^3 = 2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r$

$11qr^2 = 11 \times q \times r \times r$

$$66pq^2r^3 \div 11qr^2 = \frac{2\times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r} = 6pqr$$

(iv) $34\,x^3y^3z^3 = 2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z$

$51\,xy^2z^3 = 3 \times 17 \times x \times y \times y \times z \times z \times z$

$$34\,x^3y^3z^3 \div 51\,xy^2z^3 = \frac{2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times y\times y\times z\times z\times z}$$

$$= \frac{2}{3}x^2y$$

(v) $12a^8b^8 = 2 \times 2 \times 3 \times a^8 \times b^8$

$6a^6b^4 = 2 \times 3 \times a^6 \times b^4$

$$12a^8b^8 \div \left(-6a^6b^4\right) = \frac{2\times2\times3\times a^8\times b^8}{-2\times3\times a^6\times b^4} = -2a^2b^4$$

Question 2:

Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Answer:

(i) $5x^2 - 6x = x(5x - 6)$

$$\left(5x^2 - 6x\right) \div 3x = \frac{x\left(5x-6\right)}{3x} = \frac{1}{3}\left(5x-6\right)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$\left(3y^8 - 4y^6 + 5y^4\right) \div y^4 = \frac{y^4\left(3y^4 - 4y^2 + 5\right)}{y^4} = 3y^4 - 4y^2 + 5$$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8\left(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3\right) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2\left(x+y+z\right)}{4x^2y^2z^2} = 2\left(x+y+z\right)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$\left(x^3 + 2x^2 + 3x\right) \div 2x = \frac{x\left(x^2+2x+3\right)}{2x} = \frac{1}{2}\left(x^2+2x+3\right)$$

(v) $p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3\left(q^3-p^3\right)}{p^3q^3} = q^3 - p^3$$

Question 3:

Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Answer:

(i) $$(10x-25)\div 5 = \frac{2\times 5\times x-5\times 5}{5}=\frac{5(2x-5)}{5}=2x-5$$

(ii) $$(10x-25)\div(2x-5)=\frac{2\times 5\times x-5\times 5}{(2x-5)}=\frac{5(2x-5)}{2x-5}=5$$

(iii) $$10y(6y+21)\div 5(2y+7)=\frac{2\times 5\times y[2\times 3\times y+3\times 7]}{5(2y+7)}$$

$$=\frac{2\times 5\times y\times 3(2y+7)}{5(2y+7)}=6y$$

(iv) $$9x^2y^2(3z-24)\div 27xy(z-8)=\frac{9x^2y^2[3\times z-2\times 2\times 2\times 3]}{27xy(z-8)}$$

$$=\frac{xy\times 3(z-8)}{3(z-8)}=xy$$

(v) $96\,abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

$$=\frac{96abc(3\times a-3\times 4)(5\times b-2\times 3\times 5)}{144(a-4)(b-6)}$$

$$=\frac{2abc\times 3(a-4)\times 5(b-6)}{3(a-4)(b-6)}=10abc$$

Question 4:

Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

Answer:

(i) $5(2x+1)(3x+5) \div (2x+1) = \dfrac{5(2x+1)(3x+1)}{(2x+1)}$ $= 5(3x + 1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4) = \dfrac{2\times 13\times xy(x+5)(y-4)}{13x(y-4)}$ $= 2y(x + 5)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

$$=\frac{2\times 2\times 13\times p\times q\times r\times (p+q)\times (q+r)\times (r+p)}{2\times 2\times 2\times 13\times p\times q\times (q+r)\times (r+p)}$$

$$=\frac{1}{2}r(p+q)$$

(iv) $20(y + 4)(y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)(y^2 + 5y + 3)$

$$20(y+4)(y^2+5y+3)\div 5(y+4) = \frac{2\times 2\times 5\times (y+4)\times (y^2+5y+3)}{5\times (y+4)}$$

$$=4(y^2+5y+3)$$

(v) $x(x+1)(x+2)(x+3) \div x(x+1) = \dfrac{x(x+1)(x+2)(x+3)}{x(x+1)}$

$= (x + 2)(x + 3)$

Question 5:

Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Answer:

(i) $(y^2 + 7y + 10) = y^2 + 2y + 5y + 10$

$= y(y + 2) + 5(y + 2)$

$= (y + 2)(y + 5)$

$$\left(y^2+7y+10\right)\div\left(y+5\right)=\frac{\left(y+5\right)\left(y+2\right)}{\left(y+5\right)}=y+2$$

(ii) $m^2 - 14m - 32 = m^2 + 2m - 16m - 32$

$= m(m + 2) - 16(m + 2)$

$= (m + 2)(m - 16)$

$$\left(m^2-14m-32\right)\div\left(m+2\right)=\frac{\left(m+2\right)\left(m-16\right)}{\left(m+2\right)}=m-16$$

(iii) $5p^2 - 25p + 20 = 5(p^2 - 5p + 4)$

$= 5[p^2 - p - 4p + 4]$

$= 5[p(p - 1) - 4(p - 1)]$

$= 5(p - 1)(p - 4)$

$$\left(5p^2-25p+20\right)\div\left(p-1\right)=\frac{5\left(p-1\right)\left(p-4\right)}{\left(p-1\right)}=5\left(p-4\right)$$

(iv) $4yz(z^2 + 6z - 16) = 4yz[z^2 - 2z + 8z - 16]$

$= 4yz[z(z - 2) + 8(z - 2)]$

$= 4yz(z - 2)(z + 8)$

$$4yz\left(z^2+6z-16\right)\div 2y\left(z+8\right)=\frac{4yz\left(z-2\right)\left(z+8\right)}{2y\left(z+8\right)}=2z\left(z-2\right)$$

(v) $5pq(p^2 - q^2) = 5pq(p - q)(p + q)$

$$5pq\left(p^2-q^2\right)\div 2p\left(p+q\right)=\frac{5pq\left(p-q\right)\left(p+q\right)}{2p\left(p+q\right)}=\frac{5}{2}q\left(p-q\right)$$

(vi) $12xy(9x^2 - 16y^2) = 12xy[(3x)^2 - (4y)^2] = 12xy(3x - 4y)(3x + 4y)$

$$12xy\left(9x^2-16y^2\right)\div 4xy\left(3x+4y\right)=\frac{2\times 2\times 3\times x\times y\times\left(3x-4y\right)\times\left(3x+4y\right)}{2\times 2\times x\times y\times\left(3x+4y\right)}$$
$$=3\left(3x-4y\right)$$

(vii) $39y^3(50y^2 - 98) = 3 \times 13 \times y \times y \times y \times 2[(25y^2 - 49)]$

$= 3 \times 13 \times 2 \times y \times y \times y \times [(5y)^2 - (7)^2]$

$= 3 \times 13 \times 2 \times y \times y \times y\ (5y - 7)\ (5y + 7)$

$26y^2(5y + 7) = 2 \times 13 \times y \times y \times (5y + 7)$

$39y^3(50y^2 - 98) \div 26y^2\ (5y + 7)$

Question 3:

Find and correct the errors in the statement: $2x + 3y = 5xy$

Answer:

L.H.S $= 2x + 3y \neq$ R.H.S.

The correct statement is $2x + 3y = 2x + 3y$

Question 4:

Find and correct the errors in the statement: $x + 2x + 3x = 5x$

Answer:

L.H.S $= x + 2x + 3x = 1x + 2x + 3x = x(1 + 2 + 3) = 6x \neq$ R.H.S.

The correct statement is $x + 2x + 3x = 6x$

Question 5:

Find and correct the errors in the statement: $5y + 2y + y - 7y = 0$

Answer:

L.H.S. $= 5y + 2y + y - 7y = 8y - 7y = y \neq$ R.H.S

The correct statement is $5y + 2y + y - 7y = y$

Question 6:

Find and correct the errors in the statement: $3x + 2x = 5x^2$

Answer:

L.H.S. $= 3x + 2x = 5x \neq$ R.H.S

The correct statement is $3x + 2x = 5x$

Question 7:

Find and correct the errors in the statement: $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

Answer:

L.H.S = $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq$ R.H.S

The correct statement is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

Question 8:

Find and correct the errors in the statement: $(2x)^2 + 5x = 4x + 5x = 9x$

Answer:

L.H.S = $(2x)^2 + 5x = 4x^2 + 5x \neq$ R.H.S.

The correct statement is $(2x)^2 + 5x = 4x^2 + 5x$

Question 9:

Find and correct the errors in the statement: $(3x + 2)^2 = 3x^2 + 6x + 4$

Answer:

L.H.S. = $(3x + 2)^2 = (3x)^2 + 2(3x)(2) + (2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

$= 9x^2 + 12x + 4 \neq$ R.H.S

The correct statement is $(3x + 2)^2 = 9x^2 + 12x + 4$

Question 10:

Find and correct the errors in the following mathematical statement. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5\,(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5\,(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5\,(-3) = -9 - 15 = -24$

Answer:

(a) For $x = -3$,

$x^2 + 5x + 4 = (-3)^2 + 5\,(-3) + 4 = 9 - 15 + 4 = 13 - 15 = -2$

(b) For $x = -3$,

$x^2 - 5x + 4 = (-3)^2 - 5\,(-3) + 4 = 9 + 15 + 4 = 28$

(c) For $x = -3$,

$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6$

Question 11:

Find and correct the errors in the statement: $(y - 3)^2 = y^2 - 9$

Answer:

L.H.S = $(y-3)^2 = (y)^2 - 2(y)(3) + (3)^2$ $[(a-b)^2 = a^2 - 2ab + b^2]$

$= y^2 - 6y + 9 \neq$ R.H.S

The correct statement is $(y-3)^2 = y^2 - 6y + 9$

Question 12:

Find and correct the errors in the statement: $(z+5)^2 = z^2 + 25$

Answer:

L.H.S = $(z+5)^2 = (z)^2 + 2(z)(5) + (5)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= z^2 + 10z + 25 \neq$ R.H.S

The correct statement is $(z+5)^2 = z^2 + 10z + 25$

Question 13:

Find and correct the errors in the statement: $(2a+3b)(a-b) = 2a^2 - 3b^2$

Answer:

L.H.S. = $(2a+3b)(a-b) = 2a \times a + 3b \times a - 2a \times b - 3b \times b$

$= 2a^2 + 3ab - 2ab - 3b^2 = 2a^2 + ab - 3b^2 \neq$ R.H.S.

The correct statement is $(2a+3b)(a-b) = 2a^2 + ab - 3b^2$

Question 14:

Find and correct the errors in the statement: $(a+4)(a+2) = a^2 + 8$

Answer:

L.H.S. = $(a+4)(a+2) = (a)^2 + (4+2)(a) + 4 \times 2$

$= a^2 + 6a + 8 \neq$ R.H.S

The correct statement is $(a+4)(a+2) = a^2 + 6a + 8$

Question 15:

Find and correct the errors in the statement: $(a-4)(a-2) = a^2 - 8$

Answer:

L.H.S. = $(a-4)(a-2) = (a)^2 + [(-4) + (-2)](a) + (-4)(-2)$

$= a^2 - 6a + 8 \neq$ R.H.S.

The correct statement is $(a-4)(a-2) = a^2 - 6a + 8$

Question 16:

Find and correct the errors in the statement: $\frac{3x^2}{3x^2} = 0$

Answer:

L.H.S = $\frac{3x^2}{3x^2} = \frac{3 \times x \times x}{3 \times x \times x} = 1 \neq \text{R.H.S.}$

The correct statement is $\frac{3x^2}{3x^2} = 1$

Question 17:

Find and correct the errors in the statement: $\frac{3x^2+1}{3x^2} = 1 + 1 = 2$

Answer:

$$\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2} \neq \text{R.H.S.}$$

The correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$

Question 18:

Find and correct the errors in the statement: $\frac{3x}{3x+2} = \frac{1}{2}$

Answer:

L.H.S = $\frac{3x}{3x+2} \neq \text{R.H.S.}$

The correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

Question 19:

Find and correct the errors in the statement: $\frac{3}{4x+3} = \frac{1}{4x}$

Answer:

L.H.S. = $\frac{3}{4x+3}$ ≠ R.H.S.

The correct statement is $\frac{3}{4x+3}=\frac{3}{4x+3}$

Question 20:

Find and correct the errors in the statement: Answer. $\frac{4x+5}{4x}=5$

L.H.S. = $\frac{4x+5}{4x}=\frac{4x}{4x}+\frac{5}{4x}=1+\frac{5}{4x}\neq \text{R.H.S}$ The correct statement is $\frac{4x+5}{4x}=1+\frac{5}{4x}$

Question 21:

Find and correct the errors in the statement: Answer. $\frac{7x+5}{5}=7x$

L.H.S. = $\frac{7x+5}{5}=\frac{7x}{5}+\frac{5}{5}=\frac{7x}{5}+1\neq \text{R.H.S.}$

The correct statement is $\frac{7x+5}{5}=\frac{7x}{5}+1$

DELHI PUBLIC SCHOOL, GANDHINAGAR
MIND MAP
CH- 14 FACTORISATION

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of four different topics. The most probable questions from examination point of view are given below.

TYPE:1 Factorisation by the method of common factors and regrouping the terms

Factorise the following:

(i) $10pqr, 20pr, 30pq$ (ii) $6abc, 24ab^{2,}12ba^2$
(iii) $x^2yz + xy^2z + xyz^2$ (iv) $5x^2y - 15xy^2$
(v) $ax + bx - ay - by$ (vi) $6xy - 4y + 6 - 9x$

TYPE-II: Factorisation using identities

Factorise the following expressions:

(i) $a^4 + 2a^2b^2 + b^4$ (v) $36a^2 - 36a + 9$
(ii) $a^4 - 16b^4$ (vi) $(l + m)^2 - (l - m)^2$
(iii) $10ab + 4a + 5b + 2$ (vii) $6xy - 4y + 6 - 9x$
(iv) $x^4 - (y + z)^4$ (viii) $q^2 - 10q + 21$

TYPE III: Division in algebraic expressions

Carry out the following divisions:

(i) $34x^3y^3z^3 \div 51xy^2z^3$
(ii) $(x^3 + 2x^2 + 3x) \div 2x$
(iii) $(10x - 25) \div (2x - 5)$
(iv) $10y(6y + 21) \div 5(2y + 7)$
(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$
(vi) $5pq(p^2 - q^2) \div 2p(p + q)$
(vii) $z(5z^2 - 80) \div 5z(z + 4)$
(viii) $(x^3 - 6x^2 + 11x - 6) \div (x^2 - 4x + 3)$

TYPE IV: Find and correct the errors in following statements

(i) $(a - 4)(a - 2) = a^2 + 8$
(ii) $(y - 3)^2 = y^2 - 9$
(iii) $\frac{3x}{3x+2} = \frac{1}{2}$
(iv) $4(x - 5) = 4x - 5$

Ch-16 Playing with Numbers

Exercise - 16.1

Question 1:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \\ \hline \end{array}$$

ANSWER:

The addition of A and 5 is giving 2 i.e., a number whose ones digit is 2. This is possible only when digit A is 7. In that case, the addition of A (7) and 5 will give 12 and thus, 1 will be the carry for the next step. In the next step,

1 + 3 + 2 = 6

Therefore, the addition is as follows.

$$\begin{array}{r} 3\ 7 \\ +\ 2\ 5 \\ \hline 6\ 2 \\ \hline \end{array}$$

Clearly, B is 6.

Hence, A and B are 7 and 6 respectively.

Question 2:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} 4\ A \\ +\ 9\ 8 \\ \hline C\ B\ 3 \\ \hline \end{array}$$

ANSWER:

The addition of A and 8 is giving 3 i.e., a number whose ones digit is 3. This is possible only when digit A is 5. In that case, the addition of A and 8 will give 13 and thus, 1 will be the carry for the next step. In the next step,

1 + 4 + 9 = 14

Therefore, the addition is as follows.

$$\begin{array}{r} 45 \\ +\ 98 \\ \hline 143 \\ \hline \end{array}$$

Clearly, B and C are 4 and 1 respectively.

Hence, A, B, and C are 5, 4, and 1 respectively.

Question 3:

Find the value of the letter in the following and give reasons for the steps involved.

$$\begin{array}{r} 1A \\ \times\ A \\ \hline 9A \\ \hline \end{array}$$

ANSWER:

The multiplication of A with A itself gives a number whose ones digit is A again. This happens only when A = 1, 5, or 6.

If A = 1, then the multiplication will be 11 × 1 = 11. However, here the tens digit is given as 9. Therefore, A = 1 is not possible. Similarly, if A = 5, then the multiplication will be 15 × 5 = 75. Thus, A = 5 is also not possible.

If we take A = 6, then 16 × 6 = 96. Therefore, A should be 6.

The multiplication is as follows.

$$\begin{array}{r} 16 \\ \times\ 6 \\ \hline 96 \\ \hline \end{array}$$

Hence, the value of A is 6.

Question 4:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} AB \\ +\ 37 \\ \hline 6A \\ \hline \end{array}$$

ANSWER:

The addition of A and 3 is giving 6. There can be two cases.

(1) First step is not producing a carry

In that case, A comes to be 3 as 3 + 3 = 6. Considering the first step in which the addition of B and 7 is giving A (i.e., 3), B should be a number such that the units digit of this addition comes to be 3. It is possible only when B = 6. In this case, A = 6 + 7 = 13. However, A is a single digit number. Hence, it is not possible.

(2) First step is producing a carry

In that case, A comes to be 2 as 1 + 2 + 3 = 6. Considering the first step in which the addition of B and 7 is giving A (i.e., 2), B should be a number such that the units digit of this addition comes to be 2. It is possible only when B = 5 and 5 + 7 = 12.

$$\begin{array}{rcc} & 2 & 5 \\ + & 3 & 7 \\ \hline & 6 & 2 \\ \hline \end{array}$$

Hence, the values of A and B are 2 and 5 respectively.

Question 5:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{ccc} & A & B \\ & \times & 3 \\ \hline C & A & B \\ \hline \end{array}$$

ANSWER:

The multiplication of 3 and B gives a number whose ones digit is B again.

Hence, B must be 0 or 5.

Let B is 5.

Multiplication of first step = 3 × 5 = 15

1 will be a carry for the next step.

We have, 3 × A + 1 = CA

This is not possible for any value of A.

Hence, B must be 0 only. If B = 0, then there will be no carry for the next step.

We should obtain, 3 × A = CA

That is, the one's digit of 3 × A should be A. This is possible when A = 5 or 0.

However, A cannot be 0 as AB is a two-digit number.

Therefore, A must be 5 only. The multiplication is as follows.

$$\begin{array}{r} 50 \\ \times \quad 3 \\ \hline 150 \\ \hline \end{array}$$

Hence, the values of A, B, and C are 5, 0, and 1 respectively.

Question 6:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} A\ B \\ \times \quad 5 \\ \hline C\ A\ B \\ \hline \end{array}$$

ANSWER:

The multiplication of B and 5 is giving a number whose ones digit is B again. This is possible when B = 5 or B = 0 only.

In case of B = 5, the product, B × 5 = 5 × 5 = 25

2 will be a carry for the next step.

We have, 5 × A + 2 = CA, which is possible for A = 2 or 7

The multiplication is as follows.

$$\begin{array}{r} 25 \\ \times\ 5 \\ \hline 125 \\ \hline \end{array} \quad \begin{array}{r} 75 \\ \times\ 5 \\ \hline 375 \\ \hline \end{array}$$

If B = 0,

B × 5 = B ⇒ 0 × 5 = 0

There will not be any carry in this step.

In the next step, 5 × A = CA

It can happen only when A = 5 or A = 0

However, A cannot be 0 as AB is a two-digit number.

Hence, A can be 5 only. The multiplication is as follows.

$$\begin{array}{r} 50 \\ \times\ 5 \\ \hline 250 \\ \hline \end{array}$$

Hence, there are 3 possible values of A, B, and C.

(i) 5, 0, and 2 respectively

(ii) 2, 5, and 1 respectively

(iii) 7, 5, and 3 respectively

Question 7:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} A\ B \\ \times\ 6 \\ \hline B\ B\ B \\ \hline \end{array}$$

ANSWER:

The multiplication of 6 and B gives a number whose one's digit is B again.

It is possible only when B = 0, 2, 4, 6, or 8

If B = 0, then the product will be 0. Therefore, this value of B is not possible.

If B = 2, then B × 6 = 12 and 1 will be a carry for the next step.

6A + 1 = BB = 22 ⇒ 6A = 21 and hence, any integer value of A is not possible.

If B = 6, then B × 6 = 36 and 3 will be a carry for the next step.

6A + 3 = BB = 66 ⇒ 6A = 63 and hence, any integer value of A is not possible.

If B = 8, then B × 6 = 48 and 4 will be a carry for the next step.

6A + 4 = BB = 88 ⇒ 6A = 84 and hence, A = 14. However, A is a single digit number. Therefore, this value of A is not possible.

If B = 4, then B × 6 = 24 and 2 will be a carry for the next step.

6A + 2 = BB = 44 ⇒ 6A = 42 and hence, A = 7

The multiplication is as follows.

$$\begin{array}{r} 74 \\ \times\ \ 6 \\ \hline 444 \\ \hline \end{array}$$

Hence, the values of A and B are 7 and 4 respectively.

Question 8:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} A\ 1 \\ +\ 1\ B \\ \hline B\ 0 \\ \hline \end{array}$$

ANSWER:

The addition of 1 and B is giving 0 i.e., a number whose ones digits is 0. This is possible only when digit B is 9. In that case, the addition of 1 and B will give 10 and thus, 1 will be the carry for the next step. In the next step,

1 + A + 1 = B

Clearly, A is 7 as 1 + 7 + 1 = 9 = B

Therefore, the addition is as follows.

$$\begin{array}{r} 7\ \ 1 \\ +\ 1\ \ 9 \\ \hline 9\ \ 0 \\ \hline \end{array}$$

Hence, the values of A and B are 7 and 9 respectively.

Question 9:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} 2\ A\ B \\ +\ A\ B\ 1 \\ \hline B\ 1\ 8 \\ \hline \end{array}$$

ANSWER:

The addition of B and 1 is giving 8 i.e., a number whose ones digits is 8. This is possible only when digit B is 7. In that case, the addition of B and 1 will give 8. In the next step,

A + B = 1

Clearly, A is 4.

4 + 7 = 11 and 1 will be a carry for the next step. In the next step,

1 + 2 + A = B

1 + 2 + 4 = 7

Therefore, the addition is as follows.

$$\begin{array}{r} 2\,4\,7 \\ +\ 4\,7\,1 \\ \hline 7\,1\,8 \\ \hline \end{array}$$

Hence, the values of A and B are 4 and 7 respectively.

Question 10:

Find the values of the letters in the following and give reasons for the steps involved.

$$\begin{array}{r} 1\,2\,A \\ +6\,A\,B \\ \hline A\,0\,9 \\ \hline \end{array}$$

ANSWER:

The addition of A and B is giving 9 i.e., a number whose ones digits is 9. The sum can be 9 only as the sum of two single digit numbers cannot be 19. Therefore, there will not be any carry in this step.

In the next step, 2 + A = 0

It is possible only when A = 8

2 + 8 = 10 and 1 will be the carry for the next step.

1 + 1 + 6 = A

Clearly, A is 8. We know that the addition of A and B is giving 9. As A is 8, therefore, B is 1.

Therefore, the addition is as follows.

$$\begin{array}{r} 128 \\ +\ 681 \\ \hline 809 \\ \hline \end{array}$$

Hence, the values of A and B are 8 and 1 respectively.

Exercise - 16.2

Question 1:

If 21*y*5 is a multiple of 9, where *y* is a digit, what is the value of *y*?

ANSWER:

If a number is a multiple of 9, then the sum of its digits will be divisible by 9.

Sum of digits of 21*y*5 = 2 + 1 + *y* + 5 = 8 + *y*

Hence, 8 + *y* should be a multiple of 9.

This is possible when 8 + *y* is any one of these numbers 0, 9, 18, 27, and so on …

However, since *y* is a single digit number, this sum can be 9 only. Therefore, *y* should be 1 only.

Question 2:

If 31*z*5 is a multiple of 9, where *z* is a digit, what is the value of *z*?

You will find that there are two answers for the last problem. Why is this so?

ANSWER:

If a number is a multiple of 9, then the sum of its digits will be divisible by 9.

Sum of digits of 31*z*5 = 3 + 1 + *z* + 5 = 9 + *z*

Hence, 9 + *z* should be a multiple of 9.

This is possible when 9 + *z* is any one of these numbers 0, 9, 18, 27, and so on …

However, since *z* is a single digit number, this sum can be either 9 or 18.
Therefore, *z* should be either 0 or 9.

Question 3:

If 24*x* is a multiple of 3, where *x* is a digit, what is the value of *x*?

(Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers: 0, 3, 6, 9, 12, 15, 18…. But since x is a digit, it can only be that 6 + x = 6 or 9 or 12 or 15. Therefore, x = 0 or 3 or 6 or 9. Thus, x can have any of four different values)

ANSWER:

Since 24x is a multiple of 3, the sum of its digits is a multiple of 3.

Sum of digits of 24x = 2 + 4 + x = 6 + x

Hence, 6 + x is a multiple of 3.

This is possible when 6 + x is any one of these numbers 0, 3, 6, 9, and so on …

Since x is a single digit number, the sum of the digits can be 6 or 9 or 12 or 15 and thus, the value of x comes to 0 or 3 or 6 or 9 respectively.

Thus, x can have its value as any of the four different values 0, 3, 6, or 9.

Question 4:

If 31z5 is a multiple of 3, where z is a digit, what might be the values of z?

ANSWER:

Since 31z5 is a multiple of 3, the sum of its digits will be a multiple of 3.

That is, 3 + 1 + z + 5 = 9 + z is a multiple of 3.

This is possible when 9 + z is any one of 0, 3, 6, 9, 12, 15, 18, and so on …

Since z is a single digit number, the value of 9 + z can only be 9 or 12 or 15 or 18 and thus, the value of x comes to 0 or 3 or 6 or 9 respectively.

Thus, z can have its value as any one of the four different values 0, 3, 6, or 9.

Exercise 13.1

Question 1:

Following are the car parking charges near a railway station up to

4 hours Rs 60

8 hours Rs 100

12 hours Rs 140

24 hours Rs 180

Check if the parking charges are in direct proportion to the parking time.

Answer:

A table of the given information is formed as

Number of hours	4	8	12	24
Parking charges (in Rs)	60	100	140	180

The ratio of parking charges to the respective number of hours (Rs/ hour) can be calculated as

$$\frac{60}{4}=15,\quad \frac{100}{8}=\frac{25}{2},\ \frac{140}{12}=\frac{35}{3},\ \frac{180}{24}=\frac{15}{2}$$

As each ratio is not same, therefore, the parking charges are not in a direct proportion to the parking time.

Question 2:

A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
parts of base	8	…	…	…	…

Answer:

The given mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. For more parts of red pigments, the parts of the base will also be more.

Therefore, the parts of red pigments and the parts of base are in direct proportion.

The given information in the form of a table is as follows.

Parts of red pigment	1	4	7	12	20
Parts of base	8	x_1	x_2	x_3	x_4

According to direct proportion,

$$\frac{x_1}{4} = \frac{8}{1} \Rightarrow x_1 = 4 \times 8 = 32$$

$$\frac{x_2}{7} = \frac{8}{1} \Rightarrow x_2 = 7 \times 8 = 56$$

$$\frac{x_3}{12} = \frac{8}{1} \quad \Rightarrow x_3 = 8 \times 12 = 96$$

$$\frac{x_4}{20} = \frac{8}{1} \quad \Rightarrow x_4 = 8 \times 20 = 160$$

The table can be drawn as follows.

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

Question 3:

In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Answer:

Let the parts of red pigment required to mix with 1800 mL of base be x.

The given information in the form of a table is as follows.

Parts of red pigment	1	x
Parts of base (in mL)	75	1800

The parts of red pigment and the parts of base are in direct proportion.

Therefore, we obtain

$$\frac{1}{75}=\frac{x}{1800}$$
$$\Rightarrow x=\frac{1\times 1800}{75}$$
$$\Rightarrow x=24$$

Thus, 24 parts of red pigments should be mixed with 1800 mL of base.

Question 4:

A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Answer:

Let the number of bottles filled by the machine in five hours be *x*.

The given information in the form of a table is as follows.

Number of bottles	840	*x*
Time taken (in hours)	6	5

The number of bottles and the time taken to fill these bottles are in direct proportion.

Therefore, we obtain

$$\frac{840}{6}=\frac{x}{5}$$
$$x=\frac{840\times 5}{6}=700$$

Thus, 700 bottles will be filled in 5 hours.

Question 5:

A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Answer:

Let the actual length of bacteria be *x* cm and the enlarged length of bacteria be *y* cm, if the photograph is enlarged for 20,000 times.

The given information in the form of a table is as follows.

Length of bacteria (in cm)	5	*x*	*y*
Number of times photograph of Bacteria was enlarged	50000	1	20000

The number of times the photograph of bacteria was enlarged and the length of bacteria are in direct proportion.

Therefore, we obtain

$$\frac{5}{50,000}=\frac{x}{1}$$

$$\Rightarrow x=\frac{1}{10000}=10^{-4}$$

Hence, the actual length of bacteria is 10^{-4} cm.

Let the length of bacteria when the photograph of bacteria is enlarged 20, 000 times be *y*.

$$\frac{5}{50,000}=\frac{y}{20,000}$$

$$y=\frac{20,000\times 5}{50,000}=2$$

Hence, the enlarged length of bacteria is 2 cm.

Question 6:

In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

Answer:

Let the length of the mast of the model ship be *x* cm.

The given information in the form of a table is as follows:

-	Height of mast	Length of ship
Model ship	9 cm	*x*
Actual ship	12 m	28 m

We know that the dimensions of the actual ship and the model ship are directly proportional to each other.

Therefore, we obtain:

$$\frac{12}{9} = \frac{28}{x}$$

$$x = \frac{28 \times 9}{12} = 21$$

Thus, the length of the model ship is 21 cm.

Question 7:

Suppose 2 kg of sugar contains 9×10^6 crystals.

How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

Answer:

(i) Let the number of sugar crystals in 5 kg of sugar be *x*.

The given information in the form of a table is as follows.

Amount of sugar (in kg)	2	5
Number of crystals	9×10^6	x

The amount of sugar and the number of crystals it contains are directly proportional to each other. Therefore, we obtain

$$\frac{2}{9 \times 10^6} = \frac{5}{x}$$

$$x = \frac{5 \times 9 \times 10^6}{2} = 2.25 \times 10^7$$

Hence, the number of sugar crystals is 2.25×10^7.

(ii) Let the number of sugar crystals in 1.2 kg of sugar be *y*. The given information in the form of a table is as follows.

Amount of sugar (in kg)	2	1.2
Number of crystals	9×10^6	y

$$\frac{2}{9\times10^6}=\frac{1.2}{y}$$

$$y=\frac{1.2\times9\times10^6}{2}=5.4\times10^6$$

Hence, the number of sugar crystals is 5.4×10^6.

Question 8:

Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Answer:

Let the distance represented on the map be *x* cm.

The given information in the form of a table is as follows.

Distance covered on road in (in km)	18	72
Distance represented on map (in cm)	1	*x*

The distances covered on road and represented on map are directly proportional to each other. Therefore, we obtain

$$\frac{18}{1}=\frac{72}{x}$$

$$\Rightarrow x=\frac{72}{18}=4$$

Hence, the distance represented on the map is 4 cm.

Question 9:

A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time –

(i) the length of the shadow cast by another pole 10 m 50 cm high

(ii) the height of a pole which casts a shadow 5 m long.

Answer:

(i) Let the length of the shadow of the other pole be *x* m.

1 m = 100 cm

The given information in the form of a table is as follows.

Height of pole (in m)	5.60	10.50
Length of shadow (in m)	3.20	x

More the height of an object, more will be the length of its shadow.

Thus, the height of an object and length of its shadow are directly proportional to each other. Therefore, we obtain

$$\frac{5.60}{3.20} = \frac{10.50}{x}$$

$$\Rightarrow x = \frac{10.50 \times 3.20}{5.60} = 6$$

Hence, the length of the shadow will be 6 m.

(ii) Let the height of the pole be y m.

The given information in the form of a table is as follows.

Height of pole (in m)	5.60	y
Length of shadow (in m)	3.20	5

The height of the pole and the length of the shadow are directly proportional to each other. Therefore,

$$\frac{5.60}{3.20} = \frac{y}{5}$$

$$y = \frac{5 \times 5.60}{3.20} = 8.75$$

Thus, the height of the pole is 8.75 m or 8 m 75 cm.

Question 10:

A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Answer:

Let the distance travelled by the truck in 5 hours be x km.

We know, 1 hour = 60 minutes

$\therefore$ 5 hours = (5 × 60) minutes = 300 minutes

The given information in the form of a table is as follows.

Distance travelled (in km)	14	x
Time (in min)	25	300

The distance travelled by the truck and the time taken by the truck are directly proportional to each other. Therefore,

$$\frac{14}{25} = \frac{x}{300}$$

$$x = \frac{14 \times 300}{25} = 168$$

Hence, the distance travelled by the truck is 168 km.

Exercise 13.2

Question 1:

Which of the following are in inverse proportion?

(i) The number of workers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

(iii) Area of cultivated land and the crop harvested.

(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) The population of a country and the area of land per person.

Answer:

(i) These are in inverse proportion because if there are more workers, then it will take lesser time to complete that job.

(ii) No, these are not in inverse proportion because in more time, we may cover more distance with a uniform speed.

(iii) No, these are not in inverse proportion because in more area, more quantity of crop may be harvested.

(iv) These are in inverse proportion because with more speed, we may complete a certain distance in a lesser time.

(v) These are in inverse proportion because if the population is increasing, then the area of the land per person will be decreasing accordingly.

Question 2:

In a Television game show, the prize money of Rs 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	100000	50000	…	…	…	…	…

Answer:

A table of the given information is as follows.

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	100000	50000	x_1	x_2	x_3	x_4	x_5

From the table, we obtain

1 × 100000 = 2 × 50000 = 100000

Thus, the number of winners and the amount given to each winner are inversely proportional to each other. Therefore,

$1 \times 100000 = 4 \times x_1$

$$x_1 = \frac{100000}{4} = 25000$$

$1 \times 100000 = 5 \times x_2$

$$x_2 = \frac{100000}{5} = 20000$$

$1 \times 100000 = 8 \times x_3$

$$x_3 = \frac{100000}{8} = 12500$$

$1 \times 100000 = 10 \times x_4$

$$x_4 = \frac{100000}{10} = 10000$$

$1 \times 100000 = 20 \times x_5$

$$x_5 = \frac{100000}{20} = 5000$$

Question 3:

Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	…	…	…

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

Answer:

A table of the given information is as follows.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	x_1	x_2	x_3

From the given table, we obtain

4 × 90° = 360° = 6 × 60°

Thus, the number of spokes and the angle between a pair of consecutive spokes are inversely proportional to each other. Therefore,

4 × 90° = x_1 × 8

$$x_1 = \frac{4 \times 90^\circ}{8} = 45^\circ$$

Similarly, $x_2 = \frac{4 \times 90^\circ}{10} = 36^\circ$ and $x_3 = \frac{4 \times 90^\circ}{12} = 30^\circ$

Thus, the following table is obtained.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(i) Yes, the number of spokes and the angles formed between the pairs of consecutive spokes are in inverse proportion.

(ii)Let the angle between a pair of consecutive spokes on a wheel with 15 spokes be *x*. Therefore,

$4 \times 90° = 15 \times x$

$$x = \frac{4 \times 90°}{15} = 24°$$

Hence, the angle between a pair of consecutive spokes of a wheel, which has 15 spokes in it, is 24°.

(iii) Let the number of spokes in a wheel, which has 40º angles between a pair of consecutive spokes, be *y*.

Therefore,

$4 \times 90° = y \times 40°$

$$y = \frac{4 \times 90}{40} = 9$$

Hence, the number of spokes in such a wheel is 9.

Question 4:

If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Answer:

Number of remaining children = 24 − 4 = 20

Let the number of sweets which each of the 20 students will get, be *x*.

The following table is obtained.

Number of students	24	20

Number of sweets	5	x

If the number of students is lesser, then each student will get more number of sweets.

Since this is a case of inverse proportion,

$24 \times 5 = 20 \times x$

$$x = \frac{24 \times 5}{20} = 6$$

Hence, each student will get 6 sweets.

Question 5:

A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Answer:

Let the number of days that the food will last if there were 10 more animals in the cattle be x. The following table is obtained.

Number of animals	20	20 + 10 = 30
Number of days	6	x

More the number of animals, lesser will be the number of days for which the food will last.

Hence, the number of days the food will last and the number of animals are inversely proportional to each other.

Therefore,

$20 \times 6 = 30 \times x$

$$x = \frac{20 \times 6}{30} = 4$$

Thus, the food will last for 4 days.

Question 6:

A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Answer:

Let the number of days required by 4 persons to complete the job be x.

The following table is obtained.

Number of days	4	x
Number of persons	3	4

If the number of persons is more, then it will take lesser time to complete the job.

Hence, the number of days and the number of persons required to complete the job are inversely proportional to each other.

Therefore,

$4 \times 3 = x \times 4$

$$x = \frac{4 \times 3}{4} = 3$$

Thus, the number of days required to complete the job is 3.

Question 7:

A batch of bottles was packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Answer:

Let the number of boxes filled, by using 20 bottles in each box, be x.

The following table is obtained.

Number of bottles	12	20
Number of boxes	25	x

More the number of bottles, lesser will be the number of boxes.

Hence, the number of bottles and the number of boxes required to pack these are inversely proportional to each other.

Therefore,

$12 \times 25 = 20 \times x$

$$x = \frac{12 \times 25}{20} = 15$$

Hence, the number of boxes required to pack these bottles is 15.

Question 8:

A factory required 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Answer:

Let the number of machines required to produce articles in 54 days be *x*. The following table is obtained.

Number of machines	42	x
Number of days	63	54

More the number of machines, lesser will be the number of days that it will take to produce the given number of articles. Thus, this is a case of inverse proportion.

Therefore,

$42 \times 63 = 54 \times x$

$$x = \frac{42 \times 63}{54} = 49$$

Hence, the required number of machines to produce the given number of articles in 54 days is 49.

Question 9:

A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. how long will it take when the car travels at the speed of 80 km/h?

Answer:

Let the time taken by the car to reach the destination, while travelling with a speed of 80 km/hr, be *x* hours.

The following table is obtained.

Speed (in km/hr)	60	80

Time taken (in hours)	2	x

More the speed of the car, lesser will be the time taken by it to reach the destination. Hence, the speed of the car and the time taken by the car are inversely proportional to each other. Therefore,

$60 \times 2 = 80 \times x$

$$x = \frac{60 \times 2}{80} = \frac{3}{2} = 1\frac{1}{2}$$

The time required by the car to reach the given destination is $1\frac{1}{2}$ hours.

Question 10:

Two persons could fit new windows in house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

Answer:

(i) Let the number of days required by 1 man to fit all the windows be x. The following table is obtained.

Number of persons	2	1
Number of days	3	x

Lesser the number of persons, more will be the number of days required to fit all the windows. Hence, this is a case of inverse proportion. Therefore,

$2 \times 3 = 1 \times x$

$x = 6$

Hence, the number of days taken by 1 man to fit all the windows is 6.

(ii) Let the number of persons required to fit all the windows in one day be y. The following table is formed.

Number of persons	2	Y

Number of days	3	1

Lesser the number of days, more will be the number of persons required to fit all the windows. Hence, this is a case of inverse proportion. Therefore,

$2 \times 3 = y \times 1$

$y = 6$

Hence, 6 persons are required to fit all the windows in one day.

Question 11:

A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Answer:

Let the duration of each period, when there are 9 periods a day in the school, be x minutes. The following table is obtained.

Duration of each period (in minutes)	45	x
Number of periods	8	9

If there is more number of periods a day in the school, then the duration of each period will be lesser. Hence, this is a case of inverse proportion. Therefore

$45 \times 8 = x \times 9$

$$x = \frac{45 \times 8}{9} = 40$$

Hence, in this case, the duration of each period will be 40 minutes.

<u>**DELHI PUBLIC SCHOOL, GANDHINAGAR**</u>

<u>**MIND MAP**</u>

<u>**CH.: 13 DIRECT AND INVERSE PROPORTIONS**</u>

<u>**SUBJECT: MATHEMATICS**</u> <u>**CLASS: VIII**</u>

This chapter consists of two topics. The most probable questions from examination point of view are given below.

<u>TYPE 1:PROBLEMS INVOLVING DIRECT PROPORTIONS</u>

- The cost of 5m of a particular quality of cloth is ₹210. Tabulate the cost of 2, 4, 10 and13m of the same cloth.
- The scale of a map is given as 1:30000000. Two cities are 4cm apart on the map. Find the actual distance between them.
- In the model of a ship, the mast is 9cm high, while the mast of actual ship is 12m high. If the length of the ship is 28m, how long is the model of the ship?
- A worker is paid ₹200 for 8 days work. If his total income of the month is ₹875, for many days did he work?
- Complete the following table given that "x" varies directly as "y"

x	4	9	----	----	3
y	16	----	48	36	-----

<u>TYPE 2: PROBLEMS INVOLVING INDIRECT PROPORTIONS</u>

- A work force of 420 men with a contractor can finish a piece of work in 9 months. How many extra men must he employ to complete the job in 7 months?
- In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls have joined the hostel, how long will these provisions last?
- 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if 5 pipes of the same type are used?
- Two persons could fit a window in a house in 3 days.
  (a) One of the persons fell ill before the work started. How long would the job take now?
  (b) How many persons would be needed to fit the window in one day?
- Complete the following table given that "x" varies inversely as "y"

x	12	16	----	8	----
y	----	6	4	----	0.25

Exercise 12.1

Question 1:

Evaluate

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Answer:

(i) $3^{-2}=\frac{1}{3^2}=\frac{1}{9}$ $\left(a^{-m}=\frac{1}{a^m}\right)$

(ii) $(-4)^{-2}=\frac{1}{(-4)^2}=\frac{1}{16}$ $\left(a^{-m}=\frac{1}{a^m}\right)$

(iii) $\left(\frac{1}{2}\right)^{-5}=\frac{1}{(2)^{-5}}=(2)^5=2\times2\times2\times2\times2=32$

Question 2:

Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5\div(-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4\times\left(\frac{5}{3}\right)^4$ (iv) $\left(3^{-7}\div3^{-10}\right)\times3^{-5}$

(v) $2^{-3}\times(-7)^{-3}$

Answer:

(i) $(-4)^5\div(-4)^8=(-4)^{5-8}$ $(a^m\div a^n=a^{m-n})$

$=(-4)^{-3}$

$=\frac{1}{(-4)^3}$ $\left(a^{-m}=\frac{1}{a^m}\right)$

(ii) $\left(\frac{1}{2^3}\right)^2 = \frac{1}{\left(2^3\right)^2} = \frac{1}{2^6}$ $\left(\left(a^m\right)^n = a^{mn}\right)$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1\times 3)^4 \times \frac{5^4}{3^4}$

$= (-1)^4 \times 3^4 \times \frac{5^4}{3^4}$ $\left[(ab)^m = a^m \times b^m\right]$

$= (-1)^4 \times 5^4$

$= 5^4$ $\left[(-1)^4 = 1\right]$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = (3^{-7-(-10)}) \times 3^{-5}$ $(a^m \div a^n = a^{m-n})$

$= 3^3 \times 3^{-5}$

$= 3^{3+(-5)}$ $(a^m \times a^n = a^{m+n})$

$= 3^{-2}$

$= \frac{1}{3^2}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

(v) $2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{\left[2\times(-7)\right]^3}$ $\left[a^m \times b^m = (ab)^m\right]$

$= \frac{1}{(-14)^3}$

Question 3:

Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$ (iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^{2}$

Answer:

(i) $(3^{0}+4^{-1})\times 2^{2}=\left(1+\frac{1}{4}\right)\times 2^{2}$ $\left(a^{0}=1 \text{ and } a^{-m}=\frac{1}{a^{m}}\right)$

$=\frac{5}{4}\times 4=5$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2}=[2^{-1}\times\{(2)^{2}\}^{-1}]\div 2^{-2}$

$=(2^{-1}\times 2^{-2})\div 2^{-2}$ $\left((a^{m})^{n}=a^{mn}\right)$

$=2^{-1+(-2)}\div 2^{-2}$ $(a^{m}\times a^{n}=a^{m+n})$

$=2^{-3}\div 2^{-2}$

$=2^{-3-(-2)}$ $(a^{m}\div a^{n}=a^{m-n})$

$=2^{-3+2}=2^{-1}$

$=\frac{1}{2}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=\left(\frac{2}{1}\right)^{2}+\left(\frac{3}{1}\right)^{2}+\left(\frac{4}{1}\right)^{2}$ $\left(\therefore a^{-m}=\frac{1}{a^{m}}\right)$

$=2^{2}+3^{2}+4^{2}=4+9+16=29$

(iv) $(3^{-1}+4^{-1}+5^{-1})^{0}$ $=\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)^{0}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$=1$ $(a^{0}=1)$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^{2}=\left\{\left(\frac{3}{-2}\right)^{2}\right\}^{2}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$$=\left\{\frac{3^2}{(-2)^2}\right\}^2 \qquad \left[\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\left(\frac{9}{4}\right)^2=\frac{81}{16}$$

Question 4:

Evaluate (i) $\frac{8^{-1}\times 5^3}{2^{-4}}$ (ii) $\left(5^{-1}\times 2^{-1}\right)\times 6^{-1}$

Answer:

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}=\frac{2^4\times 5^3}{8^1}$ $\qquad \left(a^{-m}=\frac{1}{a^m}\right)$

$$=\frac{2^4\times 5^3}{2^3}=2^{4-3}\times 5^3 \qquad \left(a^m\div a^n=a^{m-n}\right)$$

$$=2\times 125=250$$

(ii) $(5^{-1}\times 2^{-1})\times 6^{-1}=\left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6}$ $\qquad \left(a^{-m}=\frac{1}{a^m}\right)$

$$=\frac{1}{10}\times\frac{1}{6}=\frac{1}{60}$$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer:

$5^m \div 5^{-3} = 5^5$

$5^{m-(-3)} = 5^5$ $(a^m \div a^n = a^{m-n})$

$5^{m+3} = 5^5$

Since the powers have same bases on both sides, their respective exponents must be equal.

$m + 3 = 5$

$m = 5 - 3$

$m = 2$

Question 6:

Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Answer:

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}^{-1}$ $\left(a^{-m}=\frac{1}{a^m}\right)$

$=\{3-4\}^{-1}=(-1)^{-1}=\frac{1}{-1}=-1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$ $\left[\left(\frac{a}{b}\right)^{m}=\frac{a^m}{b^m}\right]$

$=\frac{8^7}{5^7}\times\frac{5^4}{8^4}$ $\left(a^{-m}=\frac{1}{a^m}\right)$

$=\frac{8^{7-4}}{5^{7-4}}$ $\left(a^m \div a^n = a^{m-n}\right)$

$=\frac{8^3}{5^3}=\frac{512}{125}$

Question 7:

Simplify. (i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}\,(t\neq 0)$ (ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Answer:

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}=\frac{5^2\times t^{-4}}{5^{-3}\times 5\times 2\times t^{-8}}$

$$= \frac{5^2 \times t^{-4}}{5^{-3+1} \times 2 \times t^{-8}} \qquad (a^m \times a^n = a^{m+n})$$

$$= \frac{5^2 \times t^{-4}}{5^{-2} \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-2)} t^{-4-(-8)}}{2} \qquad (a^m \div a^n = a^{m-n})$$

$$= \frac{5^4 t^4}{2} = \frac{625\, t^4}{2}$$

(ii) $$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \qquad \left[(a \times b)^m = a^m \times b^m\right]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)} \qquad (a^m \div a^n = a^{m-n})$$

$$= 3^0 \times 2^0 \times 5^5 \qquad (a^0 = 1)$$

$$= 5^5$$

Exercise 12.2

Question 1:

Express the following numbers in standard form.

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

Answer:

(i) 0.0000000000085 = 8.5×10^{-12}

(ii) 0.00000000000942 = 9.42×10^{-12}

(iii) 6020000000000000 = 6.02×10^{15}

(iv) 0.00000000837 = 8.37×10^{-9}

(v) 31860000000 = 3.186×10^{10}

Question 2:

Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^{4}

(iii) 3×10^{-8} (iv) 1.0001×10^{9}

(v) 5.8×10^{12} (vi) 3.61492×10^{6}

Answer:

(i) 3.02×10^{-6} = 0.00000302

(ii) 4.5×10^{4} = 45000

(iii) 3×10^{-8} = 0.00000003

(iv) 1.0001×10^{9} = 1000100000

(v) 5.8×10^{12} = 5800000000000

(vi) 3.61492×10^{6} = 3614920

Question 3:

Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer:

(i) $\frac{1}{1000000} = 1 \times 10^{-6}$

(ii) 0.000, 000, 000, 000, 000, 000, 16 = 1.6×10^{-19}

(iii) 0.0000005 = 5×10^{-7}

(iv) 0.00001275 = 1.275×10^{-5}

(v) 0.07 = 7×10^{-2}

Question 4:

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer:

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5 × 20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5 × 0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

= (100 + 0.080) mm

= 100.08 mm

= 1.0008×10^{2} m

DELHI PUBLIC SCHOOL, GANDHINAGAR
MIND MAP
CH.: 12 EXPONENTS AND POWERS

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of two topics. The most probable questions from examination point of view are given below.

TYPE 1:PROBLEMS INVOLVING LAWS OF EXPONENTS

- Simplify and express the result in exponential form:
  (1) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$
  (2) $((3)^{-7} \div (3)^{-11}) \times (3)^{-5}$
  (3) $\left\{\left(\frac{7}{3}\right)\right\}^{-3}$
  (4) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$
- Simplify/Evaluate

(1) $(3^{-1} + 4^{-1} + 5^{-1})^0$
(2) $\frac{(8)^{-1} \times (5)^3}{(2)^{-4}}$
(3) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$
(4) $\frac{25 \times (a)^{-4}}{(5)^{-3} \times 10 \times (a)^{-8}},\ (a \neq 0)$

- Find the value
  (1) $Find\ the\ value\ of$ m $for\ which$ $(5)^m \div (5)^{-3} = (5)^5$
  (2) $Find\ the\ value\ of$ x $for\ which$ $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$

TYPE 2: USE OF EXPONENTS IN EXPRESSING AND COMPARING LARGE NUMBERS

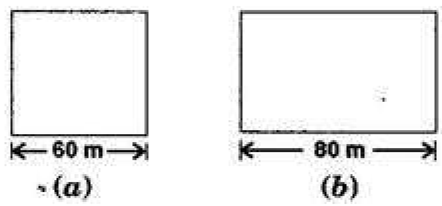
- Express the following in standard form:
  (1) 0.0000021
  (2) 216000000
  (3) $0.0000537 \times (10)^6$
  (4) $1234 \times (10)^{-4}$
- Express the numbers in usual form:

(1) $1.00001 \times (10)^9$
(2) $3.002 \times (10)^{-9}$
(3) $3.25 \times (10)^{-7}$

CH.: 11 MENSURATION

Exercise 11.1 Page No: 171

1. **A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area?**



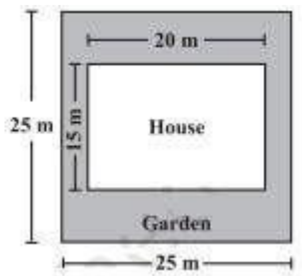
Solution:
Side of a square = 60 m (Given)
And the length of rectangular field, l = 80 m (Given)
According to question,
Perimeter of rectangular field = Perimeter of square field
2(l+b) = 4×Side (using formulas)

2(80+b) = 4×60
160+2b = 240
b = 40
Breadth of the rectangle is 40 m.

Now, Area of Square field
= $(\text{side})^2$
= $(60)^2$ = 3600 m^2
And Area of Rectangular field
= length×breadth = 80×40
= 3200 m^2

Hence, area of square field is larger.

2. Mrs.Kaushik has a square plot with the measurement as shown in the figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of Rs. 55 per m^2.



Solution:

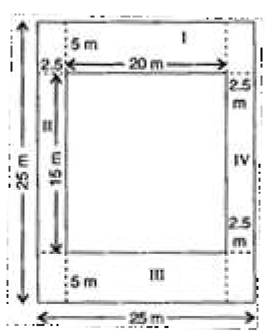
Side of a square plot = 25 m

Formula: Area of square plot = square of a side = $(\text{side})^2$

$= (25)^2 = 625$

Therefore the area of a square plot is 625 m^2

Length of the house = 20 m and



Breadth of the house = 15 m

∴ Area of the house = length×breadth

= 20×15 = 300 m^2

Area of garden = Area of square plot – Area of house

= 625–300 = 325 m^2

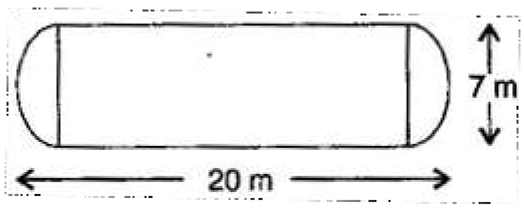
∵ Cost of developing the garden per sq. m is Rs. 55

∴ Cost of developing the garden 325 sq. m = Rs. 55×325

= Rs. 17,875

Hence total cost of developing a garden around is Rs. 17,875.

3. The shape of a garden is rectangular in the middle and semi-circular at the ends as shown in the diagram. Find the area and the perimeter of this garden [Length of rectangle is 20 – (3.5 + 3.5 meters]



Solution::

Given: Total length = 20 m

Diameter of semi circle = 7 m

∴ Radius of semi circle = 7/2 = 3.5 m

Length of rectangular field

= 20-(3.5+3.5) = 20-7 = 13 m

Breadth of the rectangular field = 7 m

∴ Area of rectangular field = l×b

= 13×7= 91m^2

Area of two semi circles = $2\times(1/2)\times\pi\times r^2$

$= 2\times(1/2)\times22/7\times3.5\times3.5$
$= 38.5\ m^2$

Area of garden = 91+38.5 = 129.5 m^2
Now Perimeter of two semi circles = $2\pi r = 2\times(22/7)\times3.5 = 22$ m

And Perimeter of garden = 22+13+13

= 48 m. Answer

4. A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2? [If required you can split the tiles in whatever way you want to fill up the corners]

Solution:

Given: Base of flooring tile = 24 cm = 0.24 m

Corresponding height of a flooring tile= 10 cm = 0.10 m

Now Area of flooring tile= Base×Altitude

= 0.24×0.10

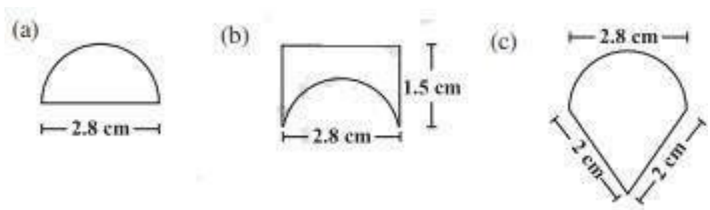
= 0.024
Area of flooring tile is 0.024m^2

∴ Number of tiles required to cover the floor= Area of floor/Area of one tile = 1080/0.024

= 45000 tiles

Hence 45000 tiles are required to cover the floor.

5. An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a longer round? Remember, circumference of a circle can be obtained by using the expression $C = 2\pi r$ **,where** r **is the radius of the circle.**



Solution:

(a) Radius = Diameter/2 = 2.8/2 cm = 1.4 cm

Circumference of semi-circle = πr

= (22/7)×1.4 = 4.4

Circumference of semi-circle is 4.4 cm

Total distance covered by the ant= Circumference of semi -circle+Diameter

= 4.4+2.8 = 7.2 cm

(b) Diameter of semi-circle = 2.8 cm
Radius = Diameter/2 = 2.8/2 = 1.4 cm

Circumference of semi-circle = πr

= (22/7)×1.4 = 4.4 cm

Total distance covered by the ant= 1.5+2.8+1.5+4.4 = 10.2 cm

(c) Diameter of semi-circle = 2.8 cm
Radius = Diameter/2 = 2.8/2

= 1.4 cm

Circumference of semi-circle = π r

= (22/7)×1.4

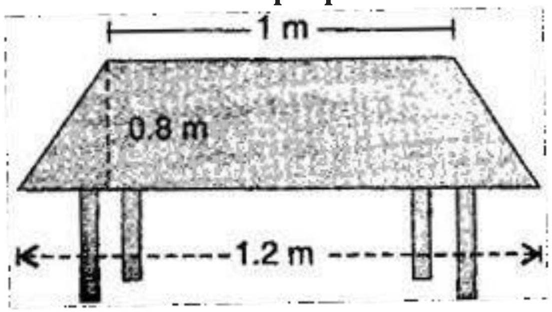
= 4.4 cm

Total distance covered by the ant = 2+2+4.4 = 8.4 cm

After analyzing results of three figures, we concluded that for figure (b) food piece, the ant would take a longer round.

Exercise 11.2

1. The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.



Solution: One parallel side of the trapezium (a) = 1 m

And second side (b) = 1.2 m and

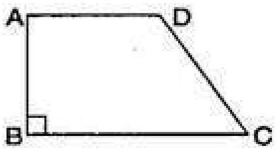
height (h) = 0.8 m

Area of top surface of the table= (½)×(a+b)h

= (½)×(1+1.2)0.8

= (½)×2.2×0.8 = 0.88
Area of top surface of the table is 0.88 m^2 .

2. The area of a trapezium is 34 cm^2 and the length of one of the parallel sides is 10 cm and its height is 4 cm Find the length of the other parallel side.



Solution: Let the length of the other parallel side be b.

Length of one parallel side, a = 10 cm

height, (h) = 4 cm and

Area of a trapezium is 34 cm^2

Formula for, Area of trapezium = $(1/2)\times(a+b)h$

$34 = \frac{1}{2}(10+b)\times4$

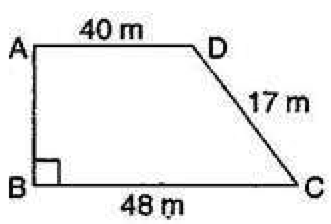
$34 = 2\times(10+b)$

After simplifying, b = 7

Hence another required parallel side is 7 cm.

3. Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.

Solution:



Given: BC = 48 m, CD = 17 m,
AD = 40 m and perimeter = 120 m

$\because$ Perimeter of trapezium ABCD

= AB+BC+CD+DA

$\Rightarrow$ 120 = AB+48+17+40

$\Rightarrow$ 120 = AB = 105

$\Rightarrow$ AB = 120−105 = 15 m

Now, Area of the field= (½)×(BC+AD)×AB

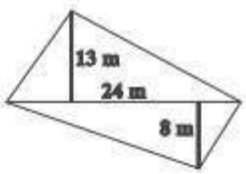
= (½)×(48 +40)×15

= (½)×88×15

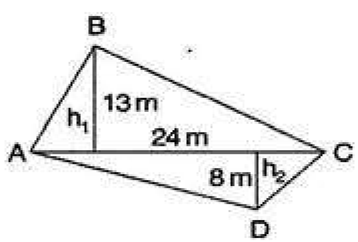
= 660

Hence, area of the field ABCD is $660m^2$.

4. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.



Solution:



Consider, h_1 = 13 m, h_2 = 8 m and AC = 24 m

Area of quadrilateral ABCD = Area of triangle ABC+Area of triangle ADC

= ½(bh_1)+ ½(bh_2)

= ½ ×b(h_1+h_2)= (½)×24×(13+8)

$= (\frac{1}{2}) \times 24 \times 21 = 252$

Hence, required area of the field is 252 m^2

5. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Solution:

Given: d1 = 7.5 cm and d2 = 12 cm

We know that,Area of rhombus = $(\frac{1}{2}) \times d1 \times d2$

$= (\frac{1}{2}) \times 7.5 \times 12 = 45$

Therefore, area of rhombus is 45 cm^2 .

6. Find the area of a rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of the diagonals is 8 cm long, find the length of the other diagonal.

Solution: Since a rhombus is also a kind of a parallelogram.

Formula for Area of rhombus = Base×Altitude

Putting values, we have

Area of rhombus = $6 \times 4 = 24$

Area of rhombus is 24 cm^2

Also, Formula for Area of rhombus = $(\frac{1}{2}) \times d_1 d_2$

After substituting the values, we get

$24 = (\frac{1}{2}) \times 8 \times d_2$

$d_2 = 6$

Hence, the length of the other diagonal is 6 cm.

7. The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs. 4.

Solution: Length of one diagonal, $d_1 = 45$ cm and $d_2 = 30$ cm

∵ Area of one tile = $(½)d_1d_2$

= $(½)×45×30 = 675$

Area of one tile is 675 cm^2

∴ Area of 3000 tiles is

= $675×3000 = 2025000$ cm^2

= 2025000/10000

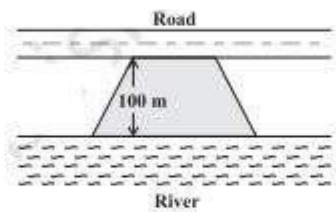
= 202.50 m^2 [∵ $1m^2 = 10000$ cm^2]

∵ Cost of polishing the floor per sq. meter = 4

∴ Cost of polishing the floor per 202.50 sq. meter = 4×202.50 = 810

Hence the total cost of polishing the floor is Rs. 810.

8. Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.



Solution:

Perpendicular distance (h) = 100 m (Given)

Area of the trapezium shaped field = 10500 m^2 (Given)

Let side along the road be 'x' m and side along the river = 2x m

∴ Area of the trapezium field = (½)×(a+b)×h

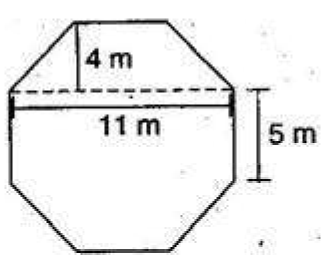
10500 = (½)×(x+2x)×100

10500 = 3x×50

After simplifying, we have x = 70, which means side along the river is 70 m

Hence, the side along the river = 2x = 2(70) = 140 m.

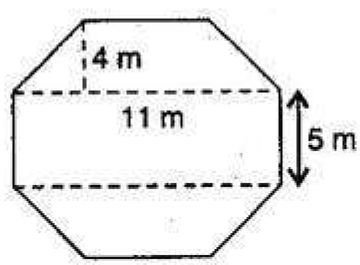
9. Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



Solution:

Octagon having eight equal sides, each 5 m. (given)

Divide the octagon as show in the below figure, 2 trapeziums whose parallel and perpendicular sides are 11 m and 4 m respectively and 3rd one is rectangle having length and breadth 11 m and 5 m respectively.



Now, Area of two trapeziums = 2 [(½)×(a+b)×h]

= 2×(½)×(11+5)×4

= 4×16 = 64

Area of two trapeziums is 64 m^2

Also, Area of rectangle = length×breadth

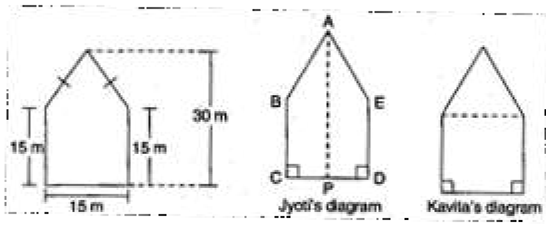
= 11×5 = 55

Area of rectangle is 55 m^2

∴ Total area of octagon = 64+55

= 119 m^2

10. There is a pentagonal shaped park as shown in the figure.
For finding its area Jyoti and Kavita divided it in two different ways.



Find the area of this park using both ways. Can you suggest some other way of finding its area?

Solution:

First way: By Jyoti's diagram,

Area of pentagon = Area of trapezium ABCP + Area of trapezium AEDP

$= (½)(AP+BC)\times CP+(1/2)\times(ED+AP)\times DP$

$= (½)(30+15)\times CP+(1/2)\times(15+30)\times DP$

$= (½)\times(30+15)\times(CP+DP)$

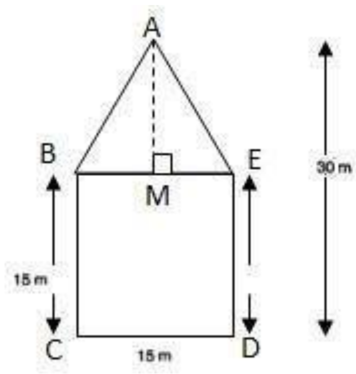
$= (½)\times 45\times CD$

$= (1/2)\times 45\times 15$

$=337.5\ m^2$

Area of pentagon is $337.5\ m^2$

Second way: By Kavita's diagram



Here, a perpendicular AM drawn to BE.

AM = 30–15 = 15 m

Area of pentagon = Area of triangle ABE+Area of square BCDE (from above figure)

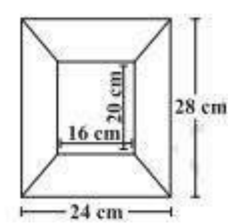
= $(\frac{1}{2})\times 15\times 15+(15\times 15)$

= 112.5+225.0

= 337.5

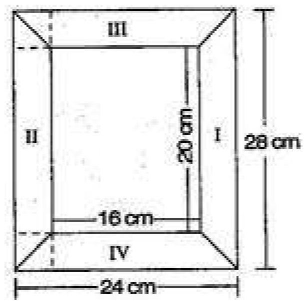
Hence, total area of pentagon shaped park = 337.5 m^2

11. Diagram of the adjacent picture frame has outer dimensions = 24 cm×28 cm and inner dimensions 16 cm×20 cm. Find the area of each section of the frame, if the width of each section is same.



Solution:

Divide given figure into 4 parts, as shown below:



Here two of given figures (I) and (II) are similar in dimensions.

And also figures (III) and (IV) are similar in dimensions.

∴ Area of figure (I) = Area of trapezium

= $(½)\times(a+b)\times h$

= $(½)\times(28+20)\times 4$

= $(½)\times 48\times 4 = 96$

Area of figure (I) = 96 cm^2

Also,Area of figure (II) = 96 cm^2

Now, Area of figure (III) = Area of trapezium

= $(½)\times(a+b)\times h$

= $(½)\times(24+16)4$

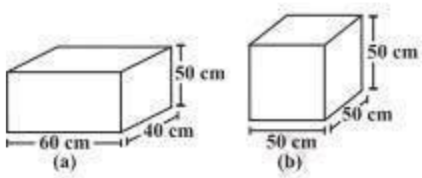
= $(½)\times 40\times 4 = 80$

Area of figure (III) is 80 cm^2

Also, Area of figure (IV) = 80 cm^2

Exercise 11.3 Page No:186

1. There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?



Solution:(a) Given: Length of cuboidal box (l) = 60 cm

Breadth of cuboidal box (b) = 40 cm

Height of cuboidal box (h) = 50 cm

$\therefore$ Total surface area of cuboidal box = 2×(lb+bh+hl)

= 2×(60×40+40×50+50×60)

= 2×(2400+2000+3000)

= 14800 cm^2

(b) Length of cubical box (l) = 50 cm
Breadth of cubicalbox (b) = 50 cm
Height of cubicalbox (h) = 50 cm

$\therefore$ Total surface area of cubical box = 6(side)^2

= 6(50×50)

= 6×2500

= 15000

Surface area of the cubical box is 15000 cm^2

From the result of (a) and (b), cuboidal box requires the lesser amount of material to make.

2. A suitcase with measures 80 cm x 48 cm x 24 cm is to be covered with a tarpaulin cloth. How many meters of tarpaulin of width 96 cm is required to cover 100 such suitcases?

Solution: Length of suitcase box, l = 80 cm,

Breadth of suitcase box, b= 48 cm

And Height of cuboidal box , h = 24 cm

Total surface area of suitcase box = 2(lb+bh+hl)

= 2(80×48+48×24+24×80)

= 2 (3840+1152+1920)

= 2×6912

= 13824

Total surface area of suitcase box is 13824 cm^2

Area of Tarpaulin cloth = Surface area of suitcase

l×b = 13824

l ×96 = 13824

l = 144

Required tarpaulin for 100 suitcases = 144×100 = 14400 cm = 144 m

Hence tarpaulin cloth required to cover 100 suitcases is 144 m.

3. Find the side of a cube whose surface area is 600cm^2 .

Solution: Surface area of cube = 600 cm^2 (Given)

Formula for surface area of a cube = $6(side)^2$

Substituting the values, we get

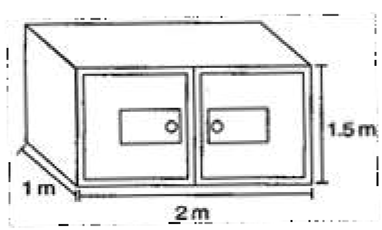
$6(side)^2 = 600$

$(side)^2 = 100$

Or side = ±10

Since side cannot be negative, the measure of each side of a cube is 10 cm

4. Rukshar painted the outside of the cabinet of measure 1 m ×2 m ×1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet?



Solution:Length of cabinet, l = 2 m, Breadth of cabinet, b = 1 m and Height of cabinet, h = 1.5 m

Surface area of cabinet = lb+2(bh+hl)

= 2×1+2(1×1.5+1.5×2)

= 2+2(1.5+3.0)

= 2+9.0

= 11

Required surface area of cabinet is $11m^2$.

5. Daniel is paining the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m^2of area is painted. How many cans of paint will she need to paint the room?

Solution: Length of wall, l = 15 m, Breadth of wall, b = 10 m and Height of wall, h = 7 m

Total Surface area of classroom = lb+2(bh+hl)

= 15×10+2(10×7+7×15)

= 150+2(70+105)

= 150+350

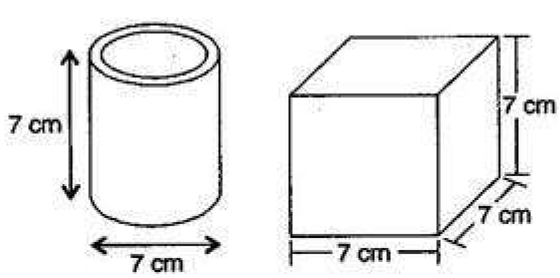
= 500

Now, Required number of cans = Area of hall/Area of one can

= 500/100 = 5

Therefore, 5 cans are required to paint the room.

6. Describe how the two figures below are alike and how they are different. Which box has larger lateral surface areas?



Solution:

Diameter of cylinder = 7 cm (Given)

Radius of cylinder, r = 7/2 cm

Height of cylinder, h = 7 cm

Lateral surface area of cylinder = 2πrh

= 2×(22/7)×(7/2)×7 = 154

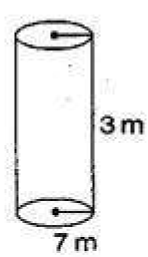
So, Lateral surface area of cylinder is154 cm^2
Now, lateral surface area of cube = 4 $(side)^2$=4×7^2 = 4×49 = 196

Lateral surface area of cube is 196 cm^2

Hence, the cube has larger lateral surface area.

7. A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

Solution:



Radius of cylindrical tank, r = 7 m

Height of cylindrical tank , h = 3 m

Total surface area of cylindrical tank = 2πr(h+r)

$= 2\times(22/7)\times7(3+7)$

$= 44\times10 = 440$

Therefore, 440 m^2 metal sheet is required.

8. The lateral surface area of a hollow cylinder is 4224cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

Solution: Lateral surface area of hollow cylinder = 4224 cm^2

Height of hollow cylinder, h = 33 cm and say r be the radius of the hollow cylinder

Curved surface area of hollow cylinder = $2\pi rh$

$4224 = 2\times\pi\times r\times33$

$r = (4224)/(2\pi\times33)$

$r = 64/\pi$

Now, Length of rectangular sheet, $l = 2\pi r$

$l = 2\pi\times(64/\pi) = 128$ (using value of r)

So the length of the rectangular sheet is 128 cm.

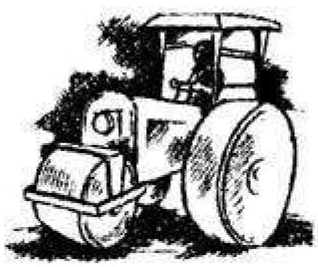
Also, Perimeter of rectangular sheet = $2(l+b)$

$= 2(128+33)$

$= 322$

The perimeter of rectangular sheet is 322 cm.

9. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length 1 m.



Solution:

Diameter of road roller, d = 84 cm

Radius of road roller, r = d/2 = 84/2 = 42 cm

Length of road roller, h = 1 m = 100 cm

Formula for Curved surface area of road roller = $2\pi rh$
$= 2\times(22/7)\times42\times100 = 26400$
Curved surface area of road roller is 26400 cm^2
Again, Area covered by road roller in 750 revolutions = $26400\times750 cm^2$

= 1,98,00,000cm^2

= 1980 m^2 [$\because$ 1 m^2= 10,000 cm^2]

Hence the area of the road is 1980 m^2.

10. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in figure). If the label is placed 2 cm from top and bottom, what is the area of the label?

Solution: Diameter of cylindrical container , d = 14 cm

Radius of cylindrical container, r = d/2 = 14/2 = 7 cm

Height of cylindrical container = 20 cm

Height of the label, say h = 20–2–2 (from the figure)

= 16 cm

Curved surface area of label = 2πrh

= $2\times(22/7)\times7\times16$
= 704

Hence, the area of the label is 704 cm^2.

<u>Exercise 11.4</u> Page No: 191

1. Given a cylindrical tank, in which situation will you find surface are and in which situation volume.

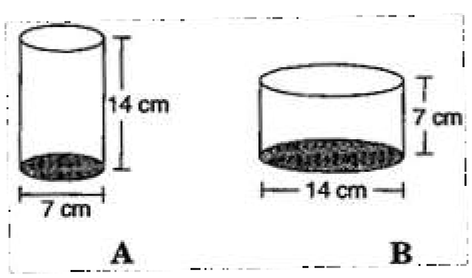
(a) To find how much it can hold.
(b) Number of cement bags required to plaster it.
(c) To find the number of smaller tanks that can be filled with water from it.

Solution: We find area when a region covered by a boundary, such as outer and inner surface area of a cylinder, a cone, a sphere and surface of wall or floor.

When the amount of space occupied by an object such as water, milk, coffee, tea, etc., then we have to find out volume of the object.

(a) Volume (b) Surface area (c) Volume

2. Diameter of cylinder A is 7 cm and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area.



Solution: Yes, we can say that volume of cylinder B is greater, since radius of cylinder B is greater than that of cylinder A.

Find Volume for cylinders A and B

Diameter of cylinder A = 7 cm

Radius of cylinder A = 7/2 cm

And Height of cylinder A = 14 cm

Volume of cylinder A = $\pi r^2 h$

= $(22/7)\times(7/2)\times(7/2)\times14 = 539$

Volume of cylinder A is 539 cm^3

Now, Diameter of cylinder B = 14 cm

Radius of cylinder B = 14/2 = 7 cm

And Height of cylinder B = 7 cm

Volume of cylinder B = $\pi r^2 h$

= $(22/7)\times7\times7\times7 = 1078$

Volume of cylinder B is 1078 cm^3

Find surface area for cylinders A and B

Surface area of cylinder A = $2\pi r(r+h)$

= $2 \times 22/7 \times 7/2 \times (7/2 + 14) = 385$

Surface area of cylinder A is385 cm^2

Surface area of cylinder B = $2\pi r(r+h)$

= $2\times(22/7)\times7(7+7) = 616$

Surface area of cylinder B is 616 cm^2

Yes, cylinder with greater volume also has greater surface area.

3. Find the height of a cuboid whose base area is 180 cm^2 and volume is 900 cm^3?

Solution: Given, Base area of cuboid = 180 cm^2 and Volume of cuboid = 900 cm^3

We know that, Volume of cuboid = lbh

900 = 180×h (substituting the values)

h= 900/180 = 5

Hence the height of cuboid is 5 cm.

4. A cuboid is of dimensions 60 cm×54 cm×30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?

Solution: Given, Length of cuboid, l = 60 cm, Breadth of cuboid, b = 54 cm and

Height of cuboid, h = 30 cm

We know that, Volume of cuboid = lbh = 60 ×54×30 = 97200 cm^3

And Volume of cube = $(Side)^3$
= 6×6×6 = 216 cm^3

Also, Number of small cubes = volume of cuboid / volume of cube

= 97200/216

= 450

Hence , required cubes are 450.

5. Find the height of the cylinder whose volume if 1.54 m^3 and diameter of the base is 140 cm.

Solution:

Given: Volume of cylinder = 1.54 $m^3$and Diameter of cylinder = 140 cm
$\therefore$ Radius (r)= d/2 = 140/2 = 70 cm

Volume of cylinder = $\pi r^2 h$

1.54 = (22/7)×0.7×0.7×h

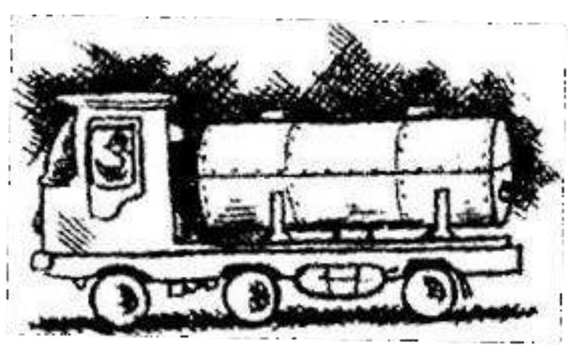
After simplifying, we get the value of h that is

h = (1.54×7)/(22×0.7×0.7)

h = 1

Hence, height of the cylinder is 1 m.

6. A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in liters that can be stored in the tank.



Solution: Given, Radius of cylindrical tank, r = 1.5 m and Height of cylindrical tank, h = 7 m

Volume of cylindrical tank, V = $\pi r^2 h$

$= (22/7)\times 1.5\times 1.5 \times 7$

$= 49.5\ cm^3$

$= 49.5\times 1000$ liters = 49500 liters

[$\because$ 1 m^3= 1000 liters]
Hence, required quantity of milk is 49500 liters.

7. If each edge of a cube is doubled,
(i) how many times will its surface area increase?
(ii) how many times will its volume increase?

Solution:(i) Let the edge of cube be " l" .

Formula for Surface area of the cube, $A = 6\ l^2$

When edge of cube is doubled, then

Surface area of the cube, say $A' = 6(2l)^2 = 6\times 4l^2 = 4(6\ l^2)$

$A' = 4A$

Hence, surface area will increase by four times.

(ii) Volume of cube, $V = l^3$
When edge of cube is doubled, then

Volume of cube, say $V' = (2l)^3 = 8(\ l^3)$

$V' = 8\times V$

Hence, volume will increase 8 times.

8. Water is pouring into a cuboidal reservoir at the rate of 60 liters per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.

Solution:

Given, volume of reservoir = 108 m^3

Rate of pouring water into cuboidal reservoir = 60 liters/minute

= 60/1000 m^3per minute

Since 1 liter = (1/1000)m^3

= (60×60)/1000 m^3 per hour

Therefore, (60×60)/1000 m^3 water filled in reservoir will take = 1 hour

Therefore 1 m^3 water filled in reservoir will take = 1000/(60×60) hours

Therefore, 108 m^3 water filled in reservoir will take = (108×1000)/(60×60) hours = 30 hours

Answer: It will take 30 hours to fill the reservoir.

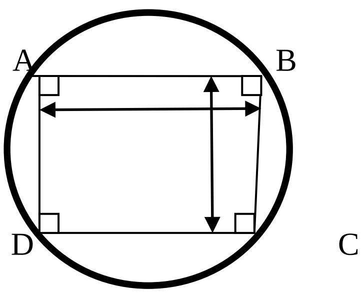
DELHI PUBLIC SCHOOL, GANDHINAGAR
MIND MAP
CH.: 11 MENSURATION

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of three topics. The most probable questions from examination point of view are given below.

TYPE:1 Areas of plane figures.(Area of rectangle, square, triangle, parallelogram & circle)

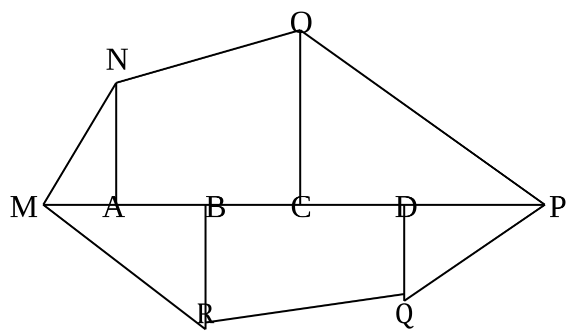
1. A flooring tile has the shape of a parallelogram whose base is 25cm and height is 10cm. How many such tiles are required to cover a floor of 3500m²? (If required you can split the tiles in whatever way you want to fill up the corners).
2. Find the area of the shaded region. Where AB =8cm & AD =6cm (use $\pi = 3.14$)



3. A rectangular grassy plot is 112m long and 78m broad. It has a gravel path 2.5m wide all around it outside. Find the area of the path and the cost of constructing it at the rate of ₹4.50 per square meter.

TOPIC-II: Areas of plane figures. (Area of trapezium, rhombus, general quadrilateral & polygon)

4. Find the area of the rhombus if length of its each side is 14cm and the altitude is 16cm.
5. The area of rhombus is 84 sq meter. If its perimeter is 40m, then find its altitude.
6. The area of trapezium is 105cm² and its height is 7cm. If one of the parallel sides is longer than other by 6cm, find the length of two parallel sides.
7. Find the height of trapezium the sum of lengths of whose bases (parallel sides) is 60cm and whose area is 600cm².
8. The diagonal of a quadrilateral shaped field is 24m and the perpendiculars dropped on it from the remaining opposite vertices are 8m and 13m. Find the area of the field.
9. Find the area of the hexagon MNOPQR shown in figure. If MP =9cm, MD =7cm, MC =6cm, MB=4cm, MA =2cm, AN =3cm, OC =5cm, DQ =2cm and BR =4cm.



10. The internal measures of a cuboidal room are 12m x 8m x 4m. Find the total cost of whitewashing the all four walls of the room.

TOPIC III: Volume of cube, cuboids and cylinder

11. The lateral surface area of a hollow cylinder is 4224 cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of the rectangular sheet.
12. Find the height of a cuboid with base area 180cm^2 and volume 900cm^3.
13. If each edge of a cube is doubled,
    (i) How many times will its surface area increase?
    (ii) How many times will its volume increase?
14. A well with 10m inside diameter is dug 14m deep. Earth taken out of it is spread all around to a width of 5m to form an embankment. Find the height of embankment.
15. A milk tank is in the form of a cylinder whose radius is 1.5m and length is 7m. Find the quantity of milk in liters that can be stored in the tank?

CHAPTER-5 DATA HANDLING

Exercise 5.1

1. For which of these would you use a histogram to show the data?

(a) The number of letters for different areas in a postman's bag.

(b) The height of competitors in an athletics meet.

(c) The number cassettes produced by 5 companies.

(d) The number of passengers boarding trains from 7.00 a.m. to 7.00 p.m. at a station. Give reason for each.

Solution:

We know that Histogram is a graphical representation of data if the data represented using class-interval.

Since the cases mentioned in options (b) and (d) can be divided into class intervals, histogram can be used to show the data.

Similarly, since the cases mentioned in options (a) and (c) cannot be divided into class intervals, histogram cannot be used to represent the data.

2. The shoppers who come to a departmental store are marked as: man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning.

W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W W M W B W G M W W W W G W M M W M W G W M G W M M B G G W.

Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

Solution:

Shopper	Tally Marks	Number of shoppers
W	卌 卌 卌 卌 卌 |||	28
M	卌 卌 卌	15
B	卌	5
G	卌 卌 ||	12
	Total	60

Frequency distribution table:

Bar-graph:

3. The weekly wages (in ₹) of 30 workers in a factory are:

830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840.

Using tally marks, make a frequency table with intervals as 800 – 810, 810 – 820 and so on.

Solution:

Class Intervals	Tally Marks	Frequency
800–810	|||	3
810–820	||	2
820–830	|	1
830–840	𝍸 ||||	9
840–850	𝍸	5
850–860	|	1
860–870	|||	3
870–880	|	1
880–890	|	1
890–900	||||	4
	Total	30

The frequency table with intervals as 800 – 810, 810 – 820 and so on, using tally marks is given below:

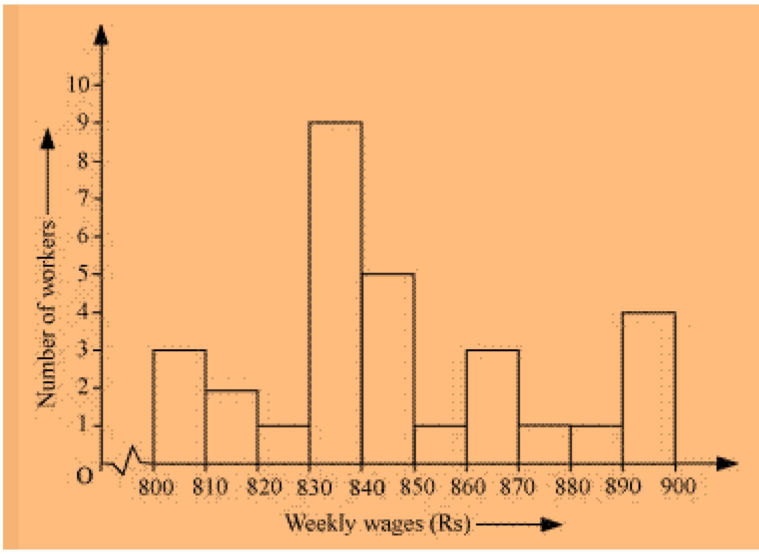
4. Draw a histogram for the frequency table made for the data in Question 3 and answer the following questions.

(i) Which group has the maximum number of workers?

(ii) How many workers earn ₹ 850 and more?

(iii) How many workers earn less than ₹ 850?

Solution:



(i) 830-840 is the group having maximum number of workers, 9, compared to other groups.

(ii) Workers earning ₹ 850 and more= 1+3+1+1+4=10

(iii) Workers earning less than ₹ 850= 3+2+1+9+5=20

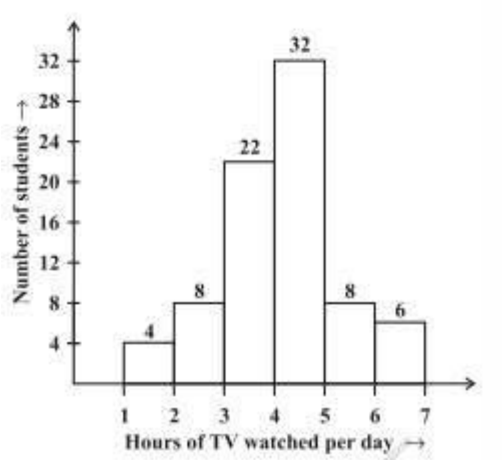
5. The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

Answer the following:

(i) For how many hours did the maximum number of students watch T.V.?

(ii) How many students watched TV for less than 4 hours?

(iii) How many students spent more than 5 hours in watching TV?



Solution:

(i) 32 students watched T.V for 4-5 hours. ∴, The maximum number of students who watched T.V. for 4 – 5 hours.

(ii) The number of students who watched T.V. less than 4 hours= 22+8+4=34

(iii) The number of students who spent more than 5 hours in watching TV

=8+6=14

Exercise 5.2

1. A survey was made to find the type of music that a certain group of young people liked in a city.

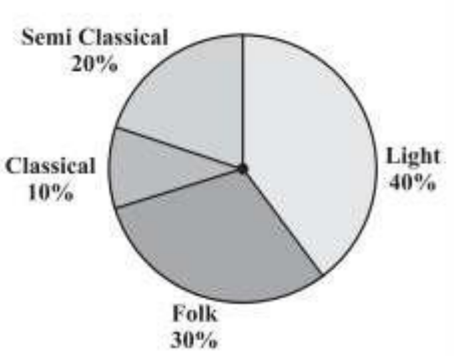
Adjoining pie chart shows the findings of this survey. From this pie chart, answer the following:

(i) If 20 people liked classical music, how many young people were surveyed?

(ii) Which type of music is liked by the maximum number of people?

(iii) If a cassette company were to make 1000 CD's, how many of each type would they

make?



Solution:

(i) 10% represents 100 people.

⟹20% represents = (100×20)/10 = 200

∴, 200 people were surveyed.

(ii) Since 40% of the total people surveyed liked light music and no other form of song liked more than that, we can conclude that Light music is liked by the maximum number of people.

(iii) CD's of classical music = (10 × 1000)/100 = 100

CD's of semi-classical music = (20 × 1000)/100 = 200

CD's of light music = (40 × 1000)/100 = 400

CD's of folk music = (30 × 1000)/100 = 300

2. A group of 360 people were asked to vote for their favourite season from the three

seasons rainy, winter and summer.

(i) Which season got the most votes?

(ii)Find the central angle of each sector.

(iii) Draw a pie chart to show this information

Season	No. of votes
Summer	90
Rainy	120
Winter	150

Solution:

(i) According to the table given in the question, Winter season got the most votes.

(ii) Central angle of summer season= (90×360)/360= 90°

Central angle of rainy season= (120×360)/360= 120°

Central angle of winter season= (150×360)/360= 150°

(iii)

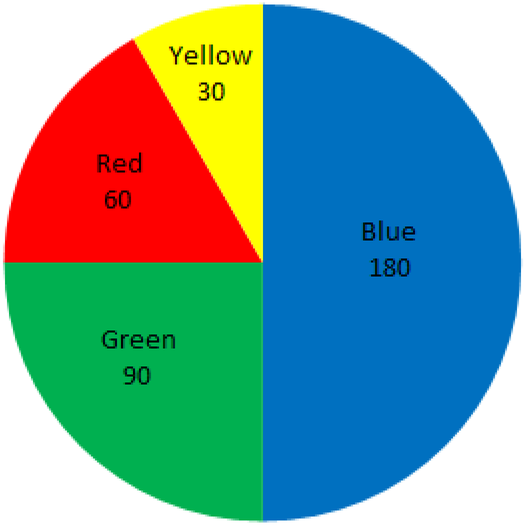
3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

Solution:

Here, central angle = 360° Total number of people = 36

Colours	No. of people	In fraction	Central angles
Blue	18	$\frac{18}{36}=\frac{1}{2}$	$\frac{1}{2}\times 360°=180°$
Green	9	$\frac{9}{36}=\frac{1}{4}$	$\frac{1}{4}\times 360°=90°$
Red	6	$\frac{6}{36}=\frac{1}{6}$	$\frac{1}{6}\times 360°=60°$
Yellow	3	$\frac{3}{36}=\frac{1}{12}$	$\frac{1}{12}\times 360°=30°$



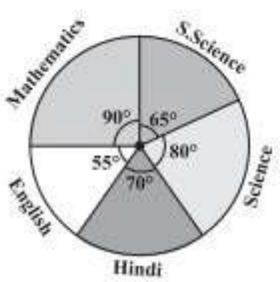
4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.

(i) In which subject did the student score 105 marks?

(Hint: for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)

(ii) How many more marks were obtained by the student in Mathematics than in Hindi?

(iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi. (Hint: Just study the central angles).



Solution:

Subject	Central Angle	Marks obtained
Mathematics	90°	$\frac{90^\circ}{360^\circ} \times 540 = 135$
Social Science	65°	$\frac{65^\circ}{360^\circ} \times 540 = 97.5$
Science	80°	$\frac{80^\circ}{360^\circ} \times 540 = 120$
Hindi	70°	$\frac{70^\circ}{360^\circ} \times 540 = 105$
English	55°	$\frac{55^\circ}{360^\circ} \times 540 = 82.5$

(i) The student scored 105 marks in Hindi.

(ii) Marks obtained in Mathematics = 135 Marks obtained in Hindi = 105 Difference = 135 – 105 = 30

Thus, 30 more marks were obtained by the student in Mathematics than in Hindi.

(iii) The sum of marks in Social Science and Mathematics = 97.5 + 135 = 232.5 The sum of marks in Science and Hindi = 120 + 105 = 225

∴, the sum of the marks in Social Science and Mathematics is more than that in Science and Hindi.

5. The number of students in a hostel, speaking different languages is given below.

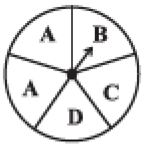
Display the data in a pie chart.

Language	No. of students	In fraction	Central Angle
Hindi	40	$\frac{40}{72}=\frac{5}{9}$	$\frac{5}{9}\times 360^\circ = 200^\circ$
English	12	$\frac{12}{72}=\frac{1}{6}$	$\frac{1}{6}\times 360^\circ = 60^\circ$
Marathi	9	$\frac{9}{72}=\frac{1}{8}$	$\frac{1}{8}\times 360^\circ = 45^\circ$
Tamil	7	$\frac{7}{72}=\frac{7}{72}$	$\frac{7}{72}\times 360^\circ = 35^\circ$
Bengali	4	$\frac{4}{72}=\frac{1}{18}$	$\frac{1}{18}\times 360^\circ = 20^\circ$
Total	72		

Exercise 5.3

1. List the outcomes you can see in these experiments.

(a) Spinning a wheel (b) Tossing two coins together



Solution:

(a) There are four letters A, B, C and D in a spinning wheel. So there are 4 outcomes.

(b) When two coins are tossed together. There are four possible outcomes HH, HT, TH, TT.

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number (b) not a prime number.

(ii) (a) a number greater than 5 (b) a number not greater than 5.

Solution:

(i) (a) Outcomes of event of getting a prime number are 2, 3 and 5.

(b) Outcomes of event of not getting a prime number are 1, 4 and 6.

(ii) (a) Outcomes of event of getting a number greater than 5 is 6.

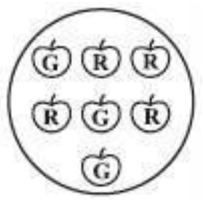
(b) Outcomes of event of not getting a number greater than 5 are 1, 2, 3, 4 and 5.

3. Find the

(a) Probability of the pointer stopping on D in (Question 1-(a)).

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards.

(c) Probability of getting a red apple. (See figure below)



Solution:

(a) In a spinning wheel, there are five pointers A, A, B, C, D. So there are five

outcomes. Pointer stops at D which is one outcome.

So the probability of the pointer stopping on D = 1/5

(b) There are 4 aces in a deck of 52 playing cards. So there are four events of getting an ace.

So, probability of getting an ace = 4/52 = 1/13

(c) Total number of apples = 7

Number of red apples = 4

Probability of getting red apple = 4/7

4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

(i) getting a number 6?

(ii) getting a number less than 6? (iii) getting a number greater than 6? (iv)getting a 1-digit number?

Solution:

(i) Outcome of getting a number 6 from ten separate slips is one.

∴, probability of getting a number 6 = 1/10

(ii) Numbers less than 6 are 1, 2, 3, 4 and 5 which are five. So there are 5 outcomes.

∴, probability of getting a number less 6 =5/10 = ½

(iii)Number greater than 6 out of ten that are 7, 8, 9, 10. So there are 4 possible outcomes.

∴, probability of getting a number greater than 6 = 4/10 = 2/5

(iv) One digit numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9 out of ten.

∴, probability of getting a 1-digit number = 9/10

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non-blue sector?

Solution:

A total of five sectors are present.

Out of the five sectors, three sectors are green.

∴, probability of getting a green sector = 3/5

Out of the five sectors, one sector is blue. Hence, Non-blue sectors = 5 – 1 = 4 sectors

∴, probability of getting a non-blue sector= 4/5

6. Find the probabilities of the events given in Question 2.

Solution:

When a die is thrown, there are total six outcomes, i.e., 1, 2, 3, 4, 5 and 6.

(i)

(a) 2, 3, 5 are the prime numbers. So there are 3 outcomes out of 6.

∴,probability of getting a prime number =3/6 = ½

(b) 1, 4, 6 are not the prime numbers. So there are 3 outcomes out of 6.

∴,probability of getting a prime number =3/6 = ½

(ii)

(c) Only 6 is greater than 5.

So there is one outcome out of 6.

∴,probability of getting a number greater than 5= 1/6

(d) Numbers not greater than 5 are 1, 2, 3, 4 and 5. So there are 5 outcomes out of 6.

∴,probability of not getting a number greater than 5= 5/6

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.: 5 DATA HANDLING

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of four different topics. The most probable questions from examination point of view are given below.

TYPE:1 FREQUENCY DISTRIBUTION TABLE & BAR GRAPH

Q.1. The blood groups of 30 students in a class are given below. Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

B, A, A, O, AB, O, A, B, O, A, B, AB, A, B, AB,

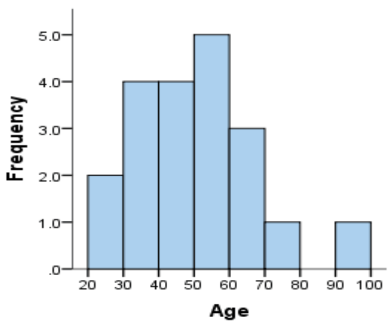
B, A, B, A, B, A, B, O, A, B, AB, O, A, B, A.

TYPE: 2 HISTOGRAM

Q.1. Draw a histogram to represent the following data. A garment exporting firm made the following amount of exports.

Year	Exports in crores of rupees
1997- 1998	6
1998- 1999	8
1999- 2000	10
2000- 2001	9
2001- 2002	12

Q.2. The following histogram represents the number of people of different age groups in a club. Find the following:



(a) How many are senior citizens(above 60)?

(b) Which age group shows the highest frequency?

(c) How many are below 50 years?

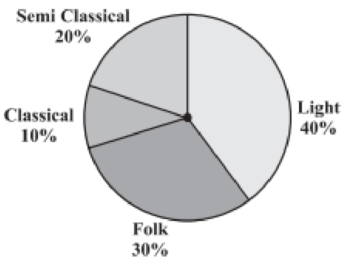
TYPE: 3 PIE CHART

Q.1 Jivan's report card states his marks as

Subject	Marks
English	60
Hindi	54
Maths	86
Science	54
Social studies	46
Total	300

Make a Pie chart exhibiting his marks in various subjects.

Q.2 A survey was made to find the type of music that a certain group of young people liked in a city. The given pie chart shows the findings of the survey. Look at the pie chart and answer the following questions:



(a) Which type of music is liked by maximum number of people?

(b) If 20 people liked classical music, how many young people were surveyed?

(c) If a cassette company were to make 1000 CD's, how many of each type would they make?

TYPE: 4 PROBABILITY

Q.1 When a die is thrown, list the outcomes of an event getting:

(a) a prime number.

(b) not a prime number.

(c) a number greater than 5.

Q.2 State the sample space and find the probability of drawing a slip labelled as 5 from a bag containing four slips labelled as 3, 5, 8 and 9.

Exercise 6.1 : Solutions of Questions on Page Number : 96

Q1 :

What will be the unit digit of the squares of the following numbers?

(i) 81 (ii) 272

(iii) 799 (iv) 3853

(v) 1234 (vi) 26387

(vii) 52698 (viii) 99880

(ix) 12796 (x) 55555

Answer :

We know that if a number has its unit's place digit as *a*, then its square will end with the unit digit of the multiplication *a* x *a*.

(i) 81

Since the given number has its unit's place digit as 1, its square will end with the unit digit of the multiplication (1 x 1 = 1) i.e., 1.

(ii) 272

Since the given number has its unit's place digit as 2, its square will end with the unit digit of the multiplication (2 x 2 = 4) i.e., 4.

(iii) 799

Since the given number has its unit's place digit as 9, its square will end with the unit digit of the multiplication (9 x 9 = 81) i.e., 1.

(iv) 3853

Since the given number has its unit's place digit as 3, its square will end with the unit digit of the multiplication (3 x 3 = 9) i.e., 9.

(v) 1234
Since the given number has its unit's place digit as 4, its square will end with the unit digit of the multiplication (4 x 4 = 16) i.e., 6.

(vi) 26387

Since the given number has its unit's place digit as 7, its square will end with the unit digit of the multiplication (7 x 7 = 49) i.e., 9.

(vii) 52698

Since the given number has its unit's place digit as 8, its square will end with the unit digit of the multiplication (8 x 8 = 64) i.e., 4. (viii) 99880

Since the given number has its unit's place digit as 0, its square will have two zeroes at the end. Therefore, the unit digit of the square of the given number is 0.

(xi) 12796

Since the given number has its unit's place digit as 6, its square will end with the unit digit of the multiplication (6 x 6 = 36) i.e., 6.

(x) 55555

Since the given number has its unit's place digit as 5, its square will end with the unit digit of the multiplication (5 x 5 = 25) i.e., 5.

Q2 :

The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453

(iii) 7928 (iv) 222222

(v) 64000 (vi) 89722

(vii) 222000 (viii) 505050

Answer :
The square of numbers may end with any one of the digits 0, 1, 5, 6, or 9. Also, a perfect square has even number of zeroes at the end of it.

(i) 1057 has its unit place digit as 7. Therefore, it cannot be a perfect square.

(ii) 23453 has its unit place digit as 3. Therefore, it cannot be a perfect square.

(iii) 7928 has its unit place digit as 8. Therefore, it cannot be a perfect square.

(iv) 222222 has its unit place digit as 2. Therefore, it cannot be a perfect square.

(v) 64000 has three zeros at the end of it. However, since a perfect square cannot end with odd number of zeroes, it is not a perfect square.

(vi) 89722 has its unit place digit as 2. Therefore, it cannot be a perfect square.

(vii) 222000 has three zeroes at the end of it. However, since a perfect square cannot end with odd number of zeroes, it is not a perfect square.

(viii) 505050 has one zero at the end of it. However, since a perfect square cannot end with odd number of zeroes, it is not a perfect square.

Q3 :

The squares of which of the following would be odd numbers?

(i) 431 (ii) 2826

(iii) 7779 (iv) 82004

Answer :

The square of an odd number is odd and the square of an even number is even. Here, 431 and 7779 are odd numbers.

Thus, the square of 431 and 7779 will be an odd number.

Q4 :

Observe the following pattern and find the missing digits.
$11^2 = 121$

$101^2 = 10201$

$1001^2 = 1002001$

$100001^2 = 1…2…1$

$10000001^2 = …$

Answer :

In the given pattern, it can be observed that the squares of the given numbers have the same number of zeroes before and after the digit 2 as it was in the original number. Therefore,

$100001^2 = 10000200001$

$10000001^2 = 100000020000001$

Q5 :

Observe the following pattern and supply the missing number.

$11^2 = 121$

$101^2 = 10201$

$10101^2 = 102030201$

$1010101^2 = …$

$…^2 = 10203040504030201$

Answer :

By following the given pattern, we obtain

$1010101^2 = 1020304030201$

$101010101^2 = 10203040504030201$

Q6 :

Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + \_^2 = 21^2$

$5^2 + \_^2 + 30^2 = 31^2$

$6^2 + 7^2 + \_^2 = \_\_^2$

Answer :

From the given pattern, it can be observed that,

(i) The third number is the product of the first two numbers.

(ii) The fourth number can be obtained by adding 1 to the third number.

Thus, the missing numbers in the pattern will be as follows.

$4^2 + 5^2 + \underline{20^2} = 21^2$

$5^2 + \underline{6^2} + 30^2 = 31^2$

$6^2 + 7^2 + \underline{42^2} = \underline{43^2}$

Q7 :

Without adding find the sum

(i) $1 + 3 + 5 + 7 + 9$
(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23$

Answer :

We know that the sum of first n odd natural numbers is n^2.

(i) Here, we have to find the sum of first five odd natural numbers.

Therefore, $1 + 3 + 5 + 7 + 9 = (5)^2 = 25$

(ii) Here, we have to find the sum of first ten odd natural numbers.

Therefore, $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = (10)^2 = 100$

(iii) Here, we have to find the sum of first twelve odd natural numbers.

Therefore, $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 +17 + 19 + 21 + 23 = (12)^2 = 144$

Q8 :

(i) Express 49 as the sum of 7 odd numbers.

(ii) Express 121 as the sum of 11odd numbers.

Answer :

We know that the sum of first n odd natural numbers is n^2.

(i) $49 = (7)^2$

Therefore, 49 is the sum of first 7 odd natural numbers.

$49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$

(ii) $121 = (11)^2$

Therefore, 121 is the sum of first 11 odd natural numbers.
$121 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

Q9 :

How many numbers lie between squares of the following numbers?

(i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

Answer :

We know that there will be $2n$ numbers in between the squares of the numbers n and $(n + 1)$.

(i) Between 12^2 and 13^2, there will be 2 x 12 = 24 numbers

(ii) Between 25^2 and 26^2, there will be 2 x 25 = 50 numbers

(iii) Between 99^2 and 100^2, there will be 2 x 99 = 198 numbers

Exercise 6.2 : Solutions of Questions on Page Number : 98

Q1 :

Find the square of the following numbers

(i) 32 (ii) 35

(iii) 86 (iv) 93

(v) 71 (vi) 46

Answer :

(i) $32^2 = (30 + 2)^2$

$= 30 (30 + 2) + 2 (30 + 2)$

$= 30^2 + 30 \times 2 + 2 \times 30 + 2^2$

$= 900 + 60 + 60 + 4$
$= 1024$

(ii) The number 35 has 5 in its unit's place. Therefore,

$35^2 = (3) (3 + 1)$ hundreds + 25

= (3 x 4) hundreds + 25

$= 1200 + 25 = 1225$

(iii) $86^2 = (80 + 6)^2$

$= 80 (80 + 6) + 6 (80 + 6)$

$= 80^2 + 80 \times 6 + 6 \times 80 + 6^2$

$= 6400 + 480 + 480 + 36$

$= 7396$

(iv) $93^2 = (90 + 3)^2$

$= 90 (90 + 3) + 3 (90 + 3)$

$= 90^2 + 90 \times 3 + 3 \times 90 + 3^2$

$= 8100 + 270 + 270 + 9$

$= 8649$

(v) $71^2 = (70 + 1)^2$

$= 70 (70 + 1) + 1 (70 + 1)$

$= 70^2 + 70 \times 1 + 1 \times 70 + 1^2$

$= 4900 + 70 + 70 + 1$

$= 5041$

(vi) $46^2 = (40 + 6)^2$

$= 40 (40 + 6) + 6 (40 + 6)$

$= 40^2 + 40 \times 6 + 6 \times 40 + 6^2$
$= 1600 + 240 + 240 + 36$

$= 2116$

Q2 :

Write a Pythagorean triplet whose one member is

(i) 6 (ii) 14

(iii) 16 (iv) 18

Answer :

For any natural number $m > 1$, $2m$, $m^2 - 1$, $m^2 + 1$ forms a Pythagorean triplet.

(i) If we take $m^2 + 1 = 6$, then $m^2 = 5$

The value of m will not be an integer.

If we take $m^2 - 1 = 6$, then $m^2 = 7$

Again the value of m is not an integer.

Let $2m = 6$ $m = 3$

Therefore, the Pythagorean triplets are 2×3, $3^2 - 1$, $3^2 + 1$ or 6, 8, and 10.

(ii) If we take $m^2 + 1 = 14$, then $m^2 = 13$

The value of m will not be an integer.

If we take $m^2 - 1 = 14$, then $m^2 = 15$

Again the value of m is not an integer.

Let $2m = 14$ $m = 7$

Thus, $m^2 - 1 = 49 - 1 = 48$ and $m^2 + 1 = 49 + 1 = 50$

Therefore, the required triplet is 14, 48, and 50.

(iii) If we take $m^2 + 1 = 16$, then $m^2 = 15$

The value of m will not be an integer.

If we take $m^2 - 1= 16$, then $m^2 = 17$

Again the value of m is not an integer.

Let $2m = 16$ $m = 8$

Thus, $m^2 - 1 = 64 - 1 = 63$ and $m^2 + 1 = 64 + 1 = 65$

Therefore, the Pythagorean triplet is 16, 63, and 65.

(iv) If we take $m^2 + 1 = 18$, $m^2 = 17$

The value of m will not be an integer.

If we take m^2 - 1 = 18, then m^2 = 19

Again the value of m is not an integer.

Let 2m =18 m = 9

Thus, m^2 - 1 = 81 - 1 = 80 and m^2 + 1 = 81 + 1 = 82

Therefore, the Pythagorean triplet is 18, 80, and 82.

Exercise 6.3 : Solutions of Questions on Page Number : 102

Q1 :

What could be the possible 'one's' digits of the square root of each of the following numbers?
(i) 9801 (ii) 99856

(iii) 998001 (iv) 657666025

Answer :

(i) If the number ends with 1, then the one's digit of the square root of that number may be 1 or 9. Therefore, one's digit of the square root of 9801 is either 1 or 9.

(ii) If the number ends with 6, then the one's digit of the square root of that number may be 4 or 6. Therefore, one's digit of the square root of 99856 is either 4 or 6.

(iii) If the number ends with 1, then the one's digit of the square root of that number may be 1 or 9. Therefore, one's digit of the square root of 998001 is either 1 or 9.

(iv) If the number ends with 5, then the one's digit of the square root of that number will be 5. Therefore, the one's digit of the square root of 657666025 is 5.

Q2 :

Without doing any calculation, find the numbers which are surely not perfect squares.

(i) 153 (ii) 257

(iii) 408 (iv) 441

Answer :

The perfect squares of a number can end with any of the digits 0, 1, 4, 5, 6, or 9 at unit's place. Also, a perfect square will end with even number of zeroes, if any.

(i) Since the number 153 has its unit's place digit as 3, it is not a perfect square.

(ii) Since the number 257 has its unit's place digit as 7, it is not a perfect square.

(iii) Since the number 408 has its unit's place digit as 8, it is not a perfect square.

(iv) Since the number 441 has its unit's place digit as 1, it is a perfect square.

Q3 :

Find the square roots of 100 and 169 by the method of repeated subtraction.

Answer :

We know that the sum of the first n odd natural numbers is n^2.

Consider $\sqrt{100}$.

(i) 100 - 1 = 99 (ii) 99 - 3 = 96 (iii) 96 - 5 = 91

(iv) 91 - 7 = 84 (v) 84 - 9 = 75 (vi) 75 - 11= 64

(vii) 64 - 13 = 51 (viii) 51 - 15 = 36 (ix) 36 - 17 = 19

(x) 19 - 19 = 0

We have subtracted successive odd numbers starting from 1 to 100, and obtained 0 at 10^{th} step.

Therefore, $\sqrt{100} = 10$

The square root of 169 can be obtained by the method of repeated subtraction as follows.

(i) 169 - 1 = 168 (ii) 168 - 3 = 165 (iii) 165 - 5 = 160

(iv) 160 - 7 = 153 (v) 153 - 9 = 144 (vi) 144 - 11 = 133

(vii) 133 - 13 = 120 (viii) 120 - 15 = 105 (ix) 105 - 17 = 88

(x) 88 - 19 = 69 (xi) 69 - 21 = 48 (xii) 48 - 23 = 25

(xiii)25 - 25 = 0

We have subtracted successive odd numbers starting from 1 to 169, and obtained 0 at 13th step.

Therefore, $\sqrt{169} = 13$

Q4 :
Find the square roots of the following numbers by the Prime Factorisation Method.

(i) 729 (ii) 400

(iii) 1764 (iv) 4096

(v) 7744 (vi) 9604

(vii) 5929 (viii) 9216

(ix) 529 (x) 8100

Answer :

(i) 729 can be factorised as follows.

3	729
3	243
3	81
3	27
3	9
3	3
	1

729 = $\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

∴ $\sqrt{729} = 3 \times 3 \times 3$ = 27

(ii) 400 can be factorised as follows.

2	400
2	200
2	100
2	50
5	25
5	5
	1

400 = $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

∴ $\sqrt{400} = 2 \times 2 \times 5$ = 20

(iii) 1764 can be factorised as follows.

2	1764
2	882
3	441
3	147
7	49
7	7
	1

1764 = 2 × 2 × 3 × 3 × 7 × 7

∴ $\sqrt{1764} = 2 \times 3 \times 7$ = 42

(iv) 4096 can be factorised as follows.

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16

2	8
2	4
2	2
	1

4096 = 2 Ãƒ

Q5 :

For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252 (ii) 180

(iii) 1008 (iv) 2028

(v) 1458 (vi) 768

Answer :

(i)252 can be factorised as follows.

2	252
2	126
3	63
3	21
7	7
	1

$252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

Here, prime factor 7 does not have its pair.

If 7 gets a pair, then the number will become a perfect square. Therefore, 252 has to be multiplied with 7 to obtain a perfect square.

$252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

Therefore, 252 × 7 = 1764 is a perfect square.

$\therefore \sqrt{1764} = 2 \times 3 \times 7 = 42$

(ii)180 can be factorised as follows.

2	180
2	90
3	45
3	15
5	5
	1

$180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

Here, prime factor 5 does not have its pair. If 5 gets a pair, then the number will become a perfect square. Therefore, 180 has to be multiplied with 5 to obtain a perfect square.

$180 \times 5 = 900 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}$

Therefore, 180 × 5 = 900 is a perfect square.

$\therefore \sqrt{900} = 2 \times 3 \times 5 = 30$

(iii)1008 can be factorised as follows.

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

1008 = $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

Here, prime factor 7 does not have its pair. If 7 gets a pair, then the number will become a perfect square. Therefore, 1008 can be multiplied with 7 to obtain a perfect square.

1008 × 7 = 7056 = $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

Therefore, 1008 × 7 = 7056 is a perfect square.

∴ $\sqrt{7056} = 2 \times 2 \times 3 \times 7$ = 84

(iv) 2028 can be factorised as follows.

2	2028
2	1014

Q6 :

For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252 (ii) 2925

(iii) 396 (iv) 2645

(v) 2800 (vi) 1620

Answer :

(i) 252 can be factorised as follows.

2	252
2	126
3	63
3	21
7	7
	1

$252 = \underline{2\times2}\times\underline{3\times3}\times7$

Here, prime factor 7 does not have its pair.

If we divide this number by 7, then the number will become a perfect square. Therefore, 252 has to be divided by 7 to obtain a perfect square.

252 ÃƒÂ·7 = 36 is a perfect square.

$$36 = \underline{2\times2}\times\underline{3\times3}$$
$$\therefore \sqrt{36} = 2\times3 = 6$$

(ii) 2925 can be factorised as follows.

3	2925
3	975
5	325
5	65
13	13
	1

$$2925 = \underline{3 \times 3} \times \underline{5 \times 5} \times 13$$

Here, prime factor 13 does not have its pair.

If we divide this number by 13, then the number will become a perfect square. Therefore, 2925 has to be divided by 13 to obtain a perfect square.

2925 ÃƒÂ·13 = 225 is a perfect square.

$$225 = \underline{3 \times 3} \times \underline{5 \times 5}$$
$$\therefore \sqrt{225} = 3 \times 5 = 15$$

(iii)396 can be factorised as follows.

2	396
2	198
3	99
3	33
11	11
	1

$396 = \underline{2 \times 2} \times \underline{3 \times 3} \times 11$

Here, prime factor 11 does not have its pair.

If we divide this number by 11, then the number will become a perfect square. Therefore, 396 has to be divided by 11 to obtain a perfect square.
396 ÃƒÂ·11 = 36 is a perfect square.

$36 = \underline{2 \times 2} \times \underline{3 \times 3}$

$\therefore \sqrt{36} = 2 \times 3 = 6$

(iv) 2645 can be factorised as follows.

5	2645

Q7 :

The students of Class VIII of a school donated Rs 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Answer :

It is given that each student donated as many rupees as the number of students of the class. Number of students in the class will be the square root of the amount donated by the students of the class.

The total amount of donation is Rs 2401.

Number of students in the class = $\sqrt{2401}$

$2401 = \underline{7 \times 7} \times \underline{7 \times 7}$

$\therefore \sqrt{2401} = 7 \times 7 = 49$

Hence, the number of students in the class is 49.

Q8 :

2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Answer :

It is given that in the garden, each row contains as many plants as the number of rows.

Hence,

Number of rows = Number of plants in each row

Total number of plants = Number of rows × Number of plants in each row

Number of rows × Number of plants in each row = 2025

(Number of rows)^2 = 2025

$\text{Number of rows} = \sqrt{2025}$

$2025 = \underline{5\times5}\times\underline{3\times3}\times\underline{3\times3}$

$\therefore \sqrt{2025} = 5\times3\times3 = 45$

Thus, the number of rows and the number of plants in each row is 45.

Q9 :

Find the smallest square number that is divisible by each of the numbers 4, 9, and 10.

Answer :

The number that will be perfectly divisible by each one of 4, 9, and 10 is their LCM. The LCM of these numbers is as follows.

2	4, 9, 10
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

LCM of 4, 9, 10 = $\underline{2 \times 2} \times \underline{3 \times 3} \times 5 = 180$

Here, prime factor 5 does not have its pair. Therefore, 180 is not a perfect square. If we multiply 180 with 5, then the number will become a perfect square. Therefore, 180 should be multiplied with 5 to obtain a perfect square.

Hence, the required square number is 180 × 5 = 900

Q10 :

Find the smallest square number that is divisible by each of the numbers 8, 15,

and 20.

Answer :

The number that is perfectly divisible by each of the numbers 8, 15, and 20 is their LCM.

2	8, 15, 20
2	4, 15, 10
2	2, 15, 5

3	1, 15, 5
5	1, 5, 5
	1, 1, 1

LCM of 8, 15, and 20 = $\underline{2 \times 2} \times 2 \times 3 \times 5 = 120$

Here, prime factors 2, 3, and 5 do not have their respective pairs. Therefore, 120 is not a perfect square.
Therefore, 120 should be multiplied by 2 × 3 × 5, i.e. 30, to obtain a perfect square.

Hence, the required square number is 120 × 2 × 3 × 5 = 3600

Exercise 6.4 : Solutions of Questions on Page Number : 107

Q1 :

Find the square root of each of the following numbers by division method.

(i) 2304 (ii) 4489

(iii) 3481 (iv) 529

(v) 3249 (vi) 1369

(vii) 5776 (viii) 7921

(ix) 576 (x) 1024

(xi) 3136 (xii) 900

Answer :

(i) The square root of 2304 can be calculated as follows.

	48
4	$\overline{23}\,\overline{04}$ -16
88	704 704
	0

$\therefore \sqrt{2304} = 48$

(ii) The square root of 4489 can be calculated as follows.

	67
6	$\overline{44}\,\overline{89}$ -36
127	889 889
	0

$\therefore \sqrt{4489} = 67$

(iii) The square root of 3481 can be calculated as follows.

	59
5	$\overline{34}\,\overline{81}$ -25

109	981 981
	0

Therefore, $\sqrt{3481} = 59$

(iv) The square root of 529 can be calculated as follows.

	23
2	$\overline{5}\,\overline{29}$ -4
43	129 129
	0

$\therefore \sqrt{529} = 23$

(v) The square root of 3249 can be calculated as follows.

	57
5	$\overline{32}\,\overline{49}$ -25
107	749 749
	0

∴

Q2 :

Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64 (ii) 144

(iii) 4489 (iv) 27225

(v) 390625

Answer :

(i) By placing bars, we obtain

$$64 = \overline{64}$$

Since there is only one bar, the square root of 64 will have only one digit in it.
(ii) By placing bars, we obtain

$$144 = \overline{1}\,\overline{44}$$

Since there are two bars, the square root of 144 will have 2 digits in it.

(iii) By placing bars, we obtain

$$4489 = \overline{44}\,\overline{89}$$

Since there are two bars, the square root of 4489 will have 2 digits in it.

(iv) By placing bars, we obtain

$$27225 = \overline{2}\,\overline{72}\,\overline{25}$$

Since there are three bars, the square root of 27225 will have three digits in it.

(v) By placing the bars, we obtain

$390625 = \overline{39}\,\overline{06}\,\overline{25}$

Since there are three bars, the square root of 390625 will have 3 digits in it.

Q3 :

Find the square root of the following decimal numbers.

(i) 2.56 (ii) 7.29

(iii) 51.84 (iv) 42.25

(v) 31.36

Answer :

(i) The square root of 2.56 can be calculated as follows.

	1. 6
1	$\overline{2}.\overline{56}$ -1
26	156 156
	0

$\therefore \sqrt{2.56} = 1.6$

(ii) The square root of 7.29 can be calculated as follows.

	2. 7
2	$\overline{7}.\overline{29}$ -4
47	329 329
	0

$\therefore \sqrt{7.29} = 2.7$

(iii) The square root of 51.84 can be calculated as follows.

	7.2
7	$\overline{51}.\overline{84}$ -49
142	284 284
	0

$\therefore \sqrt{51.84} = 7.2$

(iv) The square root of 42.25 can be calculated as follows.

	6.5
6	$\overline{42}.\overline{25}$ -36

125	625 625
	0

$\therefore \sqrt{42.25} = 6.5$

(v) The square root of 31.36 can be calculated as follows.

	5.6
5	$\overline{31}.\overline{36}$ -25
106	636 636
	0

$\therefore$

Q4 :

Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained. (i) 402 (ii) 1989
(iii) 3250 (iv) 825

(v) 4000

Answer :

(i) The square root of 402 can be calculated by long division method as follows.

	20
2	$\overline{4}\,\overline{02}$ -4
40	02 00
	2

The remainder is 2. It represents that the square of 20 is less than 402 by 2. Therefore, a perfect square will be obtained by subtracting 2 from the given number 402.

Therefore, required perfect square = 402 - 2 = 400

And, $\sqrt{400} = 20$

(ii) The square root of 1989 can be calculated by long division method as follows.

	44
4	$\overline{19}\,\overline{89}$ -16
84	389 336
	53

The remainder is 53. It represents that the square of 44 is less than 1989 by 53. Therefore, a perfect square will be obtained by subtracting 53 from the given number 1989.

Therefore, required perfect square = 1989 - 53 = 1936

And, $\sqrt{1936} = 44$

(iii) The square root of 3250 can be calculated by long division method as follows.

	57
5	$\overline{32}\,\overline{50}$ -25
107	750 749
	1

The remainder is 1. It represents that the square of 57 is less than 3250 by 1. Therefore, a perfect square can be obtained by subtracting 1 from the given number 3250.

Therefore, required perfect square = 3250 - 1 = 3249

And, $\sqrt{3249} = 57$

(iv) The square root of 825 can be calculated by long division method as follows.

	28
2	$\overline{8}\,\overline{25}$ -4

48	425 384
	41

The remainder is 41. It represents that the square of 28 is less than 825 by 41. Therefore, a perfect square can be calculated by subtracting

Q5 :

Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525 (ii) 1750

(iii) 252 (iv) 1825

(v) 6412

Answer :

(i) The square root of 525 can be calculated by long division method as follows.

	22
2	$\overline{5}\,\overline{25}$ -4
42	125 84

	41

The remainder is 41.

It represents that the square of 22 is less than 525.

Next number is 23 and $23^2 = 529$

Hence, number to be added to 525 = 23^2 - 525 = 529 - 525 = 4

The required perfect square is 529 and $\sqrt{529} = 23$

(ii) The square root of 1750 can be calculated by long division method as follows.

	41
4	$\overline{17}\,\overline{50}$ -16
81	150 81
	69

The remainder is 69.

It represents that the square of 41 is less than 1750.

The next number is 42 and $42^2 = 1764$

Hence, number to be added to 1750 = 42^2 - 1750 = 1764 - 1750 = 14

The required perfect square is 1764 and $\sqrt{1764} = 42$

(iii) The square root of 252 can be calculated by long division method as follows.

	15
1	$\overline{2}\,\overline{52}$ -1
25	152 125
	27

The remainder is 27. It represents that the square of 15 is less than 252.

The next number is 16 and $16^2 = 256$

Hence, number to be added to 252 = 16^2 - 252 = 256 - 252 = 4

The required perfect square is 256 and $\sqrt{256} = 16$

(iv) The square root of 1825 can be calculated by long division method as follows.

	42
4	$\overline{18}\,\overline{25}$ -16
82	225 164
	61

The remainder is 61. It represents that the square of 42

Q6 :

Find the length of the side of a square whose area is 441 m^2.

Answer :

Let the length of the side of the square be x m.

Area of square = $(x)^2 = 441\ m^2$

$x = \sqrt{441}$

The square root of 441 can be calculated as follows.

	21
2	$\overline{4}\,\overline{41}$ -4
41	041 41
	0

$\therefore x = 21$ m

Hence, the length of the side of the square is 21 m.

Q7 :

In a right triangle ABC, ∠B = 90°.

(a) If AB = 6 cm, BC = 8 cm, find AC

(b) If AC = 13 cm, BC = 5 cm, find AB

Answer :

(a) ΔABC is right-angled at B.

Therefore, by applying Pythagoras theorem, we obtain

$AC^2 = AB^2 + BC^2$

$AC^2 = (6\text{ cm})^2 + (8\text{ cm})^2$

$AC^2 = (36 + 64)\text{ cm}^2 = 100\text{ cm}^2$

AC= $\left(\sqrt{100}\right)\text{ cm} = \left(\sqrt{10\times10}\right)\text{ cm}$

AC = 10 cm

(b) ΔABC is right-angled at B.

Therefore, by applying Pythagoras theorem, we obtain

$AC^2 = AB^2 + BC^2$
$(13\text{ cm})^2 = (AB)^2 + (5\text{ cm})^2$

$AB^2 = (13\text{ cm})^2 - (5\text{ cm})^2 = (169 - 25)\text{ cm}^2 = 144\text{ cm}^2$

AB= $\left(\sqrt{144}\right)\text{ cm} = \left(\sqrt{12\times12}\right)\text{ cm}$

AB = 12 cm

Q8 :

A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

Answer :

It is given that the gardener has 1000 plants. The number of rows and the number of columns is the same.

We have to find the number of more plants that should be there, so that when the gardener plants them, the number of rows and columns are same.

That is, the number which should be added to 1000 to make it a perfect square has to be calculated.

The square root of 1000 can be calculated by long division method as follows.

	31
3	$\overline{10}\,\overline{00}$ -9
61	100 61
	39

The remainder is 39. It represents that the square of 31 is less than 1000.

The next number is 32 and $32^2 = 1024$

Hence, number to be added to 1000 to make it a perfect square

$= 32^2 - 1000 = 1024 - 1000 = 24$

Thus, the required number of plants is 24.

Q9 :

These are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement?

Answer :

It is given that there are 500 children in the school. They have to stand for a P.T. drill such that the number of rows is equal to the number of columns.

The number of children who will be left out in this arrangement has to be calculated. That is, the number which should be subtracted from 500 to make it a perfect square has to be calculated.

The square root of 500 can be calculated by long division method as follows.

	22
2	$\overline{5}\,\overline{00}$ -4
42	100 84
	16

The remainder is 16.
It shows that the square of 22 is less than 500 by 16. Therefore, if we subtract 16 from 500, we will obtain a perfect square.

Required perfect square = 500 - 16 = 484

Thus, the number of children who will be left out is 16.

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER 6: SQUARES AND SQUARE ROOTS

MIND MAP

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE:1 SQUARE NUMBERS AND PROPERTIES OF SQUARE NUMBERS

Q.1. Find the square of the given number.

Q.2. Express 121 as the sum of 11 odd numbers.

Q.3. Find the number of non- square numbers between $6^2$and 7^2.

Q.4. Find the Pythagorean Triplet whose one member is 16.

TYPE: 2 FINDING SQUARE ROOT

Q.1. Find square root of 5629 by the **prime factorisation** method.

Q.2. Find the square root of 2304 by **long division** method.

Q.3. Find the square root of 12.25 by **long division** method.

Q.4. Find the smallest number by which 1008 should be **multiplied** so as to get a perfect square. Also find the square root of the square number so obtained.

Q.5. Find the smallest number by which 2645 should be **divided** so as to get a perfect square. Also find the square root of the square number so obtained.

Q.6. Find the least number which must be **subtracted** from 3520 so as to get a perfect square. Also find the square root of the square number so obtained.

Q.7. Find the least number which must be **added** to 1750 so as to get a perfect square. Also find the square root of the square number so obtained.

TYPE: 3 APPLICATION OF SQUARE AND SQUARE ROOT IN REAL LIFE SITUATIONS

Q.1. There are 2401 students in a school. P.T teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

Q.2. Area of a square plot is 2304 m^2. Find the side of the square plot.

CHAPTER - 7 CUBES AND CUBE ROOTS

Exercise 7.1

1. Which of the following numbers are not perfect cubes?

(i) 216

Solution:

By resolving 216 into prime factor,

2	216
2	108
2	54
3	27
3	9
3	3
	1

216 = 2×2×2×3×3×3

By grouping the factors in triplets of equal factors, 216 = (2×2×2)×(3×3×3)

Here, 216 can be grouped into triplets of equal factors,

∴ 216 = (2×3) = 6

Hence, 216 is cube of 6.

(ii) 128

Solution:

By resolving 128 into prime factor,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

128 = 2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 128 = (2×2×2)×(2×2×2)×2

Here, 128 cannot be grouped into triplets of equal factors, we are left of with one factors 2 .

∴ 128 is not a perfect cube.

(iii) 1000

Solution:

By resolving 1000 into prime factor,

2	1000
2	500
2	250
5	125
5	25
5	5
	1

1000 = 2×2×2×5×5×5

By grouping the factors in triplets of equal factors, 1000 = (2×2×2)×(5×5×5)

Here, 1000 can be grouped into triplets of equal factors,

∴ 1000 = (2×5) = 10

Hence, 1000 is cube of 10.

(iv) 100

Solution:

By resolving 100 into prime factor,

2	100
2	50
5	25
5	5
	1

100 = 2×2×5×5

Here, 100 cannot be grouped into triplets of equal factors.

∴ 100 is not a perfect cube.

(v) 46656

Solution:

By resolving 46656 into prime factor,

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

46656 = 2×2×2×2×2×2×3×3×3×3×3×3

By grouping the factors in triplets of equal factors, 46656 = (2×2×2)×(2×2×2)×(3×3×3)×(3×3×3)

Here, 46656 can be grouped into triplets of equal factors,

∴ 46656 = (2×2×3×3) = 36

Hence, 46656 is cube of 36.

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243

Solution:

By resolving 243 into prime factor,

3	243
3	81
3	27
3	9
3	3
	1

243 = 3×3×3×3×3

By grouping the factors in triplets of equal factors, 243 = (3×3×3)×3×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will multiply 243 by 3 to get perfect cube.

(ii) 256

Solution:

By resolving 256 into prime factor,

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

256 = 2×2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 256 = (2×2×2)×(2×2×2)×2×2

Here, 2 cannot be grouped into triplets of equal factors.

∴ We will multiply 256 by 2 to get perfect cube.

(iii) 72

Solution:

By resolving 72 into prime factor,

2	72
2	36
2	18
3	9
3	3
	1

72 = 2×2×2×3×3

By grouping the factors in triplets of equal factors, 72 = (2×2×2)×3×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will multiply 72 by 3 to get perfect cube.

(iv) 675

Solution:

By resolving 675 into prime factor,

3	675
3	225
3	75
5	25
5	5
	1

675 = 3×3×3×5×5

By grouping the factors in triplets of equal factors, 675 = (3×3×3)×5×5

Here, 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 675 by 5 to get perfect cube.

(v) 100

Solution:

By resolving 100 into prime factor,

2	100
2	50
5	25
5	5
	1

100 = 2×2×5×5

Here, 2 and 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 100 by (2×5) 10 to get perfect cube.

3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) 81

Solution:

By resolving 81 into prime factor,

3	81
3	27
3	9
3	3
	1

81 = 3×3×3×3

By grouping the factors in triplets of equal factors, 81 = (3×3×3)×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will divide 81 by 3 to get perfect cube.

(ii) 128

Solution:

By resolving 128 into prime factor,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

128 = 2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 128 = (2×2×2)×(2×2×2)×2

Here, 2 cannot be grouped into triplets of equal factors.

∴ We will divide 128 by 2 to get perfect cube.

(iii) 135

Solution:

By resolving 135 into prime factor,

3	135
3	45
3	15
5	5
	1

135 = 3×3×3×5

By grouping the factors in triplets of equal factors, 135 = (3×3×3)×5

Here, 5 cannot be grouped into triplets of equal factors.

∴ We will divide 135 by 5 to get perfect cube.

(iv) 192

Solution:

By resolving 192 into prime factor,

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

192 = 2×2×2×2×2×2×3

By grouping the factors in triplets of equal factors, 192 = (2×2×2)×(2×2×2)×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will divide 192 by 3 to get perfect cube.

(v) 704

Solution:

By resolving 704 into prime factor,

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

704 = 2×2×2×2×2×2×11

By grouping the factors in triplets of equal factors, 704 = (2×2×2)×(2×2×2)×11

Here, 11 cannot be grouped into triplets of equal factors.

∴ We will divide 704 by 11 to get perfect cube.

4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Solution:

Given, side of cube is 5 cm, 2 cm and 5 cm.

∴ Volume of cube = 5×2×5 = 50

2	50
5	25
5	5
	1

50 = 2×5×5

Here, 2 , 5 and 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 50 by (2×2×5) 20 to get perfect cube. Hence, 20 cuboid is needed.

Exercise 7.2

1. Find the cube root of each of the following numbers by prime factorisation method.

(i) 64

Solution:

64 = 2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 64 = (2×2×2)×(2×2×2)

Here, 64 can be grouped into triplets of equal factors,

∴ 64 = 2×2 = 4

Hence, 4 is cube root of 64.

(ii) 512

Solution:

512 = 2×2×2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 512 = (2×2×2)×(2×2×2)×(2×2×2)

Here, 512 can be grouped into triplets of equal factors,

∴ 512 = 2×2×2 = 8

Hence, 8 is cube root of 512.

(iii) 10648

Solution:

10648 = 2×2×2×11×11×11

By grouping the factors in triplets of equal factors, 10648 = (2×2×2)×(11×11×11)

Here, 10648 can be grouped into triplets of equal factors,

∴ 10648 = 2 ×11 = 22

Hence, 22 is cube root of 10648.

(iv) 27000

Solution:

27000 = 2×2×2×3×3×3×3×5×5×5

By grouping the factors in triplets of equal factors, 27000 = (2×2×2)×(3×3×3)×(5×5×5)

Here, 27000 can be grouped into triplets of equal factors,

∴ 27000 = (2×3×5) = 30

Hence, 30 is cube root of 27000.

(v) 15625

Solution:

15625 = 5×5×5×5×5×5

By grouping the factors in triplets of equal factors, 15625 = (5×5×5)×(5×5×5)

Here, 15625 can be grouped into triplets of equal factors,

∴ 15625 = (5×5) = 25

Hence, 25 is cube root of 15625.

(vi) 13824

Solution:

13824 = 2×2×2×2×2×2×2×2×2×3×3×3

By grouping the factors in triplets of equal factors,

13824 = (2×2×2)×(2×2×2)×(2×2×2)×(3×3×3)

Here, 13824 can be grouped into triplets of equal factors,

∴ 13824 = (2×2× 2×3) = 24

Hence, 24 is cube root of 13824.

(vii) 110592

Solution:

110592 = 2×2×2×2×2×2×2×2×2×2×2×2×3×3×3

By grouping the factors in triplets of equal factors,

110592 = (2×2×2)×(2×2×2)×(2×2×2)×(2×2×2)×(3×3×3)

Here, 110592 can be grouped into triplets of equal factors,

∴ 110592 = (2×2×2×2 × 3) = 48

Hence, 48 is cube root of 110592.

(viii) 46656

Solution:

46656 = 2×2×2×2×2×2×3×3×3×3×3×3

By grouping the factors in triplets of equal factors,

46656 = (2×2×2)×(2×2×2)×(3×3×3)×(3×3×3)

Here, 46656 can be grouped into triplets of equal factors,

∴ 46656 = (2×2×3×3) = 36

Hence, 36 is cube root of 46656.

(ix) 175616

Solution:

175616 = 2×2×2×2×2×2×2×2×2×7×7×7

By grouping the factors in triplets of equal factors,

175616 = (2×2×2)×(2×2×2)×(2×2×2)×(7×7×7)

Here, 175616 can be grouped into triplets of equal factors,

∴ 175616 = (2×2×2×7) = 56

Hence, 56 is cube root of 175616.

(x) 91125

Solution:

91125 = 3×3×3×3×3×3×5×5×5

By grouping the factors in triplets of equal factors, 91125 = (3×3×3)×(3×3×3)×(5×5×5)

Here, 91125 can be grouped into triplets of equal factors,

∴ 91125 = (3×3×5) = 45

Hence, 45 is cube root of 91125.

2. State true or false.

(i) Cube of any odd number is even.

Solution:

False

(ii) A perfect cube does not end with two zeros.

Solution:

True

(iii) If cube of a number ends with 5, then its cube ends with 25.

Solution:

False

(iv) There is no perfect cube which ends with 8.

Solution:

False

(v) The cube of a two digit number may be a three digit number.

Solution:

False

(vi) The cube of a two digit number may have seven or more digits.

Solution:

False

(vii) The cube of a single digit number may be a single digit number.

Solution:

True

3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

Solution:

(i) By grouping the digits, we get 1 and 331

We know that, since, the unit digit of cube is 1, the unit digit of cube root is 1.

∴ We get 1 as unit digit of the cube root of 1331.

The cube of 1 matches with the number of second group.

∴ The ten's digit of our cube root is taken as the unit place of smallest number.

We know that, the unit's digit of the cube of a number having digit as unit's place 1 is 1.

∴ $\sqrt[3]{1331} = 11$

(ii) By grouping the digits, we get 4 and 913

We know that, since, the unit digit of cube is 3, the unit digit of cube root is 7.

∴ we get 7 as unit digit of the cube root of 4913. We know $1^3 = 1$ and $2^3 = 8$, $1 > 4 > 8$.

Thus, 1 is taken as ten digit of cube root.

∴ $\sqrt[3]{4913} = 17$

(iii) By grouping the digits, we get 12 and 167.

We know that, since, the unit digit of cube is 7, the unit digit of cube root is 3.

∴ 3 is the unit digit of the cube root of 12167 We know $2^3 = 8$ and $3^3 = 27$, $8 > 12 > 27$.

Thus, 2 is taken as ten digit of cube root.

∴ $\sqrt[3]{12167}= 23$

(iv) By grouping the digits, we get 32 and 768.

We know that, since, the unit digit of cube is 8, the unit digit of cube root is 2.

∴ 2 is the unit digit of the cube root of 32768. We know $3^3 = 27$ and $4^3 = 64$, $27 > 32 > 64$.

Thus, 3 is taken as ten digit of cube root.

∴ $\sqrt[3]{32768}= 32$

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.: 6 CUBES AND CUBE ROOTS

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE:1 CUBES

Q.1. Check whether the given number is a perfect cube or not.

Q.2. Find the smallest number by which 243 must be **multiplied** to obtain a perfect cube.

Q.3. Find the smallest number by which 135 must be **divided** to obtain a perfect cube.

TYPE: 2 FINDING CUBE ROOT

Q.1. Find the cube root of 13824 by **prime factorisation** method.

Q.2. Find the cube root of 32768 by **estimation** method.

Number	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

Exercise 8.1 : Solutions of Questions on Page Number : 119

Q1 :

Find the ratio of the following:

(a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.

(b) 5 m to 10 km

(c) 50 paise to Rs 5

Answer :

(a) Ratio of the speed of cycle to the speed of scooter $= \frac{15}{30} = 1:2$

(b) Since 1 km = 1000 m,

Required ratio $= \frac{5\text{ m}}{10\text{ km}} = \frac{5\text{ m}}{10 \times 1000\text{ m}} = 1:2000$

(c) Since Re 1 = 100 paise,

Required ratio $= \frac{50\text{ paise}}{\text{Rs }5} = \frac{50\text{ paise}}{500\text{ paise}} = 1:10$

Q2 :

Convert the following ratios to percentages.

(a) 3:4 (b) 2:3

Answer :

(a) $3:4 = \frac{3}{4} = \frac{3}{4} \times \frac{100}{100} = \frac{3}{4} \times 100\% = 75\%$

(b) $2:3=\frac{2}{3}=\frac{2}{3}\times\frac{100}{100}=\frac{2}{3}\times 100\%=\frac{200}{3}\%$

$=\left(\frac{66\times 3+2}{3}\right)\%=66\frac{2}{3}\%$

Q3 :

72% of 25 students are good in mathematics. How many are not good in mathematics?

Answer :

It is given that 72% of 25 students are good in mathematics.

Therefore,

Percentage of students who are not good in mathematics = (100 - 72)%

= 28%

∴Number of students who are not good in mathematics = $\frac{28}{100}\times 25$

= 7

Thus, 7 students are not good in mathematics.

Q4 :

A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?

Answer :

Let the total number of matches played by the team be *x*.

It is given that the team won 10 matches and the winning percentage of the team was 40%. Therefore,

$$\frac{40}{100} \times x = 10$$
$$x = 10 \times \frac{100}{40}$$
$$x = 25$$

Thus, the team played 25 matches.

Q5 :

If Chameli had Rs 600 left after spending 75% of her money, how much did she have in the beginning?

Answer :

Let the amount of money which Chameli had in the beginning be *x*.

It is given that after spending 75% of Rs *x*, she was left with Rs 600.

Therefore,

(100 - 75)% of *x* = Rs 600

Or, 25 % of *x* = Rs 600

$$\frac{25}{100} \times x = \text{Rs } 600$$
$$x = \text{Rs } \left(600 \times \frac{100}{25}\right) = \text{Rs } 2400$$

Thus, she had Rs 2400 in the beginning.

Q6 :

If 60% people in city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people are 50 lakh, find the exact number who like each type of game.

Answer :

Percentage of people who like other games = (100 - 60 - 30)%

= (100 - 90)% = 10 %

Total number of people = 50 lakh

Therefore, number of people $= \left(\frac{60}{100} \times 50\right)$ lakh who like cricket = 30 lakh

Number of people who like football $= \left(\frac{30}{100} \times 50\right)$ lakh = 15 lakh

$= \left(\frac{10}{100} \times 50\right)$ lakh

Number of people who like other games = 5 lakh

Exercise 8.2 : Solutions of Questions on Page Number : 125

Q1 :

A man got a 10% increase in his salary. If his new salary is Rs 1,54,000, find his original salary.

Answer :

Let the original salary be *x*. It is given that the new salary is Rs 1,54,000.

Original salary + Increment = New salary

However, it is given that the increment is 10% of the original salary.

Therefore,

$$x + \frac{10}{100} \times x = 154000$$
$$\frac{110x}{100} = 154000$$
$$x = \left(154000 \times \frac{100}{110}\right)$$
$$x = 140000$$

Thus, the original salary was Rs 1,40,000.

Q2 :

On Sunday 845 people went to the Zoo. On Monday only 169 people went. What is the per cent decrease in the people visiting the zoo on Monday?

Answer :

It is given that on Sunday, 845 people went to the zoo and on Monday, 169 people went.

Decrease in the number of people = 845 - 169 = 676

Percentage decrease = $\left(\frac{\text{Decrease in the number of people } \times 100}{\text{Number of people who went to zoo on sunday}}\right)\%$

$$= \left(\frac{676}{845} \times 100\right)\%$$
$$= 80\%$$

Q3 :

A shopkeeper buys 80 articles for Rs 2,400 and sells them for a profit of 16%. Find the selling price of one article.

Answer :

It is given that the shopkeeper buys 80 articles for Rs 2,400.

Cost of one article = $\text{Rs } \frac{2400}{80} = \text{Rs } 30$

Profit percent = 16

$$\text{Profit Percent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$16 = \frac{\text{Profit}}{\text{Rs } 30} \times 100$$

$$\text{Profit} = \text{Rs } \left(\frac{16 \times 30}{100}\right) = \text{Rs } 4.80$$

Selling price of one article = C.P. + Profit = Rs (30 + 4.80) = Rs 34.80

Q4 :

The cost of an article was Rs 15,500. Rs 450 were spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article.

Answer :

Total cost of an article = Cost + Overhead expenses

= Rs 15500 + Rs 450

= Rs 15950

$$\text{Profit } \% = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$15 = \frac{\text{Profit}}{\text{Rs } 15950} \times 100$$

$$\text{Profit} = \text{Rs } \left(\frac{15950 \times 15}{100}\right) = \text{Rs } 2392.50$$

∴Selling price of the article = C.P. + Profit = Rs (15950 + 2392.50)

= Rs 18342.50

Q5 :

A VCR and TV were bought for Rs 8,000 each. The shopkeeper made a loss of 4% on the VCR and a profit of 8% on the TV. Find the gain or loss percent on the whole transaction.

Answer :

C.P. of a VCR = Rs 8000

The shopkeeper made a loss of 4 % on VCR.

This means if C.P. is Rs 100, then S.P. is Rs 96.

When C.P. is Rs 8000, S.P. = $\text{Rs}\left(\frac{96}{100}\times 8000\right)$ = Rs 7680

C.P. of a TV = Rs 8000

The shopkeeper made a profit of 8 % on TV.

This means that if C.P. is Rs 100, then S.P. is Rs 108.

When C.P. is Rs 8000, S.P. = $\text{Rs}\left(\frac{108}{100}\times 8000\right)$ = Rs 8640

Total S.P. = Rs 7680 + Rs 8640 = Rs 16320

Total C.P. = Rs 8000 + Rs 8000 = Rs 16000

Since total S.P.> total C.P., there was a profit.

Profit = Rs 16320 - Rs 16000 = Rs 320

$$\text{Profit }\% = \frac{\text{Profit}}{\text{C.P.}} \times 100$$
$$= \frac{320}{16000} \times 100 = 2\%$$

Therefore, the shopkeeper had a gain of 2% on the whole transaction.

Q6 :

During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at Rs 1450 and two shirts marked at Rs 850 each?

Answer :

Total marked price = Rs (1,450 + 2 × 850) = Rs (1,450 +1,700) = Rs 3,150

Given that, discount % = 10%

Discount = $\text{Rs}\left(\frac{10}{100} \times 3150\right) = \text{Rs } 315$

Also, Discount = Marked price - Sale price

Rs 315 = Rs 3150 - Sale price

∴ Sale price = Rs (3150 - 315) = Rs 2835

Thus, the customer will have to pay Rs 2,835.

Q7 :

A milkman sold two of his buffaloes for Rs 20,000 each. On one he made a gain of 5% and on the other a loss of 10%. Find his overall gain or loss.

(Hint: Find CP of each)

Answer :

S.P. of each buffalo = Rs 20000

The milkman made a gain of 5% while selling one buffalo.

This means if C.P. is Rs 100, then S.P. is Rs 105.

C.P. of one buffalo = $\text{Rs}\left(20000\times\frac{100}{105}\right)$ = Rs 19,047.62

Also, the second buffalo was sold at a loss of 10%.

This means if C.P. is Rs 100, then S.P. is Rs 90.

∴C.P. of other buffalo = $\text{Rs}\left(20000\times\frac{100}{90}\right)$ = Rs 22222.22

Total C.P. = Rs 19047.62 + Rs 22222.22 = Rs 41269.84

Total S.P. = Rs 20000 + Rs 20000 = Rs 40000

Loss = Rs 41269.84 - Rs 40000 = Rs 1269.84

Thus, the overall loss of milkman was Rs 1,269.84.

Q8 :

The price of a TV is Rs 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it,

Answer :

On Rs 100, the tax to be paid = Rs 12

On Rs 13000, the tax to be paid will be $= \text{Rs}\left(\frac{12}{100}\times 13000\right)$

= Rs 1560

Required amount = Cost + Sales Tax = Rs 13000 + Rs 1560

= Rs 14560

Thus, Vinod will have to pay Rs 14,560 for the T.V.

Q9 :

Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is Rs 1,600, find the marked price.

Answer :

Let the marked price be *x*.

$$\text{Discount percent} = \frac{\text{Discount}}{\text{Marked price}} \times 100$$
$$20 = \frac{\text{Discount}}{x} \times 100$$
$$\text{Discount} = \frac{20}{100} \times x = \frac{1}{5}x$$

Also,

Discount = Marked price - Sale price

$$\frac{1}{5}x = x - \text{Rs } 1600$$
$$x - \frac{1}{5}x = \text{Rs } 1600$$
$$\frac{4}{5}x = \text{Rs } 1600$$
$$x = \text{Rs } \left(1600 \times \frac{5}{4}\right) = \text{Rs } 2000$$

Thus, the marked price was Rs 2000.

Q10 :

I purchased a hair-dryer for Rs 5,400 including 8% VAT. Find the price before VAT was added.

Answer :

The price includes VAT.
Thus, 8% VAT means that if the price without VAT is Rs 100, then price including VAT will be Rs 108.

When price including VAT is Rs 108, original price = Rs 100

$$\text{When price including VAT is Rs 5400, original price } = \text{Rs}\left(\frac{100}{108} \times 5400\right)$$
$$= \text{Rs } 5000$$

Thus, the price of the hair-dryer before the addition of VAT was Rs 5,000.

Q11 :

I purchased a hair-dryer for Rs 5,400 including 8% VAT. Find the price before VAT was added.

Answer :

The price includes VAT.

Thus, 8% VAT means that if the price without VAT is Rs 100, then price including VAT will be Rs 108.

When price including VAT is Rs 108, original price = Rs 100

$$\text{When price including VAT is Rs 5400, original price } = \text{Rs}\left(\frac{100}{108} \times 5400\right)$$
$$= \text{Rs } 5000$$

Thus, the price of the hair-dryer before the addition of VAT was Rs 5,000.

Exercise 8.3 : Solutions of Questions on Page Number : 133

Q1 :

Calculate the amount and compound interest on

(a) Rs 10800 for 3 years at $12\frac{1}{2}\%$ per annum compounded annually.
(b) Rs 18000 $2\frac{1}{2}$ for years at 10% per annum compounded annually.

(c) Rs 62500 $1\frac{1}{2}$ for years at 8% per annum compounded half yearly. (d) Rs 8000 for 1 year at 9% per annum compound half yearly. (You could use the year by year calculation using SI formula to verify)

(e) Rs 10000 for 1 year at 8% per annum compounded half yearly.

Answer :

(a) Principal (P) = Rs 10, 800

Rate (R) = $12\frac{1}{2}\%$ = $\frac{25}{2}$ % (annual)

Number of years (n) = 3

Amount, A = $P\left(1+\frac{R}{100}\right)^n$

$$= \text{Rs}\left[10800\left(1+\frac{25}{200}\right)^3\right]$$
$$= \text{Rs}\left[10800\left(\frac{225}{200}\right)^3\right]$$
$$= \text{Rs}\left(10800\times\frac{225}{200}\times\frac{225}{200}\times\frac{225}{200}\right)$$
$$= \text{Rs } 15377.34375$$
$$= \text{Rs } 15377.34 \qquad \text{(approximately)}$$

C.I. = A - P = Rs (15377.34 - 10800) = Rs 4,577.34

(b) Principal (P) = Rs 18,000

Rate (R) = 10% annual

Number of years (n) = $2\frac{1}{2}$ years

The amount for 2 years and 6 months can be calculated by first calculating the amount for 2 years using the compound interest formula, and then calculating the simple interest for 6 months on the amount obtained at the end of 2 years.

Firstly, the amount for 2 years has to be calculated.

$$A = \text{Rs}\left[18000\left(1+\frac{1}{10}\right)^2\right] = \text{Rs}\left(18000\times\frac{11}{10}\times\frac{11}{10}\right) = \text{Rs } 21780$$

By taking Rs 21780 as principal, the S.I.for the next $\frac{1}{2}$ year will be calculated.

$$\text{S.I.} = \text{Rs}\left(\frac{21780\times\frac{1}{2}\times 10}{100}\right) = \text{Rs } 1089$$

∴ Interest for the first 2 years = Rs (21780 - 18000) = Rs 3780

And interest for the next $\frac{1}{2}$ year = Rs 1089

∴ Total C.I. = Rs 3780 + Rs 1089 = Rs 4,869

A = P + C.I. = Rs 18000 + Rs 4869 = Rs 22,869

(c) Principal (P) = Rs 62,500

Rate = 8% per annum or 4% per half year

Number of years = $1\frac{1}{2}$

There will be 3 half years in $1\frac{1}{2}$ years.

$$A = P\left(1+\frac{R}{100}\right)^n = \text{Rs}\left[62500\left(1+\frac{4}{100}\right)^3\right]$$
$$= \text{Rs}\left(62500\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\right)$$
$$= \text{Rs } 70304$$

C.I. = A - P = Rs 70304 - Rs 62500 = Rs 7,804

(d) Principal (P) = Rs 8000

Rate of interest = 9% per annum or $\frac{9}{2}$ % per half year

Number of years = 1 year

There will be 2 half years in 1 year.

$$A = P\left(1+\frac{R}{100}\right)^n$$
$$= \text{Rs}\left[8000\left(1+\frac{9}{200}\right)^2\right]$$

$$= \text{Rs}\left[8000\left(\frac{209}{200}\right)^2\right] = \text{Rs } 8,736.20$$

C.I. = A - P = Rs 8736.20 - Rs 8000 = Rs 736.20

(e) Principal (P) = Rs 10,000

Rate = 8% per annum or 4% per half year

Number of years = 1 year

There are 2 half years in 1 year.

$$A = P\left(1+\frac{R}{100}\right)^n$$
$$= \text{Rs}\left[10000\left(1+\frac{4}{100}\right)^2\right] = \text{Rs}\left[10000\left(1+\frac{1}{25}\right)^2\right]$$
$$= \text{Rs}\left(10000\times\frac{26}{25}\times\frac{26}{25}\right) = \text{Rs } 10,816$$

C.I. = A - P = Rs 10816 - Rs 10000 = Rs 816

Q2 :

Kamala borrowed Rs 26400 from a Bank to buy a scooter at a rate of 15% p.a. compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan?

(Hint: Find A for 2 years with interest is compounded yearly and then find SI on the 2nd year

amount for $\frac{4}{12}$ years.)

Answer :

Principal (P) = Rs 26,400

Rate (R) = 15% per annum

Number of years (n) = $2\frac{4}{12}$ years

The amount for 2 years and 4 months can be calculated by first calculating the amount for 2 years using the compound interest formula, and then calculating the simple interest for 4 months on the amount obtained at the end of 2 years.

Firstly, the amount for 2 years has to be calculated.

$$A = \text{Rs}\left[26400\left(1+\frac{15}{100}\right)^2\right] = \text{Rs}\left[26400\left(1+\frac{3}{20}\right)^2\right]$$
$$= \text{Rs}\left(26400\times\frac{23}{20}\times\frac{23}{20}\right) = \text{Rs } 34,914$$

By taking Rs 34,914 as principal, the S.I. for the next $\frac{1}{3}$ years will be calculated.

$$\text{S.I.} = \text{Rs}\left(\frac{34914\times\frac{1}{3}\times 15}{100}\right) = \text{Rs } 1,745.70$$

Interest for the first two years = Rs (34914 - 26400) = Rs 8,514

And interest for the next $\frac{1}{3}$ year = Rs 1,745.70

Total C.I. = Rs (8514 + Rs 1745.70) = Rs 10,259.70

Amount = P + C.I. = Rs 26400 + Rs 10259.70 = Rs 36,659.70

Q3 :

Fabina borrows Rs 12,500 at 12% per annum for 3 years at simple interest and Radha borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much?

Answer :

Interest paid by Fabina = $\frac{P \times R \times T}{100}$

$$= \text{Rs}\left(\frac{12500 \times 12 \times 3}{100}\right) = \text{Rs } 4,500$$

Amount paid by Radha at the end of 3 years = A = $P\left(1+\frac{R}{100}\right)^n$

$$A = \text{Rs}\left[12500\left(1+\frac{10}{100}\right)^3\right]$$

$$= \text{Rs}\left(12500 \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100}\right) = \text{Rs } 16,637.50$$

C.I. = A - P = Rs 16637.50 - Rs 12500 = Rs 4,137.50

The interest paid by Fabina is Rs 4,500 and by Radha is Rs 4,137.50.

Thus, Fabina pays more interest.

Rs 4500 - Rs 4137.50 = Rs 362.50

Hence, Fabina will have to pay Rs 362.50 more.

Q4 :

I borrowed Rs 12000 from Jamshed at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compound interest, what extra amount would I have to pay?

Answer :

P = Rs 12000

R = 6% per annum

T = 2 years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \text{Rs}\left(\frac{12000 \times 6 \times 2}{100}\right) = \text{Rs } 1,440$$

To find the compound interest, the amount (A) has to be calculated.

$$A = P\left(1+\frac{R}{100}\right)^n = \text{Rs}\left[12000\left(1+\frac{6}{100}\right)^2\right]$$
$$= \text{Rs}\left[12000\left(1+\frac{3}{50}\right)^2\right] = \text{Rs}\left(12000 \times \frac{53}{50} \times \frac{53}{50}\right)$$
$$= \text{Rs } 13,483.20$$

∴ C.I. = A - P = Rs 13483.20 - Rs 12000 = Rs 1,483.20

C.I. - S.I. = Rs 1,483.20 - Rs 1,440 = Rs 43.20

Thus, the extra amount to be paid is Rs 43.20.

Q5 :

Vasudevan invested Rs 60000 at an interest rate of 12% per annum compounded half yearly. What amount would he get

(i) after 6 months?

(ii) after 1 year?

Answer :

(i) P = Rs 60,000

Rate = 12% per annum = 6% per half year

n = 6 months = 1 half year

$$A = P\left(1+\frac{R}{100}\right)^{n}$$
$$= \text{Rs}\left[60000\left(1+\frac{6}{100}\right)^{1}\right] = \text{Rs}\left(60000\times\frac{106}{100}\right) = \text{Rs } 63,600$$

(ii) There are 2 half years in 1

year. n = 2

$$A = \text{Rs}\left[60000\left(1+\frac{6}{100}\right)^{2}\right] = \text{Rs}\left(60000\times\frac{106}{100}\times\frac{106}{100}\right) = \text{Rs } 67,416$$

Q6 :

Arif took a loan of Rs 80,000 from a bank. If the rate of interest is 10% per annum, find the

difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is

(i) Compounded annually
(ii) Compounded half yearly

Answer :

(i) P = Rs 80,000

R = 10% per annum

$n = 1\frac{1}{2}$ years

The amount for 1 year and 6 months can be calculated by first calculating the amount for 1 year using the compound interest formula, and then calculating the simple interest for 6 months on the amount obtained at the end of 1 year.

Firstly, the amount for 1 year has to be calculated.

$$A = \text{Rs}\left[80000\left(1+\frac{10}{100}\right)^1\right]$$
$$= \text{Rs}\left[80000\left(1+\frac{1}{10}\right)\right] = \text{Rs}\left(80000\times\frac{11}{10}\right) = \text{Rs } 88,000$$

By taking Rs 88,000 as principal, the SI for the next $\frac{1}{2}$ year will be calculated.

$$\text{S.I.} = \frac{P\times R\times T}{100} = \text{Rs}\left(\frac{88000\times 10\times \frac{1}{2}}{100}\right) = \text{Rs } 4,400$$

Interest for the first year = Rs 88000 - Rs 80000 = Rs 8,000

And interest for the next $\frac{1}{2}$ year = Rs 4,400

Total C.I. = Rs 8000 + Rs 4,400 = Rs 1,2400

A = P + C.I. = Rs (80000 + 12400) = Rs 92,400

(ii) The interest is compounded half yearly.

Rate = 10% per annum = 5% per half year

There will be three half years in $1\frac{1}{2}$ years.

$$A = \text{Rs}\left[80000\left(1+\frac{5}{100}\right)^3\right] = \text{Rs}\left[80000\left(1+\frac{1}{20}\right)^3\right]$$
$$= \text{Rs}\left(80000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs } 92,610$$

Difference between the amounts = Rs 92,610 - Rs 92,400 = Rs 210

Q7 :

Maria invested Rs 8,000 in a business. She would be paid interest at 5% per annum compounded annually. Find.

(i) The amount credited against her name at the end of the second year

(ii) The interest for the 3rd year.

Answer :

(i) P = Rs 8,000

R = 5% per annum

n = 2 years

$$A = \text{Rs}\left[8000\left(1+\frac{5}{100}\right)^2\right] = \text{Rs}\left(8000\left(1+\frac{1}{20}\right)^2\right)$$
$$= \text{Rs}\left(8000\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs } 8,820$$

(ii) The interest for the next one year, i.e. the third year, has to be calculated.

By taking Rs 8,820 as principal, the S.I. for the next year will be calculated.

$$\text{S.I.} = \text{Rs}\left(\frac{8820\times5\times1}{100}\right) = \text{Rs } 441$$

Q8 :

Find the amount and the compound interest on Rs 10,000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly. Would this interest be more than the interest he would get if it was compounded annually?

Answer :

P = Rs 10,000

Rate = 10% per annum = 5% per half year

n = $1\frac{1}{2}$ years

There will be 3 half years in $1\frac{1}{2}$ years.

$$\text{A} = \text{Rs}\left[10000\left(1+\frac{5}{100}\right)^3\right] = \text{Rs}\left[10000\left(1+\frac{1}{20}\right)^3\right]$$
$$= \text{Rs}\left(10000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs } 11,576.25$$

C.I. = A - P

= Rs 11576.25 - Rs 10000 = Rs 1,576.25

The amount for 1 year and 6 months can be calculated by first calculating the amount for 1 year using the compound interest formula, and then calculating the simple interest for 6 months on the amount obtained at the end of 1 year.

The amount for the first year has to be calculated first.

$$A = \text{Rs}\left[10000\left(1+\frac{10}{100}\right)^1\right] = \text{Rs}\left[10000\left(1+\frac{1}{10}\right)\right]$$
$$= \text{Rs}\left(10000\times\frac{11}{10}\right) = \text{Rs } 11,000$$

By taking Rs 11,000 as the principal, the S.I. for the next $\frac{1}{2}$ year will be calculated.

$$\text{S.I.} = \text{Rs}\left(\frac{11000\times 10\times\frac{1}{2}}{100}\right) = \text{Rs } 550$$

∴ Interest for the first year = Rs 11000 - Rs 10000 = Rs 1,000

∴ Total compound interest = Rs 1000 + Rs 550 = Rs 1,550

Therefore, the interest would be more when compounded half yearly than the interest when compounded annually.

Q9 :

Find the amount which Ram will get on Rs 4,096, he gave it for 18 months at $12\frac{1}{2}\%$ per annum, interest being compounded half yearly.

Answer :

P = Rs 4,096

R = $12\frac{1}{2}\%$ per annum = $\frac{25}{4}\%$ per half year

n = 18 months

There will be 3 half years in 18 months.

Therefore,

$$A = \text{Rs}\left[4096\left(1+\frac{25}{400}\right)^3\right] = \text{Rs}\left[4096\left(1+\frac{1}{16}\right)^3\right]$$

$$= \text{Rs}\left(4096\times\frac{17}{16}\times\frac{17}{16}\times\frac{17}{16}\right) = \text{Rs } 4,913$$

Thus, the required amount is Rs 4,913.

Q10 :

The population of a place increased to 54000 in 2003 at a rate of 5% per annum

(i) find the population in 2001

(ii) what would be its population in 2005?

Answer :

(i) It is given that, population in the year 2003 = 54,000

Therefore,

54000 = $\left(1+\frac{5}{100}\right)^2$ (Population in 2001)

Population in 2001 $= 54000\times\frac{20}{21}\times\frac{20}{21}$ = 48979.59

Thus, the population in the year 2001 was approximately 48,980.

(ii) Population in 2005 = $54000\left(1+\frac{5}{100}\right)^2$

$$= 54000\left(1+\frac{1}{20}\right)^2 = 54000\times\frac{21}{20}\times\frac{21}{20} = 59,535$$

Thus, the population in the year 2005 would be 59,535.

Q11 :

In a laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5,06,000.

Answer :

The initial count of bacteria is given as 5,06,000.

Bacteria at the end of 2 hours = $506000\left(1+\frac{2.5}{100}\right)^2$

$$= 506000\left(1+\frac{1}{40}\right)^2 = 506000 \times \frac{41}{40} \times \frac{41}{40}$$
$$= 531616.25 = 5,31,616 \text{ (approx.)}$$

Thus, the count of bacteria at the end of 2 hours will be 5,31,616 (approx.).

Q12 :

A scooter was bought at Rs 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.

Answer :

Principal = Cost price of the scooter = Rs 42,000

Depreciation = 8% of Rs 42,000 per year

$$= \text{Rs}\left(\frac{42000 \times 8 \times 1}{100}\right)$$
$$= \text{Rs } 3,360$$

Value after 1 year = Rs 42000 - Rs 3360 = Rs 38,640

DELHI PUBLIC SCHOOL – GANDHINAGAR

CHAPTER 8 COMPARING QUANTITIES

MIND MAP

This chapter consists of five different topics. The most probable questions from examination point of view are given below.

TYPE:1 PERCENTAGE INCREASE AND DECREASE

Q.1. Shiv went to a restaurant and ordered for a pizza. When he saw the bill, he was surprised to notice that it was ` 330 which was 10% more than of last time. What was the price of the same pizza when he came last time?

Q.2. A computer costs ` 39600. In the last 6 months, the price had come down by 12%.What was the price of the computer six months back?

TYPE: 2 COST PRICE , SELLING PRICE, PROFIT & LOSS

Q.1. Find:

CP	SP	PROFIT	PROFIT%	LOSS	LOSS%
`3650	`2920	-	-	`	`
`4892	`4900	`		-	-
`784	`682			`	`
`9684		-	-	` 684	
	`7894			` 306	

Q.2. A coat was sold by a shopkeeper at a gain of 5%.If it had been sold for `1650 less, he would have suffered a loss of 5%. Find the cost price.

Q.3. By selling a bicycle for ` 819, Vinay loses 9%. At what price should he sell it to make a profit of 5%?

TYPE: 3 DISCOUNT , TAX

Q.1. Sheeba bought an air cooler for ` 3300 including Tax of 10%. Find the price of the air cooler before tax was added.

Q.2. The marked price of an article is ` 80. If it is sold at ` 72, What is the discount percentage?

Q.3. A television set was sold for `5760 after giving successive discounts of 10% and 20%, respectively. What was the marked price?

Q.4. A trader marks his goods 40% above the cost price. He sells them at a discount of 20%. What is his loss or gain percentage?

TYPE: 4 SIMPLE INTEREST & COMPOUND INTEREST

Q.1. Farhan deposits ` 6000 in a bank at the rate of 6% for 3 years at simple interest. Find the amount he will get back at the end of this period?

Q.2. A sum of money invested at a certain rate doubles itself in 10 years. How much time will it take to triple itself at the same rate?

Q.3. Find the difference between CI and SI on `15000 at 12% p.a for 3 years compounded annually.

Q.4. Find the amount and CI on `24000 compounded semi – annually for $1\frac{1}{2}$ years at the rate of 10% p.a.

TYPE: 5 APPLICATIONS OF COMPOUNDED INTEREST FORMULA

Q.1. The population of a city was 20,000 in the year 1997. It increased at a rate of 5% p.a. Find the population at the end of the year 2000.

Q.2. The population of a village has a constant growth of 5% p.a. If its present population is 33,075 ,What was the population two years ago?

Q.3. The price of a plot increases at a constant rate of 5% every year. Find its expected price after 3 years if the present price is ` 2,00,000.

Q.4. A car which costs ` 2,50,000 depreciates 10% every year. What will the car be worth after 3 years?

Different cases of **construction** which are explained in this chapter are as follows:

- When four **sides** and one **diagonal** are given.
- When two **diagonals** and three **sides** are given.
- When two **adjacent** sides and three **angles** are given.
- When three **sides** and two included **angles** are given.
- When other special **properties** are known.

Chapter 4 – Practical Geometry

Exercise : 4.1

Question 1:

Construct the following quadrilaterals.

(**i) Quadrilateral ABCD**

AB = 4.5 cm

BC = 5.5 cm

CD = 4 cm

AD = 6 cm

AC = 7 cm

(ii) Quadrilateral JUMP

JU = 3.5 cm

UM = 4 cm

MP = 5 cm

PJ = 4.5 cm

PU = 6.5 cm

(iii) Parallelogram MORE

OR = 6 cm

RE = 4.5 cm

EO = 7.5 cm

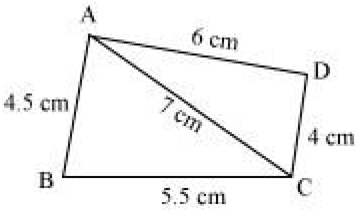
(iv) Rhombus BEST

BE = 4.5 cm

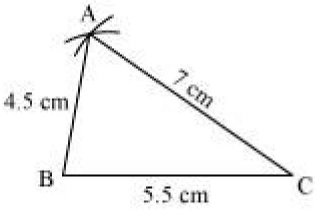
ET = 6 cm

ANSWER:

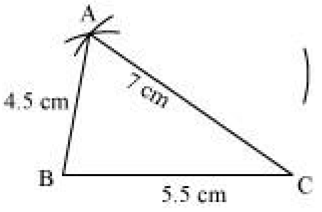
(i) Firstly, a rough sketch of this quadrilateral can be drawn as follows.



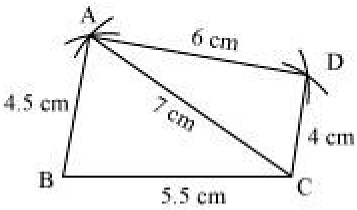
(1) ΔABC can be constructed by using the given measurements as follows.



(2) Vertex D is 6 cm away from vertex A. Therefore, while taking A as centre, draw an arc of radius 6 cm.

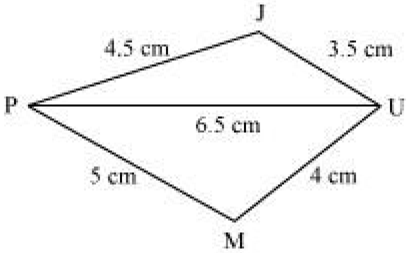


(3) Taking C as centre, draw an arc of radius 4 cm, cutting the previous arc at point D. Join D to A and C.

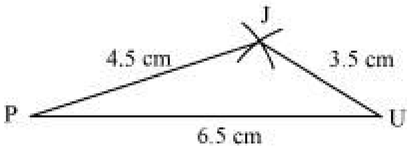


ABCD is the required quadrilateral.

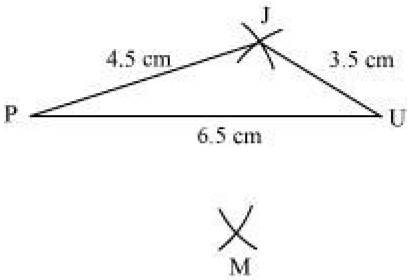
(ii) Firstly, a rough sketch of this quadrilateral can be drawn as follows.



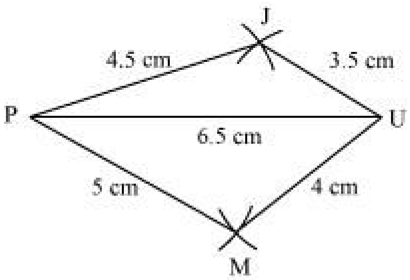
(1) Δ JUP can be constructed by using the given measurements as follows.



(2) Vertex M is 5 cm away from vertex P and 4 cm away from vertex U. Taking P and U as centres, draw arcs of radii 5 cm and 4 cm respectively. Let the point of intersection be M.



(3) Join M to P and U.

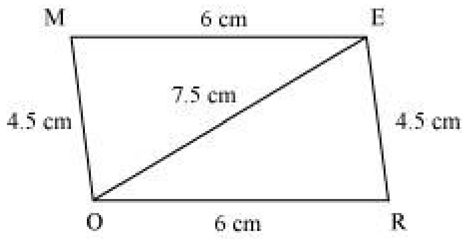


JUMP is the required quadrilateral.

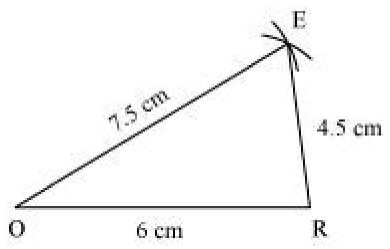
(iii) We know that opposite sides of a parallelogram are equal in length and also these are parallel to each other.

Hence, ME = OR, MO = ER

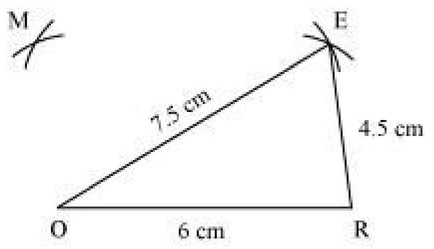
A rough sketch of this parallelogram can be drawn as follows.



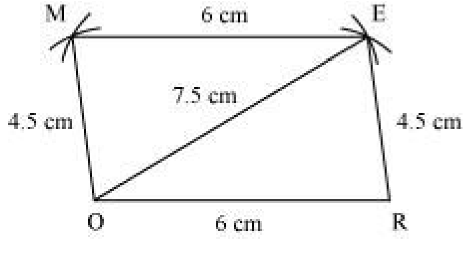
(1) Δ EOR can be constructed by using the given measurements as follows.



(2) Vertex M is 4.5 cm away from vertex O and 6 cm away from vertex E. Therefore, while taking O and E as centres, draw arcs of 4.5 cm radius and 6 cm radius respectively. These will intersect each other at point M.



(3) Join M to O and E.

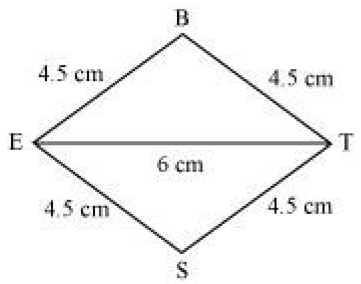


MORE is the required parallelogram.

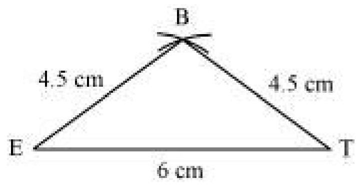
(iv) We know that all sides of a rhombus are of the same measure.

Hence, BE = ES = ST = TB

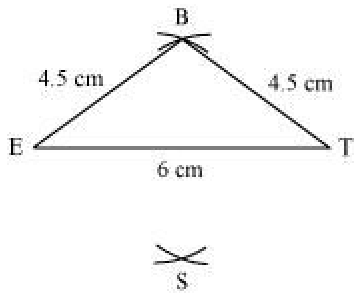
A rough sketch of this rhombus can be drawn as follows.



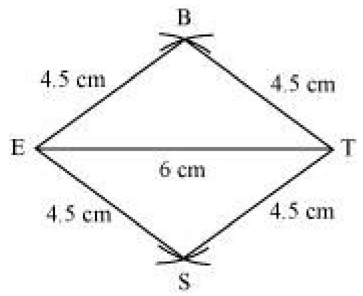
(1) Δ BET can be constructed by using the given measurements as follows.



(2) Vertex S is 4.5 cm away from vertex E and also from vertex T. Therefore, while taking E and T as centres, draw arcs of 4.5 cm radius, which will be intersecting each other at point S.



(3) Join S to E and T.



BEST is the required rhombus.

EXERCISE : 4.2

Question 1:

Construct the following quadrilaterals.

(i) Quadrilateral LIFT

LI = 4 cm

IF = 3 cm

TL = 2.5 cm

LF = 4.5 cm

IT = 4 cm

(ii) Quadrilateral GOLD

OL = 7.5 cm

GL = 6 cm

GD = 6 cm

LD = 5 cm

OD = 10 cm

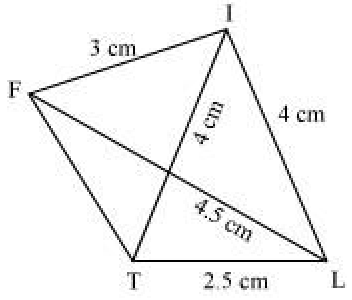
(iii) Rhombus BEND

BN = 5.6 cm

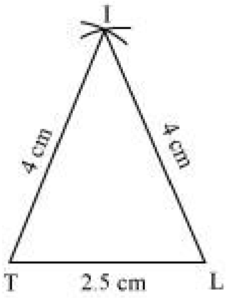
DE = 6.5 cm

ANSWER:

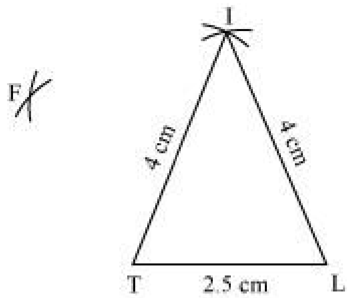
(i) A rough sketch of this quadrilateral can be drawn as follows.



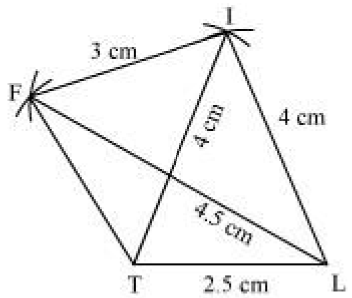
(1) Δ ITL can be constructed by using the given measurements as follows.



(2) Vertex F is 4.5 cm away from vertex L and 3 cm away from vertex I. Therefore, while taking L and I as centres, draw arcs of 4.5 cm radius and 3 cm radius respectively, which will be intersecting each other at point F.

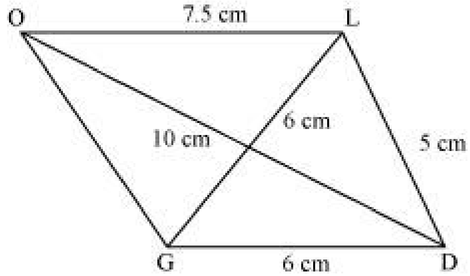


(3) Join F to T and F to I.

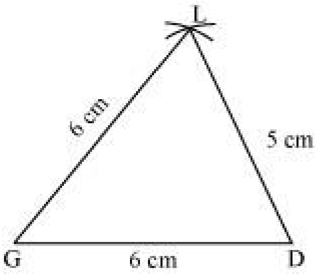


LIFT is the required quadrilateral.

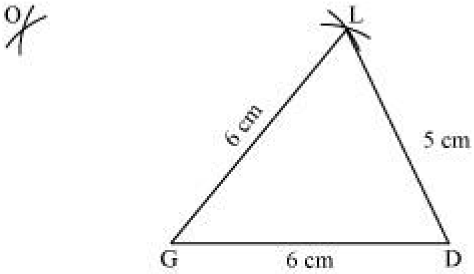
(ii)A rough sketch of this quadrilateral can be drawn as follows.



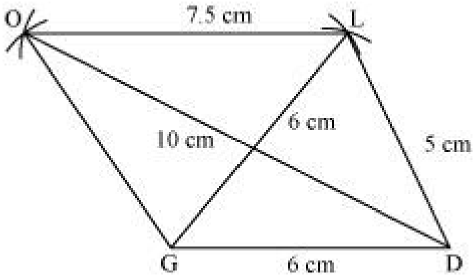
(1) Δ GDL can be constructed by using the given measurements as follows.



(2) Vertex O is 10 cm away from vertex D and 7.5 cm away from vertex L. Therefore, while taking D and L as centres, draw arcs of 10 cm radius and 7.5 cm radius respectively. These will intersect each other at point O.



(3) Join O to G and L.

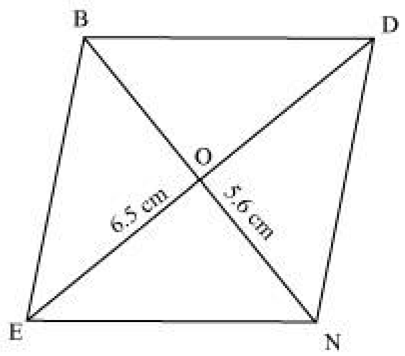


GOLD is the required quadrilateral.

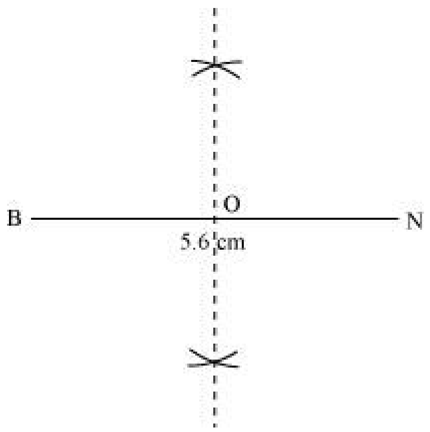
(iii) We know that the diagonals of a rhombus always bisect each other at 90º. Let us assume that these are intersecting each other at point O in this rhombus.

Hence, EO = OD = 3.25 cm

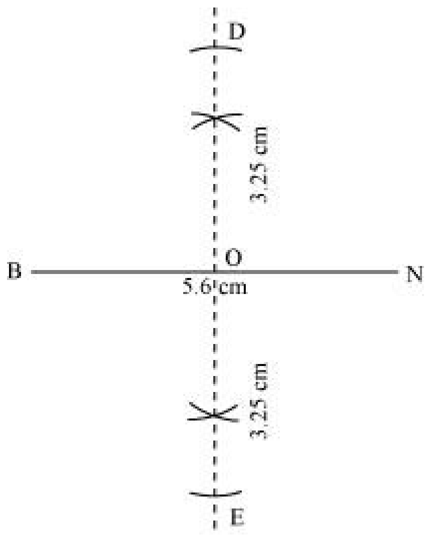
A rough sketch of this rhombus can be drawn as follows.



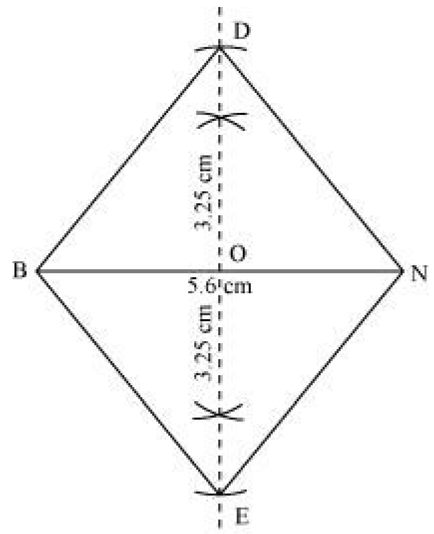
(1) Draw a line segment BN of 5.6 cm and also draw its perpendicular bisector. Let it intersect the line segment BN at point O.



(2) Taking O as centre, draw arcs of 3.25 cm radius to intersect the perpendicular bisector at point D and E.



(3) Join points D and E to points B and N.



BEND is the required quadrilateral.

Exercise : 4.3

Question 1:

Construct the following quadrilaterals.

(i) Quadrilateral MORE

MO = 6 cm

OR = 4.5 cm

∠M = 60°

∠O = 105°

∠R = 105°

(ii) Quadrilateral PLAN

PL = 4 cm

LA = 6.5 cm

∠P = 90°

∠A = 110°

∠N = 85°

(iii) Parallelogram HEAR

HE = 5 cm

EA = 6 cm

∠R = 85°

(iv) Rectangle OKAY

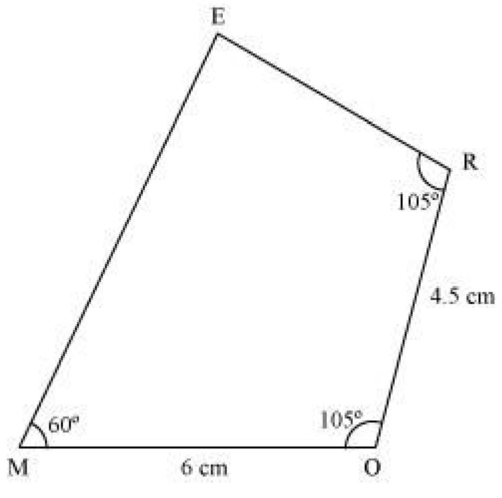
OK = 7 cm

KA = 5 cm

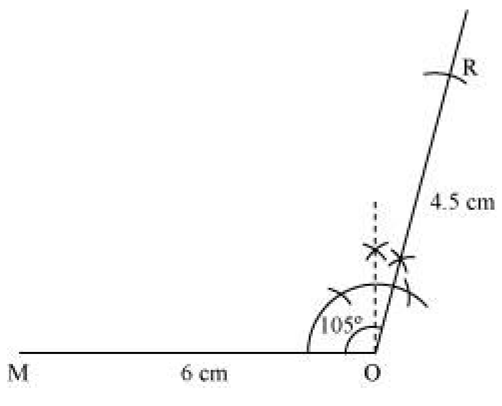
ANSWER:

(i)

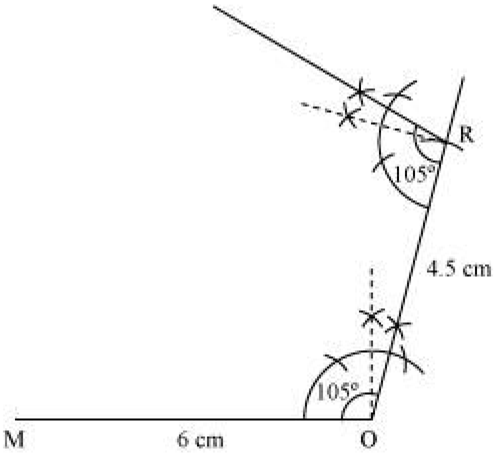
(1)A rough sketch of this quadrilateral can be drawn as follows.



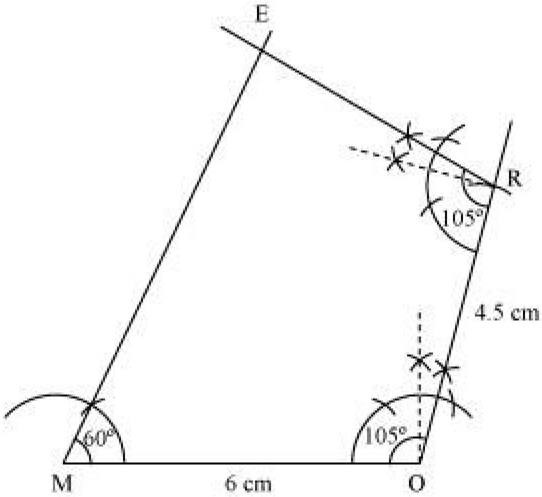
(2) Draw a line segment MO of 6 cm and an angle of 105º at point O. As vertex R is 4.5 cm away from the vertex O, cut a line segment OR of 4.5 cm from this ray.



(3) Again, draw an angle of 105º at point R.



(4) Draw an angle of 60º at point M. Let this ray meet the previously drawn ray from R at point E.



MORE is the required quadrilateral.

(ii)

(1)The sum of the angles of a quadrilateral is 360°.

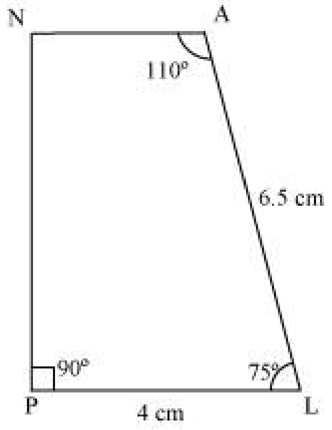
In quadrilateral PLAN, ∠P + ∠L + ∠A + ∠N = 360°

90° + ∠L + 110° + 85° = 360°

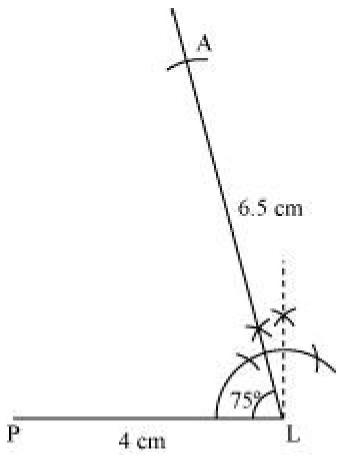
285° + ∠L = 360°

∠L = 360° − 285° = 75°

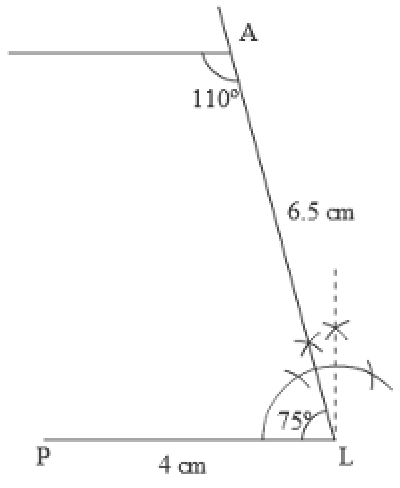
(2)A rough sketch of this quadrilateral is as follows.



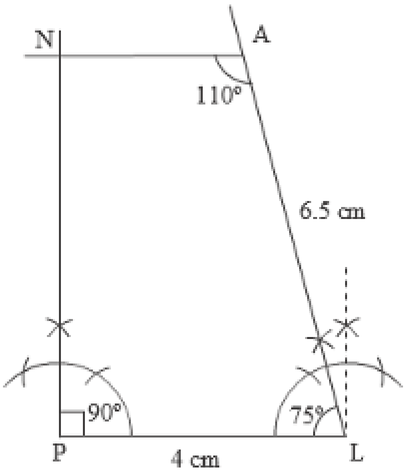
(3) Draw a line segment PL of 4 cm and draw an angle of 75º at point L. As vertex A is 6.5 cm away from vertex L, cut a line segment LA of 6.5 cm from this ray.



(4) Again draw an angle of 110º at point A.



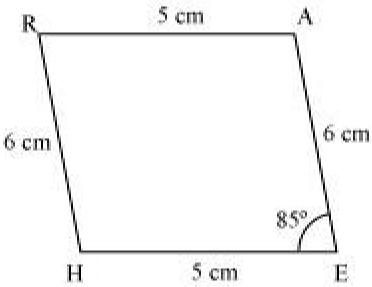
(5) Draw an angle of 90º at point P. This ray will meet the previously drawn ray from A at point N.



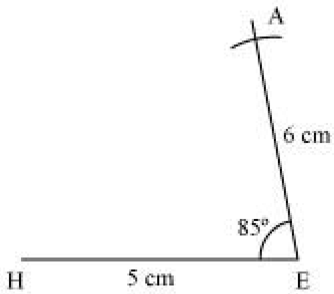
PLAN is the required quadrilateral.

(iii)

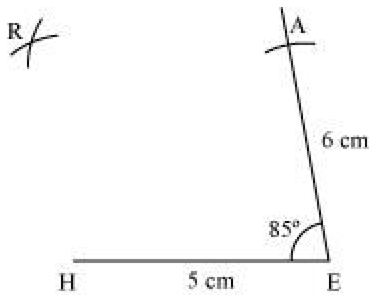
(1)Firstly, a rough sketch of this quadrilateral is as follows.



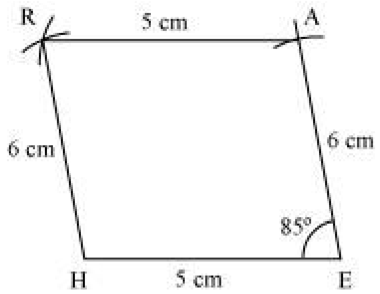
(2) Draw a line segment HE of 5 cm and an angle of 85º at point E. As vertex A is 6 cm away from vertex E, cut a line segment EA of 6 cm from this ray.



(3) Vertex R is 6 cm and 5 cm away from vertex H and A respectively. By taking radius as 6 cm and 5 cm, draw arcs from point H and A respectively. These will be intersecting each other at point R.



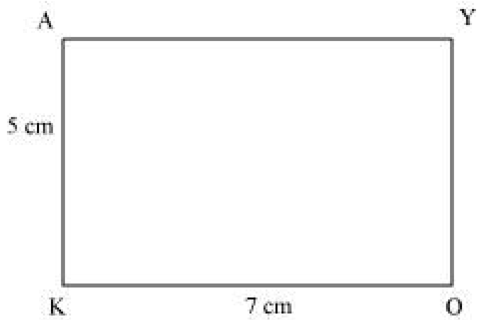
4. Join R to H and A.



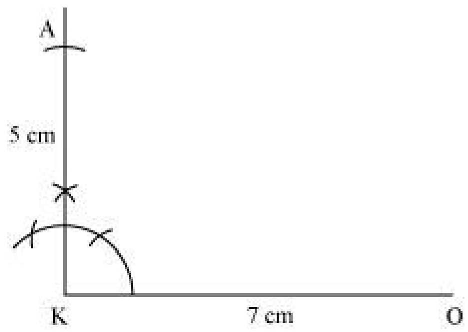
HEAR is the required quadrilateral.

(iv)

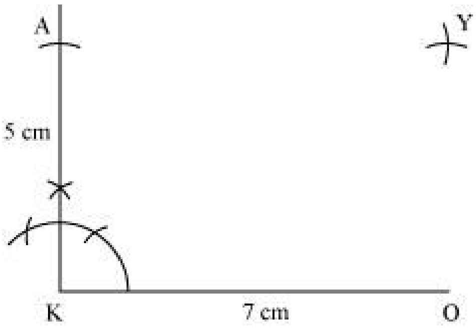
(1)A rough sketch of this quadrilateral is drawn as follows.



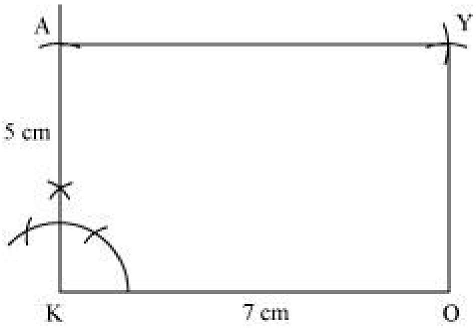
(2) Draw a line segment OK of 7 cm and an angle of 90º at point K. As vertex A is 5 cm away from vertex K, cut a line segment KA of 5 cm from this ray.



(3) Vertex Y is 5 cm and 7 cm away from vertex O and A respectively. By taking radius as 5 cm and 7 cm, draw arcs from point O and A respectively. These will be intersecting each other at point Y.



(4) Join Y to A and O.



OKAY is the required quadrilateral.

Exercise- 4.4

Question 1:

Construct the following quadrilaterals,

(i) Quadrilateral DEAR

DE = 4 cm

EA = 5 cm

AR = 4.5 cm

∠E = 60°

∠A = 90°

(ii) Quadrilateral TRUE

TR = 3.5 cm

RU = 3 cm

UE = 4 cm

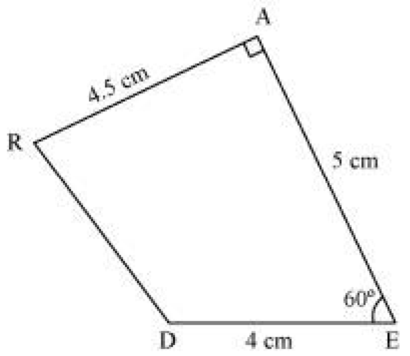
∠R = 75°

∠U = 120°

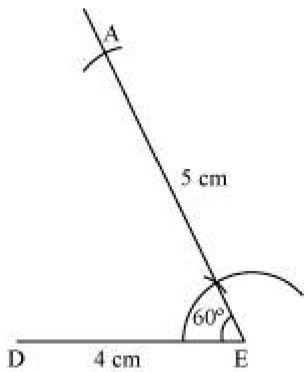
ANSWER:

(i)

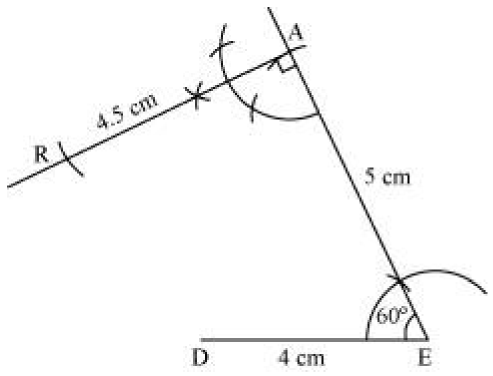
(1)A rough sketch of this quadrilateral can be drawn as follows.



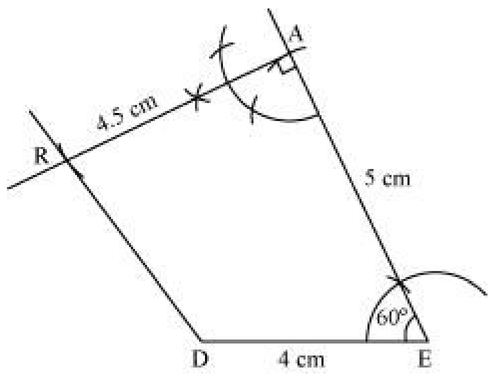
(2) Draw a line segment DE of 4 cm and an angle of 60º at point E. As vertex A is 5 cm away from vertex E, cut a line segment EA of 5 cm from this ray.



(3) Again draw an angle of 90º at point A. As vertex R is 4.5 cm away from vertex A, cut a line segment RA of 4.5 cm from this ray.



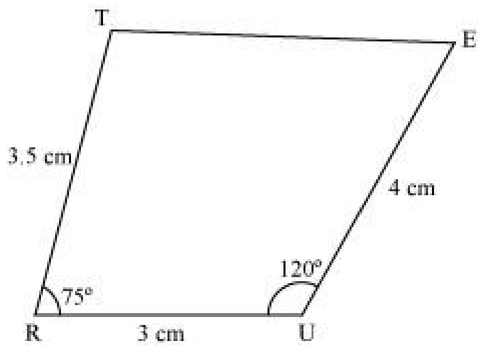
(4) Join D to R.



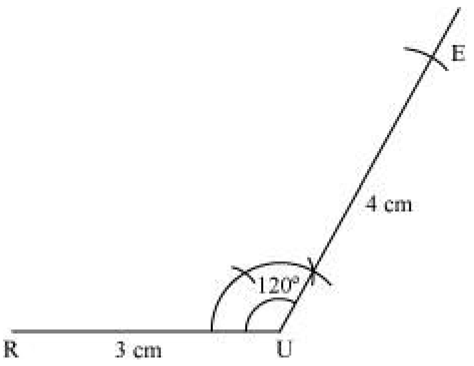
DEAR is the required quadrilateral.

(ii)

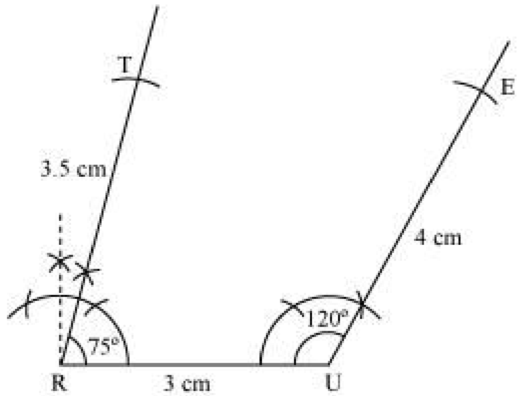
(1)A rough sketch of this quadrilateral can be drawn as follows.



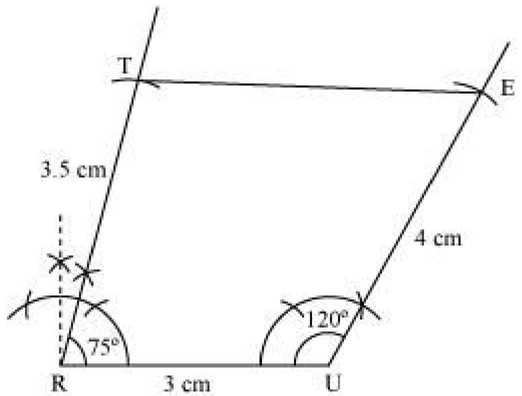
(2) Draw a line segment RU of 3 cm and an angle of 120º at point U. As vertex E is 4 cm away from vertex U, cut a line segment UE of 4 cm
from this ray.



(3) Next, draw an angle of 75º at point R. As vertex T is 3.5 cm away from vertex R, cut a line segment RT of 3.5 cm from this ray.



(4) Join T to E.



TRUE is the required quadrilateral.

Exercise - 4.5

Question 1:

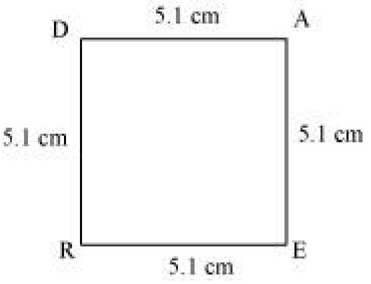
Draw the following:

The square READ with RE = 5.1 cm

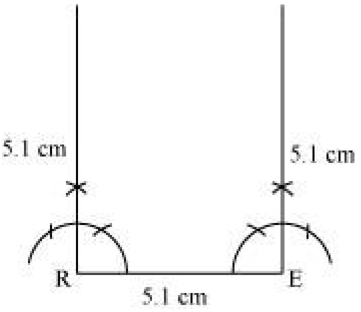
ANSWER:

All the sides of a square are of the same measure and also all the interior angles of a square are of 90º measure. Therefore, the given square READ can be drawn as follows.

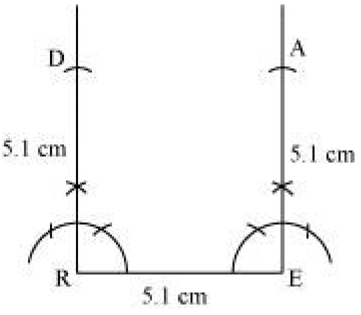
(1)A rough sketch of this square READ can be drawn as follows.



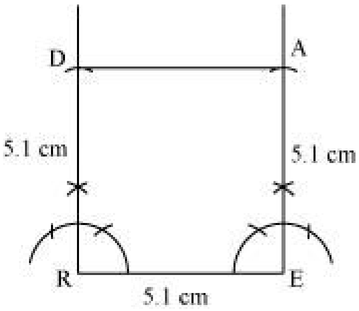
(2) Draw a line segment RE of 5.1 cm and an angle of 90º at point R and E.



(3) As vertex A and D are 5.1 cm away from vertex E and R respectively, cut line segments EA and RD, each of 5.1 cm from these rays.



(4) Join D to A.



READ is the required square.

Question 2:

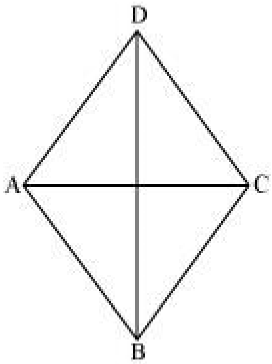
Draw the following:

A rhombus whose diagonals are 5.2 cm and 6.4 cm long.

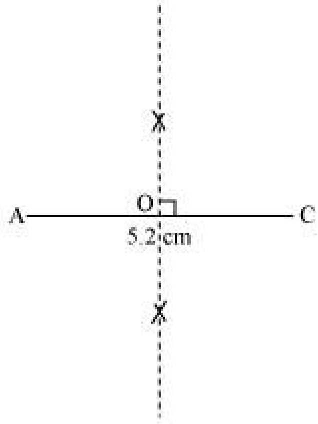
ANSWER:

In a rhombus, diagonals bisect each other at 90º. Therefore, the given rhombus ABCD can be drawn as follows.

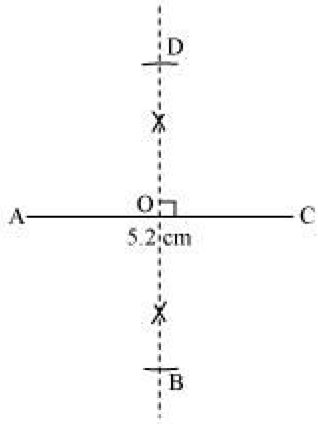
(1)A rough sketch of this rhombus ABCD is as follows.



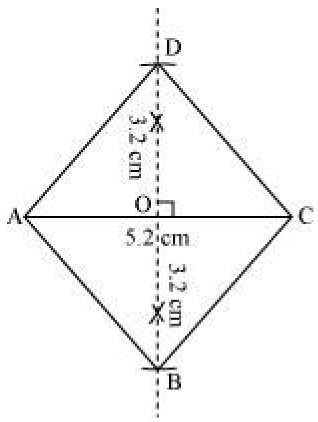
(2) Draw a line segment AC of 5.2 cm and draw its perpendicular bisector. Let it intersect the line segment AC at point O.



(3) Draw arcs of $\frac{6.4 \text{ cm}}{2} = 3.2 \text{ cm}$ on both sides of this perpendicular bisector. Let the arcs intersect the perpendicular bisector at point B and D.



(4) Join points B and D with points A and C.



ABCD is the required rhombus.

Question 3:

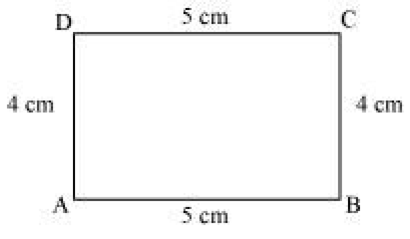
Draw the following:

A rectangle with adjacent sides of length 5 cm and 4 cm.

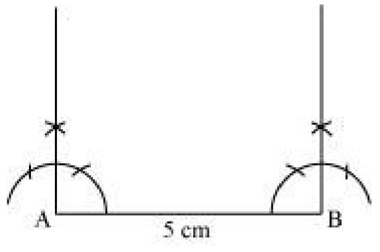
ANSWER:

Opposite sides of a rectangle have their lengths of same measure and also, all the interior angles of a rectangle are of 90º measure. The given rectangle ABCD may be drawn as follows.

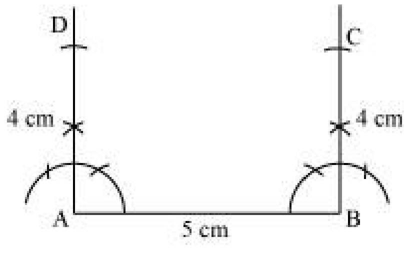
(1)A rough sketch of this rectangle ABCD can be drawn as follows.



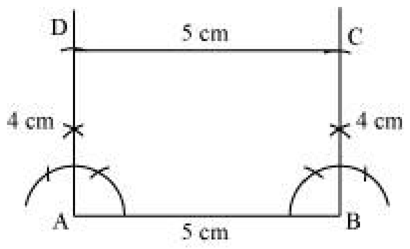
(2) Draw a line segment AB of 5 cm and an angle of 90º at point A and B.



(3) As vertex C and D are 4 cm away from vertex B and A respectively, cut line segments AD and BC, each of 4 cm, from these rays.



(4) Join D to C.



ABCD is the required rectangle.

Question 4:

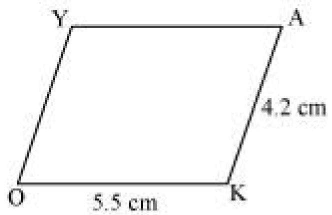
Draw the following:

A parallelogram OKAY where OK = 5.5 cm and KA = 4.2 cm.

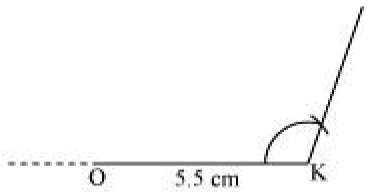
ANSWER:

Opposite sides of a parallelogram are equal and parallel to each other. The given parallelogram OKAY can be drawn as follows.

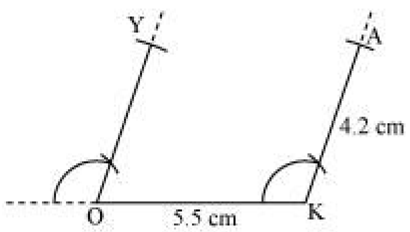
(1)A rough sketch of this parallelogram OKAY is drawn as follows.



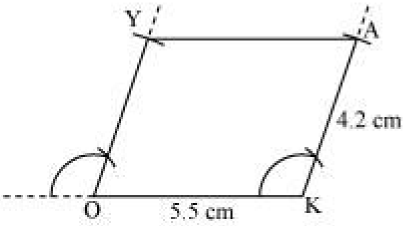
(2) Draw a line segment OK of 5.5 cm and a ray at point K at a convenient angle.



(3) Draw a ray at point O parallel to the ray at K. As the vertices, A and Y, are 4.2 cm away from the vertices K and O respectively, cut line segments KA and OY, each of 4.2 cm, from these rays.



(4) Join Y to A.

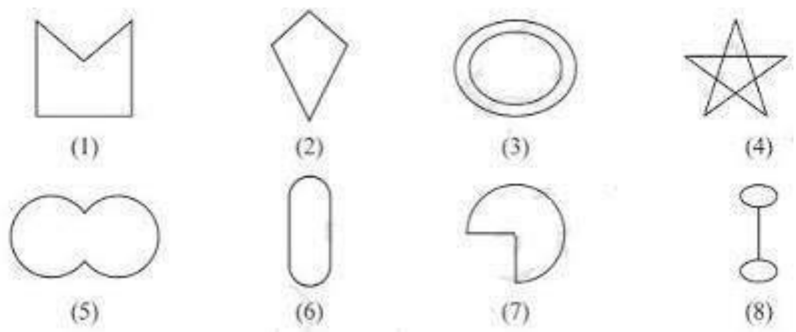


OKAY is the required parallelogram.

Class 8 Chapter 3 – Understanding Quadrilaterals

Exercise 3.1

1. Given here are some figures.



Classify each of them on the basis of the following.

Simple curve (b) Simple closed curve (c) Polygon

(d) Convex polygon (e) Concave polygon

Solution:

a) Simple curve: 1, 2, 5, 6 and 7

b) Simple closed curve: 1, 2, 5, 6 and 7

c) Polygon: 1 and 2

d) Convex polygon: 2

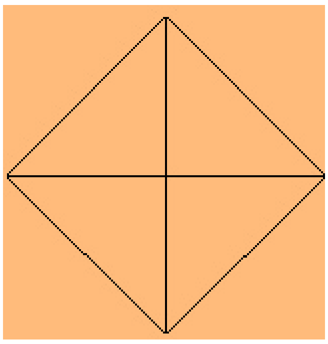
e) Concave polygon: 1

2. How many diagonals does each of the following have?

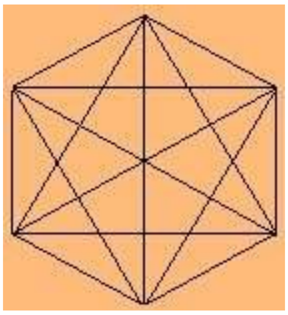
a) A convex quadrilateral (b) A regular hexagon (c) A triangle

Solution:

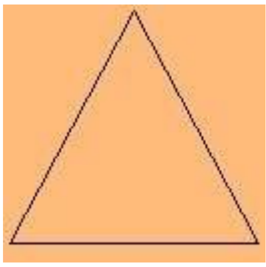
a) A convex quadrilateral: 2.



b) A regular hexagon: 9.

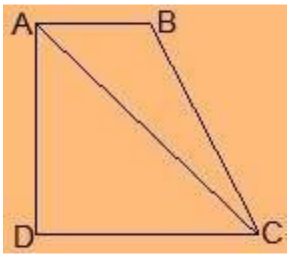


c) A triangle: 0



3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

Solution:



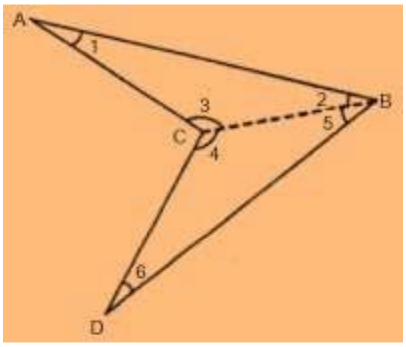
Let ABCD be a convex quadrilateral.

From the figure, we infer that the quadrilateral ABCD is formed by two triangles,

i.e. ΔADC and ΔABC.

Since, we know that sum of interior angles of triangle is 180°,

the sum of the measures of the angles is 180° + 180° = 360°



Let us take another quadrilateral ABCD which is not convex .

Join BC, Such that it divides ABCD into two triangles ΔABC and ΔBCD. In ΔABC,

∠1 + ∠2 + ∠3 = 180° (angle sum property of triangle)

In ΔBCD,

∠4 + ∠5 + ∠6 = 180° (angle sum property of triangle)

∴, ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 = 180° + 180°

⇒ ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 = 360°

⇒ ∠A + ∠B + ∠C + ∠D = 360°

Thus, this property hold if the quadrilateral is not convex.

4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

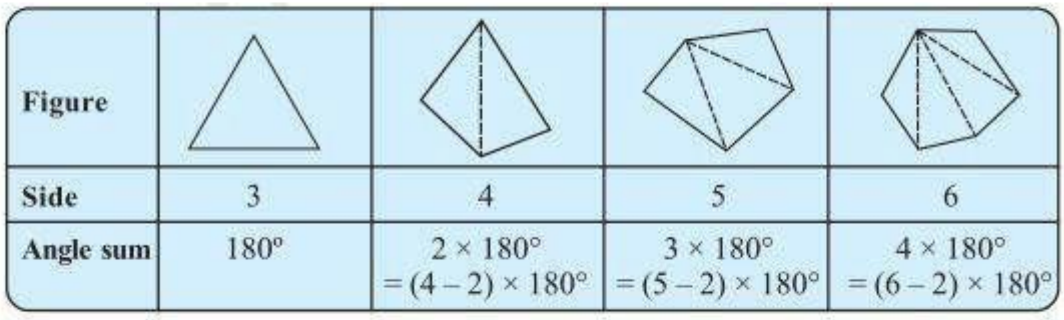


Figure				
Side	3	4	5	6
Angle sum	180°	2 × 180° = (4 – 2) × 180°	3 × 180° = (5 – 2) × 180°	4 × 180° = (6 – 2) × 180°

What can you say about the angle sum of a convex polygon with number of sides? (a) 7 (b) 8 (c) 10 (d) n

Solution:

The angle sum of a polygon having side n = (n-2)×180°

a) 7

Here, n = 7

Thus, angle sum = (7-2)×180° = 5×180° = 900°

b) 8

Here, n = 8

Thus, angle sum = (8-2)×180° = 6×180° = 1080°

c) 10

Here, n = 10

Thus, angle sum = (10-2)×180° = 8×180° = 1440°

d) n

Here, n = n

Thus, angle sum = (n-2)×180°

5. What is a regular polygon?

State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides Solution:

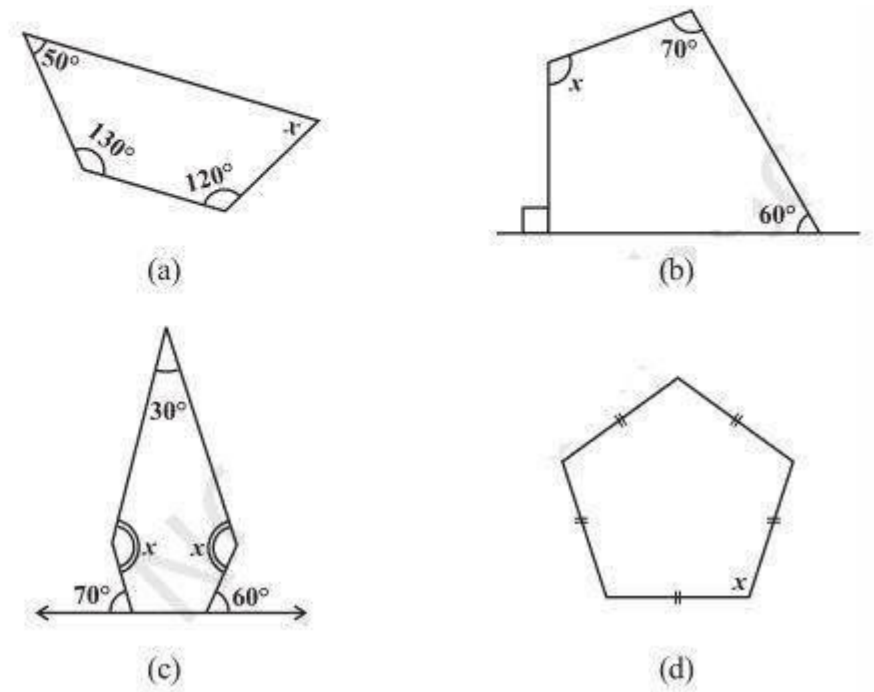
Regular polygon: A polygon having sides of equal length and angles of equal measures is called regular polygon. i.e., A regular polygon is both equilateral and equiangular.

(i) A regular polygon of 3 sides is called equilateral triangle.

(ii) A regular polygon of 4 sides is called square.

(iii) A regular polygon of 6 sides is called regular hexagon.

6. Find the angle measure x in the following figures.



Solution:

a) The figure is having 4 sides. Hence, it is a quadrilateral. Sum of angles of the quadrilateral = 360°

⇒ 50° + 130° + 120° + x = 360°

⇒ 300° + x = 360°

⇒ x = 360° – 300° = 60°

b) The figure is having 4 sides. Hence, it is a quadrilateral. Also, one side is perpendicular forming right angle.

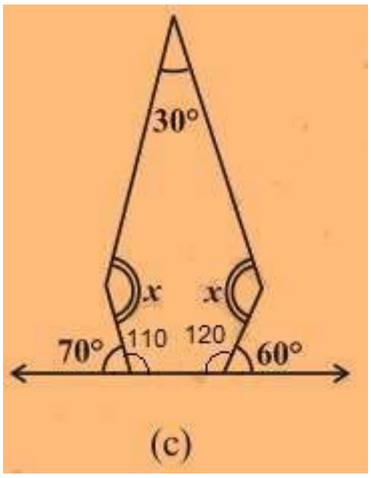
Sum of angles of the quadrilateral = 360°

⇒ 90° + 70° + 60° + x = 360°

⇒ 220° + x = 360°

⇒ x = 360° – 220° = 140°

c) The figure is having 5 sides. Hence, it is a pentagon.



Sum of angles of the pentagon = 540° Two angles at the bottom are linear pair.

∴, 180° – 70° = 110°

180° – 60° = 120°

⇒ 30° + 110° + 120° + x + x = 540°

⇒ 260° + 2x = 540°

⇒ 2x = 540° – 260° = 280°

⇒ 2x = 280°

= 140°

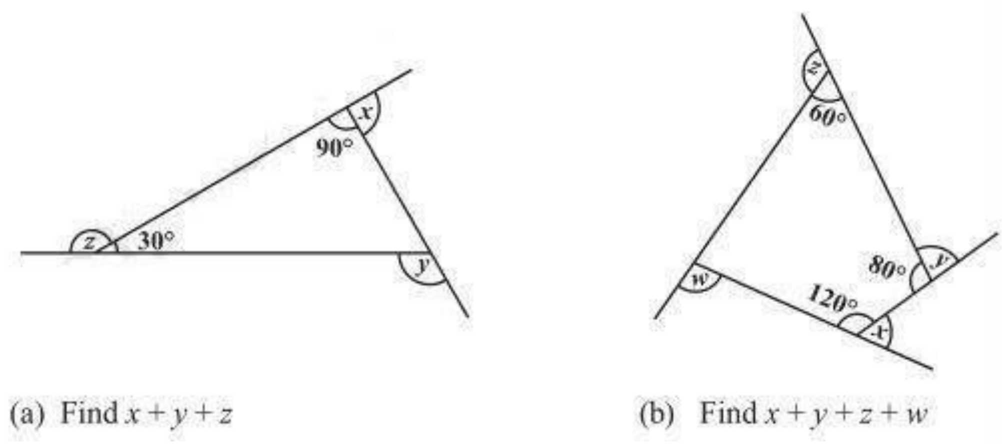
d) The figure is having 5 equal sides. Hence, it is a regular pentagon. Thus, its all angles are equal.

5x = 540°

⇒ x = 540°/5

⇒ x = 108°

7.



Solution:

a) Sum of all angles of triangle = 180°

One side of triangle = 180°- (90° + 30°) = 60°

x + 90° = 180° ⇒ x = 180° – 90° = 90°

y + 60° = 180° ⇒ y = 180° – 60° = 120°

z + 30° = 180° ⇒ z = 180° – 30° = 150°

x + y + z = 90° + 120° + 150° = 360°

b) Sum of all angles of quadrilateral = 360°

One side of quadrilateral = 360°- (60° + 80° + 120°) = 360° – 260° = 100°

x + 120° = 180° ⇒ x = 180° – 120° = 60°

y + 80° = 180° ⇒ y = 180° – 80° = 100°

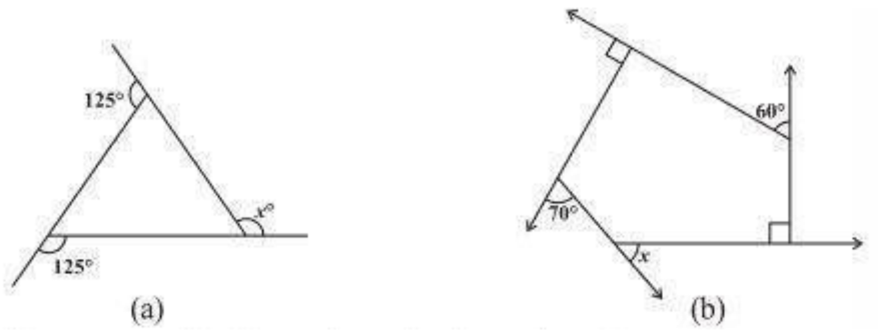
z + 60° = 180° ⇒ z = 180° – 60° = 120°

w + 100° = 180° ⇒ w = 180° – 100° = 80°

x + y + z + w = 60° + 100° + 120° + 80° = 360°

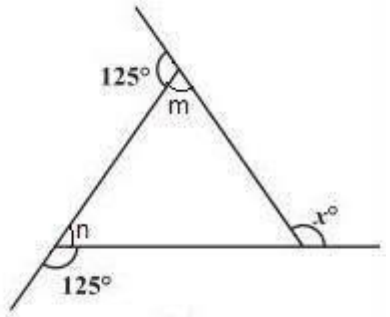
Exercise 3.2

1. Find x in the following figures.



Solution:

a)



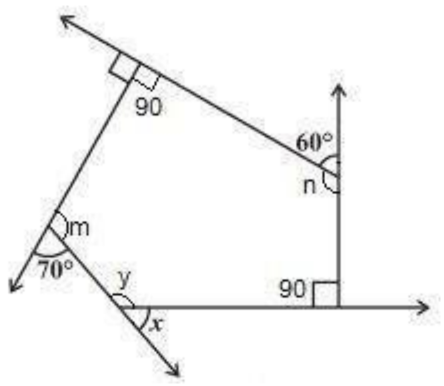
125° + m = 180° ⇒ m = 180° – 125° = 55° (Linear pair)

125° + n = 180° ⇒ n = 180° – 125° = 55° (Linear pair)

x = m + n (exterior angle of a triangle is equal to the sum of 2 opposite interior 2 angles)

⇒ x = 55° + 55° = 110°

b)



Two interior angles are right angles = 90°

70° + m = 180° ⇒ m = 180° – 70° = 110° (Linear pair)

60° + n = 180° ⇒ n = 180° – 60° = 120° (Linear pair) The figure is having five sides and is a pentagon.

Thus, sum of the angles of pentagon = 540° 90° + 90° + 110° + 120° + y = 540°

⇒ 410° + y = 540° ⇒ y = 540° – 410° = 130°

x + y = 180° (Linear pair)

⇒ x + 130° = 180°

⇒ x = 180° – 130° = 50°

2. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides Solution:

Sum of angles a regular polygon having side n = (n-2)×180°

(i) Sum of angles a regular polygon having side 9 = (9-2)×180°= 7×180° = 1260°

Each interior angle=1260/9 = 140°

Each exterior angle = 180° – 140° = 40°

Or,

Each exterior angle = sum of exterior angles/Number of angles = 360/9 = 40°

(ii) Sum of angles a regular polygon having side 15 = (15-2)×180°

= 13×180° = 2340°

Each interior angle = 2340/15 = 156°

Each exterior angle = 180° – 156° = 24°

Or,

Each exterior angle = sum of exterior angles/Number of angles = 360/15 = 24°

3. How many sides does a regular polygon have if the measure of an exterior angle is 24°? Solution:

Each exterior angle = sum of exterior angles/Number of angles

24°= 360/ Number of sides

⇒ Number of sides = 360/24 = 15

Thus, the regular polygon has 15 sides.

4. How many sides does a regular polygon have if each of its interior angles is 165°? Solution:

Interior angle = 165°

Exterior angle = 180° – 165° = 15°

Number of sides = sum of exterior angles/ exterior angles

⇒ Number of sides = 360/15 = 24

Thus, the regular polygon has 24 sides.

5.

a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?

b) Can it be an interior angle of a regular polygon? Why?

Solution:

a) Exterior angle = 22°

Number of sides = sum of exterior angles/ exterior angle

⇒ Number of sides = 360/22 = 16.36

No, we can't have a regular polygon with each exterior angle as 22° as it is not divisor of 360.

b) Interior angle = 22°

Exterior angle = 180° – 22°= 158°

No, we can't have a regular polygon with each exterior angle as 158° as it is not divisor of 360.

6.

a) What is the minimum interior angle possible for a regular polygon? Why?

b) What is the maximum exterior angle possible for a regular polygon?

Solution:

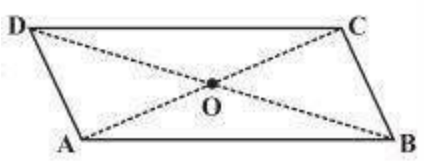
a) Equilateral triangle is regular polygon with 3 sides has the least possible minimum interior angle because the regular with minimum sides can be constructed with 3 sides at least.. Since, sum of interior angles of a triangle = 180°

Each interior angle = 180/3 = 60°

b) Equilateral triangle is regular polygon with 3 sides has the maximum exterior angle because the regular polygon with least number of sides have the maximum exterior angle possible. Maximum exterior possible = 180 – 60° = 120°

Exercise 3.3

1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.



(i) AD = …… (ii) ∠DCB = ……

(iii) OC = …… (iv) m ∠DAB + m ∠CDA = ……

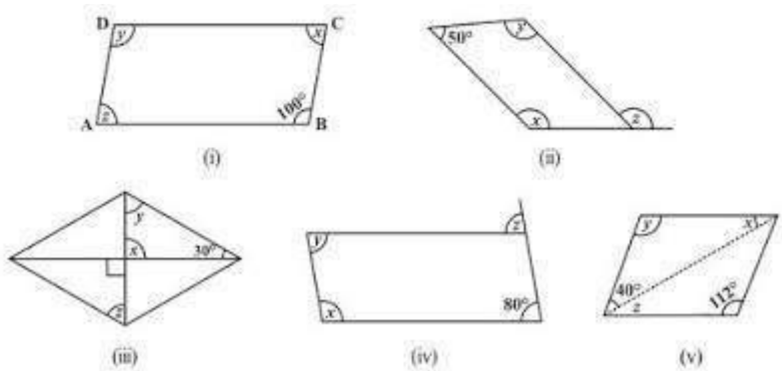
Solution:

(i) AD = BC (Opposite sides of a parallelogram are equal)

(ii) ∠DCB = ∠DAB (Opposite angles of a parallelogram are equal) (iii) OC = OA (Diagonals of a parallelogram are equal)

(iv) m ∠DAB + m ∠CDA = 180°

2. Consider the following parallelograms. Find the values of the unknown x, y, z



Solution:

(i)

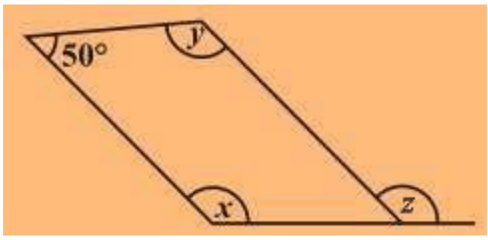
y = 100° (opposite angles of a parallelogram)

x + 100° = 180° (Adjacent angles of a parallelogram)

⇒ x = 180° – 100° = 80°

x = z = 80° (opposite angles of a parallelogram)

∴, x = 80°, y = 100° and z = 80°

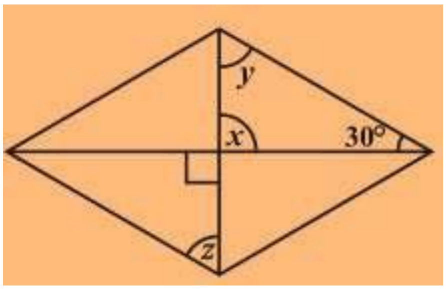


(ii)

50° + x = 180° ⇒ x = 180° – 50° = 130° (Adjacent angles of a parallelogram) x = y = 130° (opposite angles of a parallelogram)

x = z = 130° (corresponding angle)

(iii)



x = 90° (vertical opposite angles)

x + y + 30° = 180° (angle sum property of a triangle)

⇒ 90° + y + 30° = 180°

⇒ y = 180° – 120° = 60°

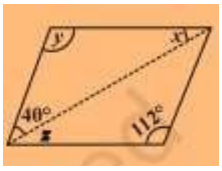
also, y = z = 60° (alternate angles)

(iv)

z = 80° (corresponding angle) z = y = 80° (alternate angles) x + y = 180° (adjacent angles)

⇒ x + 80° = 180° ⇒ x = 180° – 80° = 100°

(v)



x=28o

y = 112o z = 28o

3. Can a quadrilateral ABCD be a parallelogram if (i) ∠D + ∠B = 180°?

(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

(iii)∠A = 70° and ∠C = 65°?

Solution:

(i) Yes, a quadrilateral ABCD be a parallelogram if ∠D + ∠B = 180° but it should also fulfilled some conditions which are:

(a) The sum of the adjacent angles should be 180°.

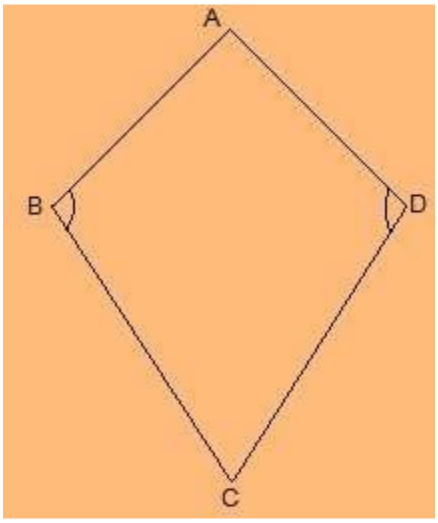
(b) Opposite angles must be equal.

(ii) No, opposite sides should be of same length. Here, AD ≠ BC

(iii) No, opposite angles should be of same measures. ∠A ≠ ∠C

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Solution:



ABCD is a figure of quadrilateral that is not a parallelogram but has exactly two opposite angles that is ∠B = ∠D of equal measure. It is not a parallelogram because ∠A ≠ ∠C.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

Solution:

Let the measures of two adjacent angles ∠A and ∠B be 3x and 2x respectively in parallelogram ABCD.

∠A + ∠B = 180°

⇒ 3x + 2x = 180°

⇒ 5x = 180°

⇒ x = 36°

We know that opposite sides of a parallelogram are equal.

∠A = ∠C = 3x = 3 × 36° = 108°

∠B = ∠D = 2x = 2 × 36° = 72°

6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Solution:

Let ABCD be a parallelogram.

Sum of adjacent angles of a parallelogram = 180°

∠A + ∠B = 180°

⇒ 2∠A = 180°

⇒ ∠A = 90°

also, 90° + ∠B = 180°

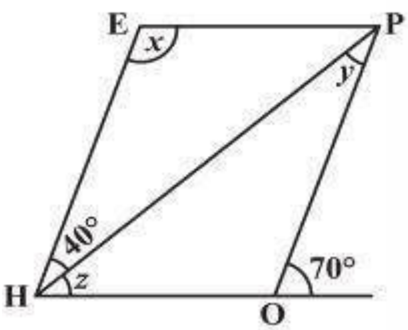
⇒ ∠B = 180° – 90° = 90°

∠A = ∠C = 90°

∠B = ∠D = 90

°

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Solution:

y = 40° (alternate interior angle)

∠P = 70° (alternate interior angle)

∠P = ∠H = 70° (opposite angles of a parallelogram)

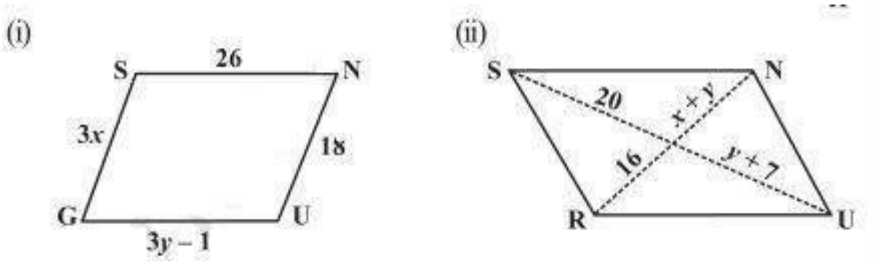
z = ∠H – 40°= 70° – 40° = 30°

∠H + x = 180°

⇒ 70° + x = 180°

⇒ x = 180° – 70° = 110°

8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)



Solution:

(i) SG = NU and SN = GU (opposite sides of a parallelogram are equal) 3x = 18

x = 18/3

⇒ x =6

3y – 1 = 26 and,

⇒ 3y = 26 + 1

⇒ y = 27/3=9

x = 6 and y = 9

(ii) 20 = y + 7 and 16 = x + y (diagonals of a parallelogram bisect each other) y + 7 = 20

⇒ y = 20 – 7 = 13 and,

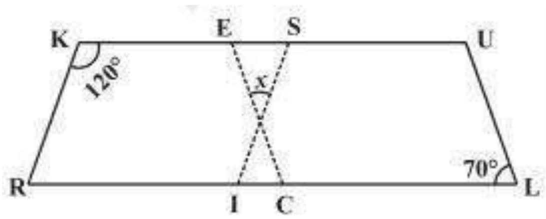
x + y = 16

⇒ x + 13 = 16

⇒ x = 16 – 13 = 3

x = 3 and y = 13

9. In the above figure both RISK and CLUE are parallelograms. Find the value of x.



Solution:

∠K + ∠R = 180° (adjacent angles of a parallelogram are supplementary)

⇒ 120° + ∠R = 180°

⇒ ∠R = 180° – 120° = 60°

also, ∠R = ∠SIL (corresponding angles)

⇒ ∠SIL = 60°

also, ∠ECR = ∠L = 70° (corresponding angles) x + 60° + 70° = 180° (angle sum of a triangle)

⇒ x + 130° = 180°

⇒ x = 180° – 130° = 50°

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)

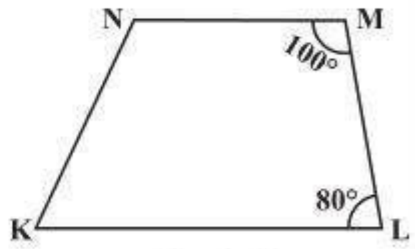


Fig 3.32

Solution:

When a transversal line intersects two lines in such a way that the sum of the adjacent angles on the same side of transversal is 180° then the lines are parallel to each other. Here, ∠M + ∠L = 100° + 80° = 180°

Thus, MN || LK

As the quadrilateral KLMN has one pair of parallel line therefore it is a trapezium. MN and LK are parallel lines.

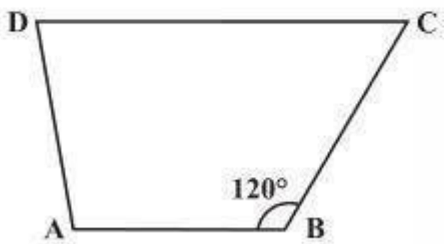


Fig 3.33

11. Find m∠C in Fig 3.33 if AB || DC ?

Solution:

m∠C + m∠B = 180° (angles on the same side of transversal)

⇒ m∠C + 120° = 180°

⇒ m∠C = 180°- 120° = 60°

12. Find the measure of ∠P and ∠S if SP || RQ ? in Fig 3.34. (If you find m∠R, is there more than one method to find m∠P?)

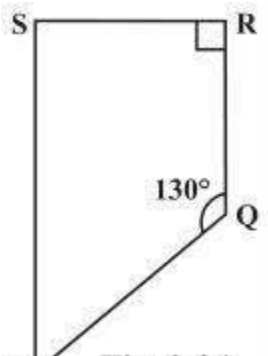


Fig 3.34

Solution:

∠P + ∠Q = 180° (angles on the same side of transversal)

⇒ ∠P + 130° = 180°

⇒ ∠P = 180° – 130° = 50°

also, ∠R + ∠S = 180° (angles on the same side of transversal)

⇒ 90° + ∠S = 180°

⇒ ∠S = 180° – 90° = 90°

Thus, ∠P = 50° and ∠S = 90°

Yes, there are more than one method to find m∠P.

PQRS is a quadrilateral. Sum of measures of all angles is 360°.

Since, we know the measurement of ∠Q, ∠R and ∠S.

∠Q = 130°, ∠R = 90° and ∠S = 90°

∠P + 130° + 90° + 90° = 360°

⇒ ∠P + 310° = 360°

⇒ ∠P = 360° – 310° = 50°

Exercise 3.4

1. State whether True or False.

(a) All rectangles are squares.

(b) All rhombuses are parallelograms.

(c) All squares are rhombuses and also rectangles.

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

Solution:

(a) False.

Because, all square are rectangles but all rectangles are not square.

(b) True

(c) True

(d) False.

Because, all squares are parallelograms as opposite sides are parallel and opposite angles are equal.

(e) False.

Because, for example, a length of the sides of a kite are not of same length.

(f) True

(g) True

(h) True

2. Identify all the quadrilaterals that have.

(a) four sides of equal length (b) four right angles

Solution:

(a) Rhombus and square have all four sides of equal length.

(b) Square and rectangle have four right angles.

3. Explain how a square is.

(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle Solution

(i) Square is a quadrilateral because it has four sides.

(ii) Square is a parallelogram because it's opposite sides are parallel and opposite angles are equal.

(iii) Square is a rhombus because all the four sides are of equal length and diagonals bisect at right angles.

(iv)Square is a rectangle because each interior angle, of the square, is 90°

4. Name the quadrilaterals whose diagonals.

(i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal Solution

(i) Parallelogram, Rhombus, Square and Rectangle

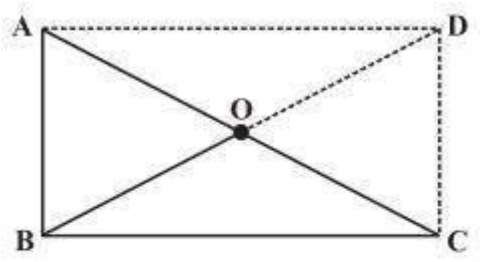
(ii) Rhombus and Square

(iii)Rectangle and Square

5. Explain why a rectangle is a convex quadrilateral. Solution

Rectangle is a convex quadrilateral because both of its diagonals lie inside the rectangle.

6. ABC is a right-angled triangle and O is the mid-point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Solution

AD and DC are drawn so that AD || BC and AB || DC

AD = BC and AB = DC

ABCD is a rectangle as opposite sides are equal and parallel to each other and all the interior angles are of 90°.

In a rectangle, diagonals are of equal length and also bisects each other.

Hence, AO = OC = BO = OD

Thus, O is equidistant from A, B and C.

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER: 3 UNDERSTANDING QUADRILATERALS

MIND MAP

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE:1 POLYGONS , ANGLE SUM PROPERTY

Q.1. Draw the following:
(a) a convex polygon (b) a concave polygon

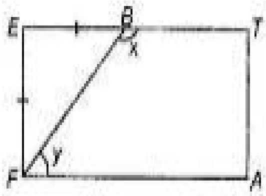
Q.2. How many diagonals does a triangle have?

Q.3. Find the number of sides of a regular polygon whose each exterior angle has a measure of 15°.

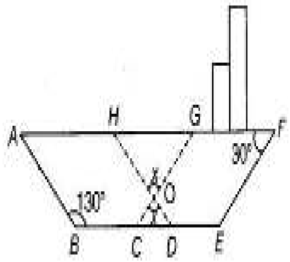
Q.4. The sum of two opposite angles of a quadrilateral is 172°. The other two angles of the quadrilateral are equal find the equal angles.

TYPE:2 KINDS OF QUADRILATERALS AND THEIR PROPERTIES

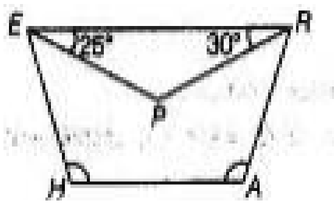
Q.1. A playground is in the form of a rectangle ATEF.
Two players are standing at the points F and B, where EF =EB.
Find the values of x and y.



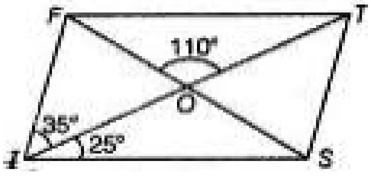
Q.2. In the following figure of a ship, ABDH and CEFG are parallelograms.
Find the value of x.



Q.3. In trapezium HARE, EP and RP are bisectors of $\angle E$ and $\angle R$, respectively.
Find $\angle HAR$ and $\angle EHA$.



Q.4. In parallelogram FIST, find, $\angle SFT$, $\angle OST$ and $\angle STO$.



Ch. 1 Rational Numbers

Exercise 1.1

Q1 :

Using appropriate properties find:

(i) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$

(ii) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

Answer :

(i)

$-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}=-\frac{2}{3}\times\frac{3}{5}-\frac{3}{5}\times\frac{1}{6}+\frac{5}{2}$

(Using commutativity of rational numbers)

$=\left(-\frac{3}{5}\right)\times\left(\frac{2}{3}+\frac{1}{6}\right)+\frac{5}{2}$ (Distributivity)

$=\left(-\frac{3}{5}\right)\times\left(\frac{2\times2+1}{6}\right)+\frac{5}{2}=\left(-\frac{3}{5}\right)\times\left(\frac{5}{6}\right)+\frac{5}{2}$

$=\left(-\frac{3}{6}\right)+\frac{5}{2}=\left(\frac{-3+5\times3}{6}\right)=\left(\frac{-3+15}{6}\right)$

$=\frac{12}{6}=2$

(ii)

$\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}=\frac{2}{5}\times\left(-\frac{3}{7}\right)+\frac{1}{14}\times\frac{2}{5}-\frac{1}{6}\times\frac{3}{2}$ (By commutativity)

$= \frac{2}{5} \times \left(-\frac{3}{7} + \frac{1}{14} \right) - \frac{1}{4}$ (By distributivity)

$= \frac{2}{5} \times \left(\frac{-3 \times 2 + 1}{14} \right) - \frac{1}{4}$

$= \frac{2}{5} \times \left(\frac{-5}{14} \right) - \frac{1}{4}$

$= -\frac{1}{7} - \frac{1}{4}$

$= \frac{-4-7}{28} = \frac{-11}{28}$

Q2 :

Write the additive inverse of each of the following:

(i) $\frac{2}{8}$ (ii) $\frac{-5}{9}$ (iii) $\frac{-6}{-5}$ (iv) $\frac{2}{-9}$ (v) $\frac{19}{-6}$

Answer :

(i) $\frac{2}{8}$

Additive inverse = $-\frac{2}{8}$

(ii) $-\frac{5}{9}$

Additive inverse = $\frac{5}{9}$

(iii) $\frac{-6}{-5} = \frac{6}{5}$

Additive inverse = $\frac{-6}{5}$

(iv) $\frac{2}{-9}=\frac{-2}{9}$

Additive inverse $=\frac{2}{9}$

(v) $\frac{19}{-6}=\frac{-19}{6}$

Additive inverse $=\frac{19}{6}$

Q3 :

Verify that - (- *x*) = *x* for.

(i) $x=\frac{11}{15}$ (ii) $x=-\frac{13}{17}$

Answer :

(i) $x=\frac{11}{15}$

The additive inverse of $x = \frac{11}{15}$ is $-x = -\frac{11}{15}$ as $\frac{11}{15} + \left(-\frac{11}{15}\right) = 0$

This equality $\frac{11}{15} + \left(-\frac{11}{15}\right) = 0$ represents that the additive inverse of $-\frac{11}{15}$ is $\frac{11}{15}$ or it can be said that $-\left(-\frac{11}{15}\right) = \frac{11}{15}$ i.e., - (- *x*) = *x*

(ii) $x = -\frac{13}{17}$

The additive inverse of $x = -\frac{13}{17}$ is $-x = \frac{13}{17}$ as $-\frac{13}{17} + \frac{13}{17} = 0$ This equality $-\frac{13}{17} + \frac{13}{17} = 0$ represents that the additive inverse of $\frac{13}{17}$ is - $\frac{13}{17}$ i.e., - (- *x*) = *x*

Q4 :

Find the multiplicative inverse of the following.

(i) -13 (ii) $\frac{-13}{19}$ (iii) $\frac{1}{5}$

(iv) $\frac{-5}{8} \times \frac{-3}{7}$ (v) $-1 \times \frac{-2}{5}$ (vi) - 1

Answer :

(i) - 13

Multiplicative inverse = - $\frac{1}{13}$

(ii) $-\frac{13}{19}$

Multiplicative inverse = $-\frac{19}{13}$

(iii) $\frac{1}{5}$

Multiplicative inverse = 5

(iv) $-\frac{5}{8}\times-\frac{3}{7}=\frac{15}{56}$

Multiplicative inverse $=\frac{56}{15}$

(v) $-1\times-\frac{2}{5}=\frac{2}{5}$

Multiplicative inverse $=\frac{5}{2}$

(vi) - 1

Multiplicative inverse = - 1

Q5 :

Name the property under multiplication used in each of the following:

(i) $\frac{-4}{5}\times1=1\times\frac{-4}{5}=-\frac{4}{5}$

(ii) $-\frac{13}{17}\times\frac{-2}{7}=\frac{-2}{7}\times\frac{-13}{17}$

(iii) $\frac{-19}{29}\times\frac{29}{-19}=1$

Answer :

(i) $-\frac{4}{5}\times1=1\times-\frac{4}{5}=-\frac{4}{5}$

1 is the multiplicative identity.

(ii) Commutativity

(iii) Multiplicative inverse

Q6 :

Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

Answer :

$$\frac{6}{13}\times\left(\text{Reciprocal of } -\frac{7}{16}\right)=\frac{6}{13}\times-\frac{16}{7}=-\frac{96}{91}$$

Q7 :

Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ as $\left(\frac{1}{3}\times6\right)\times\frac{4}{3}$.

Answer :

Associativity

Q8 :

Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

Answer :

If it is the multiplicative inverse, then the product should be 1.

However, here, the product is not 1 as

$$\frac{8}{9}\times\left(-1\frac{1}{8}\right)=\frac{8}{9}\times\left(-\frac{9}{8}\right)=-1\neq1$$

Q9 : Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

Answer :

$3\frac{1}{3} = \frac{10}{3}$

0.3 × $3\frac{1}{3}$ = 0.3 × $\frac{10}{3} = \frac{3}{10} \times \frac{10}{3} = 1$

Here, the product is 1. Hence, 0.3 is the multiplicative inverse of $3\frac{1}{3}$.

Q10 :

Write:

(i) The rational number that does not have a reciprocal.

(ii) The rational numbers that are equal to their reciprocals.

(iii) The rational number that is equal to its negative.

Answer :

(i) 0 is a rational number but its reciprocal is not defined.

(ii) 1 and -1 are the rational numbers that are equal to their reciprocals.

(iii) 0 is the rational number that is equal to its negative.

Q11 :

Fill in the blanks.

(i) Zero has___________reciprocal.

(ii) The numbers___________and___________are their own reciprocals (iii) The reciprocal of - 5 is___________.

(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is___________.

(v) The product of two rational numbers is always a___________.

(vi) The reciprocal of a positive rational number is___________.

Answer :

(i) No

(ii) 1, - 1

(iii) $-\frac{1}{5}$

(iv) x

(v) Rational number

(vi) Positive rational number

Exercise 1.2 :

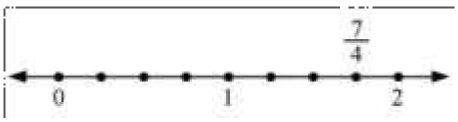
Q1 :

Represent these numbers on the number line.

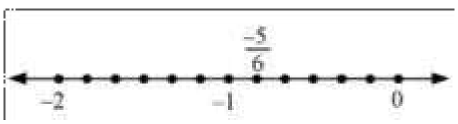
(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

Answer :

(i) $\frac{7}{4}$ can be represented on the number line as follows.



(ii) $-\frac{5}{6}$ can be represented on the number line as follows.

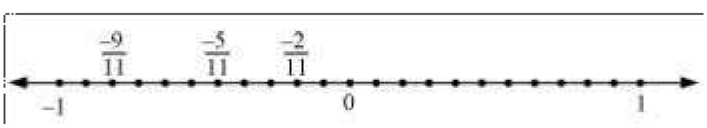


Q2 :

Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.

Answer :

$\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ can be represented on the number line as follows.



Q3 :

Write five rational numbers which are smaller than 2.

Answer :

2 can be represented as $\frac{14}{7}$

Therefore, five rational numbers smaller than 2 are

$\frac{13}{7}, \frac{12}{7}, \frac{11}{7}, \frac{10}{7}, \frac{9}{7}$

Q4 :

Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Answer :

$\frac{-2}{5}$ and $\frac{1}{2}$ can be represented as $-\frac{8}{20}$ and $\frac{10}{20}$ respectively.

Therefore, ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$ are

$-\frac{7}{20}, -\frac{6}{20}, -\frac{5}{20}, -\frac{4}{20}, -\frac{3}{20}, -\frac{2}{20}, -\frac{1}{20}, 0, \frac{1}{20}, \frac{2}{20}$

Q5 : Find five rational numbers between

(i) $\frac{2}{3}$ and $\frac{4}{5}$

(ii) $\frac{-3}{2}$ and $\frac{5}{3}$

(iii) $\frac{1}{4}$ and $\frac{1}{2}$

Answer :

(i) $\frac{2}{3}$ and $\frac{4}{5}$ can be represented $\frac{30}{45}$ and $\frac{36}{45}$ as respectively.

Therefore, five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$ are

$\frac{31}{45}, \frac{32}{45}, \frac{33}{45}, \frac{34}{45}, \frac{35}{45}$

(ii) $-\frac{3}{2}$ and $\frac{5}{3}$ can be represented as $-\frac{9}{6}$ and $\frac{10}{6}$ respectively.

Therefore, five rational numbers between $-\frac{3}{2}$ and $\frac{5}{3}$ are $-\frac{8}{6}, -\frac{7}{6}, -1, -\frac{5}{6}, -\frac{4}{6}$

(iii) $\frac{1}{4}$ and $\frac{1}{2}$ can be represented as $\frac{8}{32}$ and $\frac{16}{32}$ respectively.

Therefore, five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$ are $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

Q6 :
Write five rational numbers greater than - 2.

Answer :

- 2 can be represented as - $\frac{14}{7}$.

Therefore, five rational numbers greater than - 2 are $-\frac{13}{7}, -\frac{12}{7}, -\frac{11}{7}, -\frac{10}{7}, -\frac{9}{7}$

Q7 :

Find ten rational numbers between $\frac{3}{5}$ and .$\frac{3}{4}$

Answer :

$\frac{3}{5}$ and $\frac{3}{4}$ can be represented as $\frac{48}{80}$ and $\frac{60}{80}$ respectively.

Therefore, ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$ are

$$\frac{49}{80}, \frac{50}{80}, \frac{51}{80}, \frac{52}{80}, \frac{53}{80}, \frac{54}{80}, \frac{55}{80}, \frac{56}{80}, \frac{57}{80}, \frac{58}{80}$$

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.: 1 RATIONAL NUMBERS

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE:1 RATIONAL NUMBERS AND PROPERTIES

Q.1. Fill in the blanks. Also mention the property used.

a) $\frac{6}{13}+\left(\frac{4}{9}+\frac{7}{8}\right)=\left(\frac{6}{13}+\frac{4}{9}\right)$

b) $\frac{-3}{5}\left(\frac{3}{4}+\frac{-8}{9}\right)=\frac{-3}{5}\times\frac{3}{4}+\_\_\_\times\frac{-8}{9}$

Q.2. Show that $(-\frac{2}{5}+\frac{4}{9})+(-\frac{3}{4})=-\frac{2}{5}+\{\frac{4}{9}+(-\frac{3}{4})\}$

Q.3. Check if $-\frac{3}{5}(\frac{3}{4}+\frac{-8}{9})=(\frac{-3}{5}\times\frac{3}{4})+\{\frac{-3}{5}\times(\frac{-8}{9})\}$

Q.4. Simplify using distributive property:

a) $273\times 43+273\times 57$

b) $\frac{3}{4}\times\frac{18}{31}+\frac{3}{4}\times\frac{13}{31}$

Q.5. Simplify:

a) $(\frac{-3}{4}\times\frac{8}{15})-(\frac{2}{3}\times\frac{-3}{8})-(\frac{-4}{7}\times\frac{-14}{15})$

b) $-4\div(\frac{-2}{5})\times\frac{3}{4}$

TYPE:2 RATIONAL NUMBERS BETWEEN RATIONAL NUMBERS

Q.6. i) Find a rational number between $\frac{-8}{11}$ and $\frac{-7}{11}$

ii) Find five rational numbers between $\frac{4}{15}$ and $\frac{7}{15}$

TYPE:3 REAL LIFE APPLICATIONS

Q.7. A shirt can be stitched using $2\frac{1}{4}m$ of cloth. How much cloth is required for stitching 9 shirts.

Q.8. In a school $\frac{5}{8}$ of the students were girls. If the number of girls is 120 more than that of the boys, what is the strength of the school? How many boys are there?

Chapter 2 Linear equations in one variable

Exercise 2.1

Question 1:

Solve: $x-2=7$

Answer:

$x - 2 = 7$

Transposing 2 to R.H.S, we obtain

$x = 7 + 2 = 9$

Question 2:

Solve: $y+3=10$

Answer:

$y + 3 = 10$

Transposing 3 to R.H.S, we obtain

$y = 10 - 3 = 7$

Question 3:

Solve: $6=z+2$

Answer:

$6 = z + 2$

Transposing 2 to L.H.S, we obtain

$6 - 2 = z$

$z = 4$

Question 4:

Solve: $\frac{3}{7}+x=\frac{17}{7}$

Answer:

$\frac{3}{7}+x=\frac{17}{7}$

Transposing $\frac{3}{7}$ to R.H.S, we obtain

$$x = \frac{17}{7} - \frac{3}{7} = \frac{14}{7} = 2$$

Question 5:

Solve: $6x = 12$

Answer:

6x = 12

Dividing both sides by 6, we obtain

$$\frac{6x}{6} = \frac{12}{6}$$

x = 2

Question 6:

Solve: $\frac{t}{5} = 10$

Answer:

$$\frac{t}{5} = 10$$

Multiplying both sides by 5, we obtain

$$\frac{t}{5} \times 5 = 10 \times 5$$

$t = 50$

Question 7:

Solve: $\frac{2x}{3} = 18$

Answer:

$$\frac{2x}{3} = 18$$

Multiplying both sides by $\frac{3}{2}$, we obtain

$$\frac{2x}{3}\times\frac{3}{2}=18\times\frac{3}{2}$$

$$x=27$$

Question 8:

Solve: $1.6=\frac{y}{1.5}$

Answer:

$$1.6=\frac{y}{1.5}$$

Multiplying both sides by 1.5, we obtain

$$1.6\times1.5=\frac{y}{1.5}\times1.5$$

$$2.4=y$$

Question 9:

Solve: $7x-9=16$

Answer:

$7x - 9 = 16$

Transposing 9 to R.H.S, we obtain

$7x = 16 + 9$

$7x = 25$

Dividing both sides by 7, we obtain

$$\frac{7x}{7}=\frac{25}{7}$$

$$x=\frac{25}{7}$$

Question 10:

Solve: $14y-8=13$

Answer:

$14y - 8 = 13$

Transposing 8 to R.H.S, we obtain

$14y = 13 + 8$

$14y = 21$

Dividing both sides by 14, we obtain

$$\frac{14y}{14} = \frac{21}{14}$$

$$y = \frac{3}{2}$$

Question 11:

Solve: $17 + 6p = 9$

Answer:

$17 + 6p = 9$

Transposing 17 to R.H.S, we obtain

$6p = 9 - 17$

$6p = -8$

Dividing both sides by 6, we obtain

$$\frac{6p}{6} = -\frac{8}{6}$$

$$p = -\frac{4}{3}$$

Question 12:

Solve: $\frac{x}{3} + 1 = \frac{7}{15}$

Answer:

$$\frac{x}{3} + 1 = \frac{7}{15}$$

Transposing 1 to R.H.S, we obtain

$$\frac{x}{3} = \frac{7}{15} - 1$$

$$\frac{x}{3} = \frac{7 - 15}{15}$$

$$\frac{x}{3} = -\frac{8}{15}$$

Multiplying both sides by 3, we obtain

$$\frac{x}{3} \times 3 = -\frac{8}{15} \times 3$$

$$x = -\frac{8}{5}$$

Exercise 2.2

Question 1:

If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?

Answer:

Let the number be *x*. According to the question,

$$\left(x-\frac{1}{2}\right)\times\frac{1}{2}=\frac{1}{8}$$

On multiplying both sides by 2, we obtain

$$\left(x-\frac{1}{2}\right)\times\frac{1}{2}\times 2=\frac{1}{8}\times 2$$

$$x-\frac{1}{2}=\frac{1}{4}$$

On transposing $\frac{1}{2}$ to R.H.S, we obtain

$$x=\frac{1}{4}+\frac{1}{2}$$

$$=\frac{1+2}{4}=\frac{3}{4}$$

Therefore, the number is $\frac{3}{4}$.

Question 2:

The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

Answer:

Let the breadth be x m. The length will be $(2x + 2)$ m.

Perimeter of swimming pool = $2(l + b)$ = 154 m

$2(2x + 2 + x) = 154$

$2(3x + 2) = 154$

Dividing both sides by 2, we obtain

$$\frac{2(3x+2)}{2} = \frac{154}{2}$$

$3x + 2 = 77$

On transposing 2 to R.H.S, we obtain

$3x = 77 - 2$

$3x = 75$

On dividing both sides by 3, we obtain

$$\frac{3x}{3} = \frac{75}{3}$$

$x = 25$

$2x + 2 = 2 \times 25 + 2 = 52$

Hence, the breadth and length of the pool are 25 m and 52 m respectively.

Question 3:

The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?

Answer:

Let the length of equal sides be x cm.

Perimeter = x cm + x cm + Base = $4\frac{2}{15}$ cm

$$2x + \frac{4}{3} = \frac{62}{15}$$

On transposing $\frac{4}{3}$ to R.H.S, we obtain

$$2x = \frac{62}{15} - \frac{4}{3}$$

$$2x = \frac{62 - 4 \times 5}{15} = \frac{62 - 20}{15}$$

$$2x = \frac{42}{15}$$

On dividing both sides by 2, we obtain

$$\frac{2x}{2} = \frac{42}{15} \times \frac{1}{2}$$

$$x = \frac{7}{5} = 1\frac{2}{5}$$

Therefore, the length of equal sides is $1\frac{2}{5}$ cm.

Question 4:

Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

Answer:

Let one number be x. Therefore, the other number will be $x + 15$.

According to the question,

$x + x + 15 = 95$

$2x + 15 = 95$

On transposing 15 to R.H.S, we obtain

$2x = 95 - 15$

$2x = 80$

On dividing both sides by 2, we obtain

$$\frac{2x}{2} = \frac{80}{2}$$

$x = 40$

$x + 15 = 40 + 15 = 55$

Hence, the numbers are 40 and 55.

Question 5:

Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?

Answer:

Let the common ratio between these numbers be x. Therefore, the numbers will be $5x$ and $3x$ respectively.

Difference between these numbers = 18

$5x - 3x = 18$

$2x = 18$

Dividing both sides by 2,

$$\frac{2x}{2} = \frac{18}{2}$$

$x = 9$

First number = $5x = 5 \times 9 = 45$

Second number = $3x = 3 \times 9 = 27$

Question 6:

Three consecutive integers add up to 51. What are these integers?

Answer:

Let three consecutive integers be x, $x + 1$, and $x + 2$.

Sum of these numbers = $x + x + 1 + x + 2 = 51$

$3x + 3 = 51$

On transposing 3 to R.H.S, we obtain

$3x = 51 - 3$

$3x = 48$

On dividing both sides by 3, we obtain

$$\frac{3x}{3} = \frac{48}{3}$$

$x = 16$

$x + 1 = 17$

$x + 2 = 18$

Hence, the consecutive integers are 16, 17, and 18.

Question 7:

The sum of three consecutive multiples of 8 is 888. Find the multiples.

Answer:

Let the three consecutive multiples of 8 be $8x$, $8(x + 1)$, $8(x + 2)$.

Sum of these numbers = $8x + 8(x + 1) + 8(x + 2) = 888$

$8(x + x + 1 + x + 2) = 888$

$8(3x + 3) = 888$

On dividing both sides by 8, we obtain

$$\frac{8(3x+3)}{8} = \frac{888}{8}$$

$3x + 3 = 111$

On transposing 3 to R.H.S, we obtain

$3x = 111 - 3$

$3x = 108$

On dividing both sides by 3, we obtain

$$\frac{3x}{3} = \frac{108}{3}$$

$x = 36$

First multiple = $8x = 8 \times 36 = 288$

Second multiple = $8(x + 1) = 8 \times (36 + 1) = 8 \times 37 = 296$

Third multiple = $8(x + 2) = 8 \times (36 + 2) = 8 \times 38 = 304$

Hence, the required numbers are 288, 296, and 304.

Question 8:

Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.

Answer:

Let three consecutive integers be x, $x + 1$, $x + 2$. According to the question,

$2x + 3(x + 1) + 4(x + 2) = 74$

$2x + 3x + 3 + 4x + 8 = 74$

$9x + 11 = 74$

On transposing 11 to R.H.S, we obtain

$9x = 74 - 11$

$9x = 63$

On dividing both sides by 9, we obtain

$$\frac{9x}{9} = \frac{63}{9}$$

$x = 7$

$x + 1 = 7 + 1 = 8$

$x + 2 = 7 + 2 = 9$

Hence, the numbers are 7, 8, and 9.

Question 9:

The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?

Answer:

Let common ratio between Rahul's age and Haroon's age be x.

Therefore, age of Rahul and Haroon will be $5x$ years and $7x$ years respectively. After 4 years, the age of Rahul and Haroon will be $(5x + 4)$ years and $(7x + 4)$ years respectively.

According to the given question, after 4 years, the sum of the ages of Rahul and Haroon is 56 years.

$\therefore (5x + 4 + 7x + 4) = 56$

$12x + 8 = 56$

On transposing 8 to R.H.S, we obtain

$12x = 56 - 8$

$12x = 48$

On dividing both sides by 12, we obtain

$$\frac{12x}{12} = \frac{48}{12}$$

$x = 4$

Rahul's age = $5x$ years = (5×4) years = 20 years

Haroon's age = $7x$ years = (7×4) years = 28 years

Question 10:

The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?

Answer:

Let the common ratio between the number of boys and numbers of girls be x.

Number of boys = $7x$

Number of girls = $5x$

According to the given question,

Number of boys = Number of girls + 8

$\therefore 7x = 5x + 8$

On transposing $5x$ to L.H.S, we obtain

$7x - 5x = 8$

$2x = 8$

On dividing both sides by 2, we obtain

$$\frac{2x}{2} = \frac{8}{2}$$

$x = 4$

Number of boys = $7x = 7 \times 4 = 28$

Number of girls = $5x = 5 \times 4 = 20$

Hence, total class strength = 28 + 20 = 48 students

Question 11:

Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?

Answer:

Let Baichung's father's age be x years. Therefore, Baichung's age and Baichung's grandfather's age will be $(x - 29)$ years and $(x + 26)$ years respectively.

According to the given question, the sum of the ages of these 3 people is 135 years.

$\therefore x + x - 29 + x + 26 = 135$

$3x - 3 = 135$

On transposing 3 to R.H.S, we obtain

$3x = 135 + 3$

$3x = 138$

On dividing both sides by 3, we obtain

$$\frac{3x}{3} = \frac{138}{3}$$

$x = 46$

Baichung's father's age = x years = 46 years

Baichung's age = ($x - 29$) years = (46 − 29) years = 17 years

Baichung's grandfather's age = ($x + 26$) years = (46 + 26) years = 72 years

Question 12:

Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?

Answer:

Let Ravi's present age be x years.

Fifteen years later, Ravi's age = 4 × His present age

$x + 15 = 4x$

On transposing x to R.H.S, we obtain

$15 = 4x - x$

$15 = 3x$

On dividing both sides by 3, we obtain

$$\frac{15}{3} = \frac{3x}{3}$$

$5 = x$

Hence, Ravi's present age = 5 years

Question 13:

A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

Answer:

Let the number be *x*.

According to the given question,

$$\frac{5}{2}x+\frac{2}{3}=-\frac{7}{12}$$

On transposing $\frac{2}{3}$ to R.H.S, we obtain

$$\frac{5}{2}x=-\frac{7}{12}-\frac{2}{3}$$

$$\frac{5}{2}x=\frac{-7-(2\times 4)}{12}$$

$$\frac{5}{2}x=-\frac{15}{12}$$

On multiplying both sides by $\frac{2}{5}$, we obtain

$$x=-\frac{15}{12}\times\frac{2}{5}=-\frac{1}{2}$$

Hence, the rational number is $-\frac{1}{2}$.

Question 14:

Lakshmi is a cashier in a bank. She has currency notes of denominations Rs 100, Rs 50 and Rs 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is Rs 4, 00,000. How many notes of each denomination does she have?

Answer:

Let the common ratio between the numbers of notes of different denominations be x.

Therefore, numbers of Rs 100 notes, Rs 50 notes, and Rs 10 notes will be$2x$, $3x$, and $5x$ respectively.

Amount of Rs 100 notes = Rs $(100 \times 2x)$ = Rs $200x$

Amount of Rs 50 notes = Rs $(50 \times 3x)$= Rs $150x$

Amount of Rs 10 notes = Rs $(10 \times 5x)$ = Rs $50x$

It is given that total amount is Rs 400000.

$\therefore\ 200x + 150x + 50x = 400000$

$\ast\ 400x = 400000$

On dividing both sides by 400, we obtain

$x = 1000$

Number of Rs 100 notes = $2x = 2 \times 1000 = 2000$

Number of Rs 50 notes = $3x = 3 \times 1000 = 3000$

Number of Rs 10 notes = $5x = 5 \times 1000 = 5000$

Question 15:

I have a total of Rs 300 in coins of denomination Re 1, Rs 2 and Rs 5. The number of Rs 2 coins is 3 times the number of Rs 5 coins. The total number of coins is 160. How many coins of each denomination are with me?

Answer:

Let the number of Rs 5 coins be x.

Number of Rs 2 coins = 3 × Number of Rs 5 coins = $3x$

Number of Re 1 coins = 160 − (Number of coins of Rs 5 and of Rs 2)

$= 160 - (3x + x) = 160 - 4x$

Amount of Re 1 coins = Rs $[1 \times (160 - 4x)]$ = Rs $(160 - 4x)$

Amount of Rs 2 coins = Rs $(2 \times 3x)$= Rs $6x$

Amount of Rs 5 coins = Rs $(5 \times x)$ = Rs $5x$

It is given that the total amount is Rs 300.

$\therefore\ 160 - 4x + 6x + 5x = 300$

$160 + 7x = 300$

On transposing 160 to R.H.S, we obtain

$7x = 300 - 160$

$7x = 140$

On dividing both sides by 7, we obtain

$$\frac{7x}{7} = \frac{140}{7}$$

$x = 20$

Number of Re 1 coins $= 160 - 4x = 160 - 4 \times 20 = 160 - 80 = 80$

Number of Rs 2 coins $= 3x = 3 \times 20 = 60$

Number of Rs 5 coins $= x = 20$

Question 16:

The organizers of an essay competition decide that a winner in the competition gets a prize of Rs 100 and a participant who does not win gets a prize of Rs 25. The total prize money distributed is Rs 3000. Find the number of winners, if the total number of participants is 63.

Answer:

Let the number of winners be x. Therefore, the number of participants who did not win will be $63 - x$.

Amount given to the winners = Rs $(100 \times x)$ = Rs $100x$

Amount given to the participants who did not win = Rs $[25(63 - x)]$

= Rs $(1575 - 25x)$

According to the given question,

$100x + 1575 - 25x = 3000$

On transposing 1575 to R.H.S, we obtain

$75x = 3000 - 1575$

$75x = 1425$

On dividing both sides by 75, we obtain

$$\frac{75x}{75} = \frac{1425}{75}$$

$x = 19$

Hence, number of winners = 19

Exercise 2.3

Question 1:

Solve and check result: $3x = 2x + 18$

Answer:

$3x = 2x + 18$

On transposing $2x$ to L.H.S, we obtain

$3x - 2x = 18$

$x = 18$

L.H.S $= 3x = 3 \times 18 = 54$

R.H.S $= 2x + 18 = 2 \times 18 + 18 = 36 + 18 = 54$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 2:

Solve and check result: $5t - 3 = 3t - 5$

Answer:

$5t - 3 = 3t - 5$

On transposing $3t$ to L.H.S and -3 to R.H.S, we obtain

$5t - 3t = -5 - (-3)$

$2t = -2$

On dividing both sides by 2, we obtain

$t = -1$

L.H.S $= 5t - 3 = 5 \times (-1) - 3 = -8$

R.H.S $= 3t - 5 = 3 \times (-1) - 5 = -3 - 5 = -8$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 3:

Solve and check result: $5x + 9 = 5 + 3x$

Answer:

$5x + 9 = 5 + 3x$

On transposing $3x$ to L.H.S and 9 to R.H.S, we obtain

$5x - 3x = 5 - 9$

$2x = -4$

On dividing both sides by 2, we obtain

$x = -2$

L.H.S = $5x + 9 = 5 \times (-2) + 9 = -10 + 9 = -1$

R.H.S = $5 + 3x = 5 + 3 \times (-2) = 5 - 6 = -1$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 4:

Solve and check result: $4z + 3 = 6 + 2z$

Answer:

$4z + 3 = 6 + 2z$

On transposing $2z$ to L.H.S and 3 to R.H.S, we obtain

$4z - 2z = 6 - 3$

$2z = 3$

Dividing both sides by 2, we obtain

$$z = \frac{3}{2}$$

L.H.S = $4z + 3 = 4 \times \left(\frac{3}{2}\right) + 3 = 6 + 3 = 9$

R.H.S = $6 + 2z = 6 + 2 \times \left(\frac{3}{2}\right) = 6 + 3 = 9$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 5:

Solve and check result: $2x - 1 = 14 - x$

Answer:

$2x - 1 = 14 - x$

Transposing x to L.H.S and 1 to R.H.S, we obtain

$2x + x = 14 + 1$

$3x = 15$

Dividing both sides by 3, we obtain

$x = 5$

L.H.S = $2x - 1 = 2 \times (5) - 1 = 10 - 1 = 9$

R.H.S = $14 - x = 14 - 5 = 9$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 6:

Solve and check result: $8x + 4 = 3(x - 1) + 7$

Answer:

$8x + 4 = 3(x - 1) + 7$

$8x + 4 = 3x - 3 + 7$

Transposing $3x$ to L.H.S and 4 to R.H.S, we obtain

$8x - 3x = -3 + 7 - 4$

$5x = -7 + 7$

$x = 0$

L.H.S = $8x + 4 = 8 \times (0) + 4 = 4$

R.H.S = $3(x - 1) + 7 = 3(0 - 1) + 7 = -3 + 7 = 4$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 7:

Solve and check result: $x = \frac{4}{5}(x + 10)$

Answer:

$x = \frac{4}{5}(x + 10)$

Multiplying both sides by 5, we obtain

$5x = 4(x + 10)$

$5x = 4x + 40$

Transposing $4x$ to L.H.S, we obtain

$5x - 4x = 40$

$x = 40$

L.H.S = $x = 40$

R.H.S = $\frac{4}{5}(x+10)$ = $\frac{4}{5}(40+10) = \frac{4}{5} \times 50 = 40$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 8:

Solve and check result: $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

Answer:

$$\frac{2x}{3} + 1 = \frac{7x}{15} + 3$$

Transposing $\frac{7x}{15}$ to L.H.S and 1 to R.H.S, we obtain

$$\frac{2x}{3} - \frac{7x}{15} = 3 - 1$$

$$\frac{5 \times 2x - 7x}{15} = 2$$

$$\frac{3x}{15} = 2$$

$$\frac{x}{5} = 2$$

Multiplying both sides by 5, we obtain

$x = 10$

L.H.S = $\frac{2x}{3} + 1$ = $\frac{2 \times 10}{3} + 1 = \frac{2 \times 10 + 1 \times 3}{3} = \frac{23}{3}$

R.H.S = $\frac{7x}{15} + 3 = \frac{7 \times 10}{15} + 3 = \frac{7 \times 2}{3} + 3 = \frac{14}{3} + 3 = \frac{14 + 3 \times 3}{3} = \frac{23}{3}$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 9:

Solve and check result: $2y+\frac{5}{3}=\frac{26}{3}-y$

Answer:

$$2y+\frac{5}{3}=\frac{26}{3}-y$$

Transposing *y* to L.H.S and $\frac{5}{3}$ to R.H.S, we obtain

$$2y+y=\frac{26}{3}-\frac{5}{3}$$

$$3y=\frac{21}{3}=7$$

Dividing both sides by 3, we obtain

$$y=\frac{7}{3}$$

L.H.S = $2y+\frac{5}{3}=2\times\frac{7}{3}+\frac{5}{3}=\frac{14}{3}+\frac{5}{3}=\frac{19}{3}$

R.H.S = $\frac{26}{3}-y=\frac{26}{3}-\frac{7}{3}=\frac{19}{3}$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Question 10:

Solve and check result: $3m=5m-\frac{8}{5}$

Answer:

$$3m=5m-\frac{8}{5}$$

Transposing 5*m* to L.H.S, we obtain

$$3m-5m=-\frac{8}{5}$$

$$-2m=-\frac{8}{5}$$

Dividing both sides by −2, we obtain

$$m=\frac{4}{5}$$

L.H.S = $3m = 3\times\frac{4}{5}=\frac{12}{5}$

R.H.S = $5m-\frac{8}{5}=5\times\frac{4}{5}-\frac{8}{5}=\frac{12}{5}$

L.H.S. = R.H.S.

Hence, the result obtained above is correct.

Exercise 2.4

Question 1:

Amina thinks of a number and subtracts $\frac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

Answer:

Let the number be x.

According to the given question,

$8\left(x-\frac{5}{2}\right) = 3x$

$8x - 20 = 3x$

Transposing $3x$ to L.H.S and −20 to R.H.S, we obtain

$8x - 3x = 20$

$5x = 20$

Dividing both sides by 5, we obtain

$x = 4$

Hence, the number is 4.

Question 2:

A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

Answer:

Let the numbers be x and $5x$. According to the question,

$21 + 5x = 2(x + 21)$

$21 + 5x = 2x + 42$

Transposing $2x$ to L.H.S and 21 to R.H.S, we obtain

$5x - 2x = 42 - 21$

$3x = 21$

Dividing both sides by 3, we obtain

$x = 7$

$5x = 5 \times 7 = 35$

Hence, the numbers are 7 and 35 respectively.

Question 3:

Sum of the digits of a two digit number is 9. When we interchange the digits it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?

Answer:

Let the digits at tens place and ones place be x and $9 - x$ respectively.

Therefore, original number $= 10x + (9 - x) = 9x + 9$

On interchanging the digits, the digits at ones place and tens place will be x and $9 - x$ respectively.

Therefore, new number after interchanging the digits $= 10(9 - x) + x$

$= 90 - 10x + x$

$= 90 - 9x$

According to the given question,

New number = Original number + 27

$90 - 9x = 9x + 9 + 27$

$90 - 9x = 9x + 36$

Transposing $9x$ to R.H.S and 36 to L.H.S, we obtain

$90 - 36 = 18x$

$54 = 18x$

Dividing both sides by 18, we obtain

$3 = x$ and $9 - x = 6$

Hence, the digits at tens place and ones place of the number are 3 and 6 respectively.

Therefore, the two-digit number is $9x + 9 = 9 \times 3 + 9 = 36$

Question 4:

One of the two digits of a two digit number is three times the other digit. If you interchange the digit of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?

Answer:

Let the digits at tens place and ones place be x and $3x$ respectively.

Therefore, original number = $10x + 3x = 13x$

On interchanging the digits, the digits at ones place and tens place will be x and $3x$ respectively.

Number after interchanging = $10 \times 3x + x = 30x + x = 31x$

According to the given question,

Original number + New number = 88

$13x + 31x = 88$

$44x = 88$

Dividing both sides by 44, we obtain

$x = 2$

Therefore, original number = $13x = 13 \times 2 = 26$

By considering the tens place and ones place as $3x$ and x respectively, the two-digit number obtained is 62.

Therefore, the two-digit number may be 26 or 62.

Question 5:

Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of this mother's present age. What are their present ages?

Answer:

Let Shobo's age be x years. Therefore, his mother's age will be $6x$ years.

According to the given question,

$$\text{After 5 years, Shobo's age} = \frac{\text{Shobo's mother's present age}}{3}$$

$x + 5 = \frac{6x}{3}$

$x + 5 = 2x$

Transposing x to R.H.S, we obtain

$5 = 2x - x$

$5 = x$

$6x = 6 \times 5 = 30$

Therefore, the present ages of Shobo and Shobo's mother will be 5 years and 30 years respectively.

Question 6:

There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate Rs 100 per metre it will cost the village panchayat Rs 75, 000 to fence the plot. What are the dimensions of the plot?

Answer:

Let the common ratio between the length and breadth of the rectangular plot be x.

Hence, the length and breadth of the rectangular plot will be $11x$ m and $4x$ m respectively.

Perimeter of the plot = 2(Length + Breadth) $= \left[2\left(11x + 4x\right)\right]$ m $= 30x$ m

It is given that the cost of fencing the plot at the rate of Rs 100 per metre is Rs 75, 000.

$\therefore$ 100 × Perimeter = 75000

$100 \times 30x = 75000$

$3000x = 75000$

Dividing both sides by 3000, we obtain

$x = 25$

Length = $11x$ m = (11 × 25) m = 275 m

Breadth = $4x$ m = (4 × 25) m = 100 m

Hence, the dimensions of the plot are 275 m and 100 m respectively.

Question 7:

Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him Rs 50 per metre and trouser material that costs him Rs 90 per metre. For every 2 meters of the trouser material he buys 3 metres of the shirt material. He sells the materials at 12% and 10% profit respectively. His total sale is Rs 36660. How much trouser material did he buy?

Answer:

Let $2x$ m of trouser material and $3x$ m of shirt material be bought by him.

Per metre selling price of trouser material = $\text{Rs}\left(90+\frac{90\times 12}{100}\right)$ = Rs 100.80

Per metre selling price of shirt material = $\text{Rs}\left(50+\frac{50\times 10}{100}\right)$ = Rs 55

Given that, total amount of selling = Rs 36660

$100.80 \times (2x) + 55 \times (3x) = 36660$

$201.60x + 165x = 36660$

$366.60x = 36660$

Dividing both sides by 366.60, we obtain

$x = 100$

Trouser material = $2x$ m = (2×100) m = 200 m

Question 8:

Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

Answer:

Let the number of deer be x.

Number of deer grazing in the field = $\frac{x}{2}$

$$\text{Number of deer playing nearby} = \frac{3}{4} \times \text{Number of remaining deer}$$

$$= \frac{3}{4} \times \left(x - \frac{x}{2}\right) = \frac{3}{4} \times \frac{x}{2} = \frac{3x}{8}$$

Number of deer drinking water from the pond = 9

$$x - \left(\frac{x}{2} + \frac{3x}{8}\right) = 9$$

$$x - \left(\frac{4x + 3x}{8}\right) = 9$$

$$x - \frac{7x}{8} = 9$$

$$\frac{x}{8} = 9$$

Multiplying both sides by 8, we obtain

$x = 72$

Hence, the total number of deer in the herd is 72.

Question 9:

A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages

Answer:

Let the granddaughter's age be x years. Therefore, grandfather's age will be $10x$ years.

According to the question,

Grandfather's age = Granddaughter's age + 54 years

$10x = x + 54$

Transposing x to L.H.S, we obtain

$10x - x = 54$

$9x = 54$

$x = 6$

Granddaughter's age = x years = 6 years

Grandfather’s age = $10x$ years = (10×6) years = 60 years

Question 10:

Aman’s age is three times his son’s age. Ten years ago he was five times his son’s age. Find their present ages.

Answer:

Let Aman’s son’s age be x years. Therefore, Aman’s age will be $3x$ years. Ten years ago, their age was $(x - 10)$ years and $(3x - 10)$ years respectively.

According to the question,

10 years ago, Aman’s age = 5 × Aman’s son’s age 10 years ago

$3x - 10 = 5(x - 10)$

$3x - 10 = 5x - 50$

Transposing $3x$ to R.H.S and 50 to L.H.S, we obtain

$50 - 10 = 5x - 3x$

$40 = 2x$

Dividing both sides by 2, we obtain

$20 = x$

Aman’s son’s age = x years = 20 years

Aman’s age = $3x$ years = (3×20) years = 60 years

Exercise 2.5

Question 1:

Solve the linear equation $\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}$

Answer:

$$\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}$$

L.C.M. of the denominators, 2, 3, 4, and 5, is 60.

Multiplying both sides by 60, we obtain

$$60\left(\frac{x}{2}-\frac{1}{5}\right)=60\left(\frac{x}{3}+\frac{1}{4}\right)$$

∗ $30x - 12 = 20x + 15$ (Opening the brackets)

∗ $30x - 20x = 15 + 12$

∗ $10x = 27$

$$\Rightarrow x=\frac{27}{10}$$

Question 2:

Solve the linear equation $\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$

Answer:

$$\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$$

L.C.M. of the denominators, 2, 4, and 6, is 12.

Multiplying both sides by 12, we obtain

$6n - 9n + 10n = 252$

∗ $7n = 252$

$$\Rightarrow n=\frac{252}{7}$$

$$\Rightarrow n=36$$

Solve the linear equation $x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}$

Answer:

$$x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}$$

L.C.M. of the denominators, 2, 3, and 6, is 6.

Multiplying both sides by 6, we obtain

$6x + 42 - 16x = 17 - 15x$

$\Rightarrow 6x - 16x + 15x = 17 - 42$

$\Rightarrow 5x = -25$

$$\Rightarrow x=\frac{-25}{5}$$

$$\Rightarrow x=-5$$

Question 4:

Solve the linear equation $\frac{x-5}{3}=\frac{x-3}{5}$

Answer:

$$\frac{x-5}{3}=\frac{x-3}{5}$$

L.C.M. of the denominators, 3 and 5, is 15.

Multiplying both sides by 15, we obtain

$5(x - 5) = 3(x - 3)$

$\Rightarrow 5x - 25 = 3x - 9$ (Opening the brackets)

$\Rightarrow 5x - 3x = 25 - 9$

$\Rightarrow 2x = 16$

$$\Rightarrow x=\frac{16}{2}$$

$$\Rightarrow x=8$$

Solve the linear equation $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$

Answer:

$$\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$$

L.C.M. of the denominators, 3 and 4, is 12.

Multiplying both sides by 12, we obtain

$3(3t - 2) - 4(2t + 3) = 8 - 12t$

∗ $9t - 6 - 8t - 12 = 8 - 12t$ (Opening the brackets)

∗ $9t - 8t + 12t = 8 + 6 + 12$

∗ $13t = 26$

$$\Rightarrow t=\frac{26}{13}$$

$$\Rightarrow t=2$$

Question 6:

Solve the linear equation $m-\frac{m-1}{2}=1-\frac{m-2}{3}$

Answer:

$$m-\frac{m-1}{2}=1-\frac{m-2}{3}$$

L.C.M. of the denominators, 2 and 3, is 6.

Multiplying both sides by 6, we obtain

$6m - 3(m - 1) = 6 - 2(m - 2)$

∗ $6m - 3m + 3 = 6 - 2m + 4$ (Opening the brackets)

∗ $6m - 3m + 2m = 6 + 4 - 3$

∗ $5m = 7$

$$\Rightarrow m=\frac{7}{5}$$

Question 7:

Simplify and solve the linear equation $3(t-3)=5(2t+1)$

Answer:

3(t − 3) = 5(2t + 1)

∗ 3t − 9 = 10t + 5 (Opening the brackets)

∗ −9 − 5 = 10t − 3t

□ −14 = 7t

$\Rightarrow t=\frac{-14}{7}$

$\Rightarrow t=-2$

Question 8:

Simplify and solve the linear equation $15(y-4)-2(y-9)+5(y+6)=0$

Answer:

15(y − 4) − 2(y − 9) + 5(y + 6) = 0

□ 15y − 60 − 2y + 18 + 5y + 30 = 0 (Opening the brackets)

□ 18y − 12 = 0

□ 18y = 12

$\Rightarrow y=\frac{12}{18}=\frac{2}{3}$

Question 9:

Simplify and solve the linear equation $3(5z-7)-2(9z-11)=4(8z-13)-17$

Answer:

3(5z − 7) − 2(9z − 11) = 4(8z − 13)−17

□ 15z − 21 − 18z + 22 = 32z − 52 − 17 (Opening the brackets)

□ −3z + 1 = 32z − 69

□ −3z − 32z = −69 − 1

□ −35z = −70

$\Rightarrow z = \frac{70}{35} = 2$

Question 10:

Simplify and solve the linear equation $0.25(4f - 3) = 0.05(10f - 9)$

Answer:

0.25(4*f* − 3) = 0.05(10*f* − 9)

$\frac{1}{4}(4f - 3) = \frac{1}{20}(10f - 9)$

Multiplying both sides by 20, we obtain

5(4*f* − 3) = 10*f* − 9

□ 20*f* − 15 = 10*f* − 9 (Opening the brackets)

□ 20*f* − 10*f* = − 9 + 15

□ 10*f* = 6

$\Rightarrow f = \frac{3}{5} = 0.6$

Exercise 2.6

Question 1:

Solve: $\frac{8x-3}{3x}=2$

Answer:

$$\frac{8x-3}{3x}=2$$

On multiplying both sides by 3*x*, we obtain

$8x - 3 = 6x$

□ $8x - 6x = 3$

□ $2x = 3$

$\Rightarrow x=\frac{3}{2}$

Solve: $\frac{9x}{7-6x}=15$

Answer:

$$\frac{9x}{7-6x}=15$$

Onmultiplying both sides by $7 - 6x$, we obtain

$9x = 15(7 - 6x)$

□ $9x = 105 - 90x$

□ $9x + 90x = 105$

□ $99x = 105$

$\Rightarrow x=\frac{105}{99}=\frac{35}{33}$

Question 3:

Solve: $\frac{z}{z+15}=\frac{4}{9}$

Answer:

$$\frac{z}{z+15}=\frac{4}{9}$$

On multiplying both sides by 9(z + 15), we obtain

$9z = 4(z + 15)$

□ $9z = 4z + 60$

□ $9z - 4z = 60$

□ $5z = 60$

□ $z = 12$

Question 4:

Solve: $\frac{3y+4}{2-6y}=\frac{-2}{5}$

Answer:

$$\frac{3y+4}{2-6y}=-\frac{2}{5}$$

On multiplying both sides by 5(2 − 6y), we obtain

$5(3y + 4) = -2(2 - 6y)$

□ $15y + 20 = -4 + 12y$

□ $15y - 12y = -4 - 20$

□ $3y = -24$

□ $y = -8$

Question 5:

Solve: $\frac{7y+4}{y+2}=\frac{-4}{3}$

Answer:

$$\frac{7y+4}{y+2}=-\frac{4}{3}$$

On multiplying both sides by 3(y + 2), we obtain

$3(7y + 4) = -4(y + 2)$

□ $21y + 12 = -4y - 8$

□ $21y + 4y = -8 - 12$

□ $25y = -20$

$\Rightarrow y = -\frac{4}{5}$

Question 6:

The ages of Hari and Harry are in the ratio 5:7. Four years from now the ratio of their ages will be 3:4. Find their present ages.

Answer:

Let the common ratio between their ages be x. Therefore, Hari's age and Harry's age will be $5x$ years and $7x$ years respectively and four years later, their ages will be $(5x + 4)$ years and $(7x + 4)$ years respectively.

According to the situation given in the question,

$$\frac{5x+4}{7x+4} = \frac{3}{4}$$

$$\Rightarrow 4(5x+4) = 3(7x+4)$$

$$\Rightarrow 20x + 16 = 21x + 12$$

$$\Rightarrow 16 - 12 = 21x - 20x$$

$$\Rightarrow 4 = x$$

Hari's age = $5x$ years = (5×4) years = 20 years

Harry's age = $7x$ years = (7×4) years = 28 years

Therefore, Hari's age and Harry's age are 20 years and 28 years respectively.

Question 7:

The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

Answer:

Let the numerator of the rational number be x. Therefore, its denominator will be $x + 8$.

The rational number will be $\frac{x}{x+8}$. According to the question,

$$\frac{x+17}{x+8-1}=\frac{3}{2}$$

$$\Rightarrow \frac{x+17}{x+7}=\frac{3}{2}$$

□ $2(x + 17) = 3(x + 7)$

□ $2x + 34 = 3x + 21$

□ $34 - 21 = 3x - 2x$

$\Rightarrow 13 = x$

Numerator of the rational number = x = 13

Denominator of the rational number = $x + 8 = 13 + 8 = 21$

$$=\frac{13}{21}$$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH. 2 LINEAR EQUATIONS IN ONE VARIABLE

MIND MAP

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

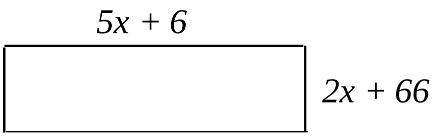
TYPE:1 SOLVING EQUATIONS WHICH HAVE LINEAR EXPRESSIONS ON ONE SIDE AND NUMBERS ON THE OTHER SIDE AND ITS APPLICATIONS

Q.1. Solve:

a) $0.4\,(3x-1)-\ 0.5x=1$

b) $\frac{2x+20}{25}=4$

Q.2. For what value of x is the perimeter of shape 186 cm?



Q.3. Radha takes some flowers in a basket and visits three temples one- by- one.At each temple she offers one half of the flowers from the basket. If she is left with 3 flowers at the end, then find the number of flowers she had in the beginning.

TYPE:2 SOLVING EQUATIONS HAVING VARIABLES ON BOTH SIDES AND ITS APPLICATIONS

1.1. Solve:

a) $m-\frac{m-1}{2}=1-\frac{m-2}{3}$

b) $0.25\,(4x-5)=0.75x+8$

2.2. Five years ago , a man was 7 times as old as his son. Five years hence, he will be three times as old as his son. Find their present ages.

3.3. The digit in the tens place of a two digit number is three times the digit in the unit's place. If the digits are reversed , the new number will be 36 less than the original number. Find the number.

TYPE:3 REDUCING EQUATIONS TO SIMPLER FORMS

Q.1. Solve: (a) $\frac{2+3y}{1+5y}=\frac{4}{3}$ (b) $\frac{2}{x+9}=\frac{-7}{4x-3}$

CH. 9 AGEBRAIC EXPRESSIONS AND IDENTITIES

Exercise 9.1 Page No: 140

1. **Identify the terms, their coefficients for each of the following expressions.**
   (i) $5xyz^2 - 3zy$
   (ii) $1 + x + x^2$
   (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$
   (iv) $3 - pq + qr - p$
   (v) $(x/2) + (y/2) - xy$
   (vi) $0.3a - 0.6ab + 0.5b$

Solution :

Sl. No.	Expression	Term	Coefficient
i)	$5xyz^2 - 3zy$	Term: $5xyz^2$ Term: $-3zy$	5 -3
ii)	$1 + x + x^2$	Term: *1* Term: x *Term* x^2	1 1 1
iii)	$4x^2y^2 - 4x^2y^2z^2 + z^2$	Term: $4x^2y^2$ Term: $-4x^2y^2z^2$ *Term :* z^2	4 -4 1
iv)	$3 - pq + qr - p$	3 -pq qr -p	3 -1 1 -1
v)	$(x/2) + (y/2) - xy$	x/2 Y/2 -xy	½ 1/2 -1
vi)	$0.3a - 0.6ab + 0.5b$	0.3a -0.6ab 0.5b	0.3 -0.6 0.5

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?
$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y – 3y^2$, $2y – 3y^2 + 4y^3$, $5x – 4y + 3xy$, $4z – 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Solution:
Let us first define the classifications of these 3 polynomials:
Monomials, Contain only one term.
Binomials, Contain only two terms.
Trinomials, Contain only three terms.

$x + y$	two terms	Binomial
1000	one term	Monomial
$x + x^2 + x^3 + x^4$	four terms	Polynomial, and it does not fit in listed three categories
$2y – 3y^2$	two terms	Binomial
$2y – 3y^2 + 4y^3$	three terms	Trinomial
$5x – 4y + 3xy$	three terms	Trinomial
$4z – 15z^2$	two terms	Binomial
$ab + bc + cd + da$	four terms	Polynomial, and it does not fit in listed three categories
pqr	one term	Monomial
$p^2q + pq^2$	two terms	Binomial
$2p + 2q$	two terms	Binomial

3. Add the following.
(i) $ab – bc, bc – ca, ca – ab$
(ii) $a – b + ab, b – c + bc, c – a + ac$
(iii) $2p^2q^2 – 3pq + 4, 5 + 7pq – 3p^2q^2$
(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

Solution:

i) $(ab - bc) + (bc - ca) + (ca-ab)$
$= ab - bc + bc - ca + ca - ab$
$= ab - ab – bc + bc – ca + ca$
$= 0$

ii) $(a - b + ab) + (b - c + bc) + (c - a + ac)$
$= a - b + ab + b - c + bc + c - a + ac$
$= a - a +b – b +c – c + ab + bc + ca$
$=0 + 0 + 0 + ab + bc + ca$
$= ab + bc + ca$

iii) $2p^2q^2 – 3pq + 4, 5 + 7pq – 3p^2q^2$
$= (2p^2q^2 - 3pq + 4) + (5 + 7pq - 3p^2q^2)$
$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$
$= - p^2q^2 + 4pq + 9$

iv) $(l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl)$
$= l^2 + l^2 + m^2 + m^2 + n^2 + n^2 + 2lm + 2mn + 2nl$
$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

4. (a) Subtract $4a - 7ab + 3b + 12$ **from** $12a - 9ab + 5b - 3$

(b) Subtract $3xy + 5yz - 7zx$ **from** $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ **from** $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Solution:

(a) $(12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$

$= 12a - 9ab + 5b - 3 - 4a + 7ab - 3b - 12$

$= 12a - 4a - 9ab + 7ab + 5b - 3b - 3 - 12$

$= 8a - 2ab + 2b - 15$

b) $(5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)$

$= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx$

$= 5xy - 3xy - 2yz - 5yz - 2zx + 7zx + 10xyz$

$= 2xy - 7yz + 5zx + 10xyz$

c) $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

$= 18 + 10 - 3p + 8p - 11q - 7q + 5pq + 3pq - 2pq^2 - 5pq^2 + 5p^2q - 4p^2q$

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

1. Find the product of the following pairs of monomials.

(i) **4, 7p**
(ii) **- 4p, 7p**
(iii) **– 4p, 7pq**
(iv) **$4p^3$, - 3p**
(v) **4p, 0**

Solution:

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7)(p \times pq) = -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3)(p^3 \times p) = -12p^4$

(v) $4p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q) ; (10m, 5n) ; ($20x^2$, $5y^2$) ; (4x, $3x^2$) ; (3mn, 4np)

Solution:

Area of rectangle = Length x breadth. So, it is multiplication of two monomials.
The results can be written in square units.

(i) $p \times q = pq$

(ii) $10m \times 5n = 50mn$

(iii) $20x^2 \times 5y^2 = 100x^2y^2$

(iv) $4x \times 3x^2 = 12x^3$

(v) $3mn \times 4np = 12mn^2p$

3. Complete the following table of products:

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Solution:

First monomial / Second monomial	2x	-5y	$3x^2$	-4xY	$7x^2Y$	$-9x^2y^2$
2x	$4x^2$	-10xy	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
-5y	-10xy	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
-4xy	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27 x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

***(i)** $5a, 3a^2, 7a^4$*
***(ii)** $2p, 4q, 8r$*
(iii) $xy, 2x^2y, 2xy^2$
***(iv)** $a, 2b, 3c$*

Solution:

Volume of rectangle = length x breadth x height. To evaluate volume of rectangular boxes, multiply all the monomials.

(i) $5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)\ (a \times a^2 \times a^4) = 105a^7$

(ii) $2p \times 4q \times 8r = (2 \times 4 \times 8)\ (p \times q \times r) = 64pqr$

(iii) $y \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2)\ (x \times x^2 \times x \times y \times y \times y^2) = 4x^4y^4$

(iv) $a \times 2b \times 3c = (1 \times 2 \times 3)\ (a \times b \times c) = 6abc$

5. Obtain the product of

(i) xy, yz, zx
(ii) $a, -a^2, a^3$
(iii) $2, 4y, 8y^2, 16y^3$
(iv) $a, 2b, 3c, 6abc$
(v) $m, -mn, mnp$

Solution:

(i) $xy \times yz \times zx = x^2y^2z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2b^2c^2$

(v) $m \times -mn \times mnp = -m^3n^2p$

Exercise 9.3 Page No: 146

1. Carry out the multiplication of the expressions in each of the following pairs.
(i) $4p, q + r$
(ii) $ab, a - b$
(iii) $a + b, 7a^2b^2$
(iv) $a^2 - 9, 4a$
(v) $pq + qr + rp, 0$

Solution:

(i) $4p(q + r) = 4pq + 4pr$

(ii) $ab(a - b) = a^2b - ab^2$

(iii) $(a + b)(7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9)(4a) = 4a^3 – 36a$

(v) $(pq + qr + rp) \times 0 = 0$ (Anything multiplied by zero is zero)

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

Solution:

	First expression	Second expression	Product
(i)	a	b + c + d	$a(b+c+d)$ $= a\times b + a\times c + a\times d$ $= ab + ac + ad$
(ii)	x + y - 5	5xy	$5xy(x + y - 5)$ $= 5xy * x + 5xy * y - 5xy * 5$ $= 5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$p(6p^2 - 7p + 5)$ $= p\times 6p^2 - p\times 7p + p\times 5$ $= 6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$P^2 - q^2$	4p^2 q^2 * (p^2 – q^2) =4 p^4 q^2 - 4p^2 q^4
(v)	a + b + c	abc	$abc(a + b + c)$ $= abc \times a + abc \times b + abc \times c$ $= a^2bc + ab^2c + abc^2$

3. Find the product.
i) a^2 x $(2a^{22})$ x $(4a^{26})$
ii) (2/3 xy) ×(-9/10 x^2y^2)
(iii) (-10/3 pq^3/) × (6/5 p^3q)
(iv) (x) × (x^2) × (x^3) × (x^4)

Solution:

i) a^2 x $(2a^{22})$ x $(4a^{26})$ = (2 × 4) (a^2 × a^22 × a^26) = 8 × a^(2+22+26) = $8a^{50}$

ii) (2xy/3) ×(-9x^2y^2/10)
=(2/3 × -9/10) (x × x^2 × y × y^2)
= (-3/5 x^3y^3)

iii) (-10pq^3/3) ×(6p^3q/5)
= (-10/3 × 6/5) (p × p^3 × q^3 × q)
= (-4p^4q^4)

iv) (x) x (x^2) x (x^3) x (x^4)
= x^ (1+2+3+8)
= x^{10}

4. (a) Simplify 3x (4x – 5) + 3 and find its values for (i) x = 3 (ii) x =1/2

(b) Simplify *a (a²+ a + 1) + 5* and find its value for (i) *a = 0*, (ii) *a = 1* (iii) *a = – 1*.

Solution:

a) 3x (4x – 5) + 3

=3x (4x) – 3x(5) +3

=12x^2 - 15x + 3

(i) Putting x=3 in the equation we gets
12x^2 - 15x + 3 =12(3^2) – 15 (3) +3
= 108 – 45 + 3 = 66

(ii) Putting x=1/2 in the equation we get

12x^2 - 15x + 3 = 12 (1/2)^2 – 15 (1/2) + 3

=12 (1/4) - 15/2 +3

= 3 - 15/2 + 3

$= 6- 15/2$
$= (12- 15) /2$
$= -3/2$

b) $a(a^2 +a +1)+5$
$=a^* a\hat{}2 + a^*a + a^*1 + 5$
$=a^3+a^2+a+ 5$

(i) putting a= 0 in the equation we get
$0^3+0^2+0+5=5$

(ii) putting a=1 in the equation we get
$1^3 + 1^2 + 1+5 = 1 + 1 + 1+5 = 8$

(iii) Putting a = -1 in the equation we get
$(-1)\_^3+(-1)^2 + (-1)+5 = -1 + 1 – 1+5 = 4$

5.* (a) Add: *p (p – q), q (q – r) and r (r – p)
(b)* Add: *2x (z – x – y)* and *2y (z – y – x)
(c)* Subtract: *3l (l – 4 m + 5 n)* from *4l (10 n – 3 m + 2 l)
(d)* Subtract: *3a (a + b + c) – 2 b (a – b + c)* from *4c (– a + b + c)

Solution:

a) p (p – q) + q (q – r) + r (r – p)

$= (p^2 - pq) + (q^2 - qr) + (r^2 - pr)$
$= p^2 + q^2 + r^2 - pq - qr - pr$

b) *2x (z – x – y) + 2y (z – y – x)*

$= (2xz - 2x^2 - 2xy) + (2yz - 2y^2 - 2xy)$
$= 2xz - 4xy + 2yz - 2x^2 - 2y^2$

c) *4l (10 n – 3 m + 2 l) - 3l (l – 4 m + 5 n)*
$=(40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$
$= 40ln - 12lm + 8l^2 - 3l^2 +12lm -15 ln$
$= 25 ln + 5l^2$

d) *4c (– a + b + c) - (3a (a + b + c) – 2 b (a – b + c))*
$= (-4ac + 4bc + 4c^2) - (3a^2 + 3ab + 3ac – (2ab – 2b\hat{}2 + 2bc))$
$=-4ac + 4bc + 4c^2 - (3a^2 + 3ab + 3ac - 2ab + 2b\hat{}2 - 2bc)$

$= -4ac + 4bc + 4c^2 - 3a^2 - 3ab - 3ac +2ab – 2b\hat{}2 + 2bc$
$= -7ac + 6bc + 4c^2 - 3a^2 – ab – 2b\hat{}2$

1. Multiply the binomials.

i) (2x + 5) and (4x – 3)

(ii) (y – 8) and (3y – 4)

(iii) (2.5l – 0.5m) and (2.5l + 0.5m)

(iv) (a + 3b) and (x + 5)

(v) $(2pq + 3q^2)$ and $(3pq – 2q^2)$

(vi) (3/4 a^2 + 3b^2) and 4(a^2 - 2/3 b^2)

Solution :

i) $(2x + 5)(4x - 3)$

$= 2x * 4x - 2x * 3 + 5 * 4x - 5 * 3$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + 14x - 15$

ii) (y - 8)(3y - 4)

= y * 3y - 4y - 8 * 3y + 32

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$

(iii) (2.5l - 0.5m)(2.5l + 0.5m)

= 2.5l * 2.5 l + 2.5l * 0.5m – 0.5m * 2.5l - 0.5m * 0.5m

= 6.25l^2 + 1.25 lm – 1.25 lm - 0.25 m^2

= 6.25l^2 - 0.25 m^2

iv) (a + 3b) (x + 5)

= ax + 5a + 3bx + 15b

v) $(2pq + 3q^2)(3pq – 2q^2)$

$= 2pq * 3pq - 2pq * 2q^2 + 3q^2 * 3pq - 3q^2 * 2q^2$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 – 6q^4$

(vi) (3/4 a^2 + 3b^2) and 4(a^2 - 2/3 b^2)

= (3/4 a^2 + 3b^2) * 4(a^2 - 2/3 b^2)

=(3/4 a^2 + 3b^2) * (4a^2 - 8/3 b^2)

=3/4 a^2 * (4a^2 - 8/3 b^2) + 3b^2 * (4a^2 - 8/3 b^2)

=3/4 a^2 * 4a^2 -3/4 a^2 * 8/3 b^2 + 3b^2 * 4a^2 - 3b^2* 8/3 b^2

=3a^4 – 2a^2b^2 + 12 a^2 b^2 - 8b^4

= 3a^4 + 10a^2 b^2 – 8b^4

2. Find the product.

(i) $(5 – 2x) (3 + x)$
(ii) $(x + 7y) (7x – y)$
(iii) $(a^2+ b) (a + b^2)$
(iv) $(p^2 – q^2) (2p + q)$

Solution:

i) $(5 – 2x) (3 + x)$

$= 5 (3 + x) - 2x (3 + x)$
$=15 + 5x - 6x - 2x^2$
$= 15 - x -2 x^2$

(ii) $(x + 7y) (7x – y)$

$= x(7x-y) + 7y (7x-y)$

$=7x^2 - xy + 49xy - 7y^2$
$= 7x^2 - 7y^2 + 48xy$

iii) $(a^2+ b) (a + b^2)$
$= a^2 (a + b^2) + b(a + b^2)$
$= a^3 + a^2$b^2 + ab + b^3
$= a^3 + b^3 + a^2$b^2 + ab

iv) $(p^2– q^2) (2p + q)$

$= p^2 (2p + q) - q^2 (2p + q)$
$=2p^3 + p^2q - 2pq^2 - q^3$
$= 2p^3 - q^3 + p^2q - 2pq^2$

3. Simplify.
(i) $(x^2– 5) (x + 5) + 25$
(ii) $(a^2+ 5) (b^3+ 3) + 5$
(iii)$(t + s^2)(t^2 – s)$
(iv) $(a + b) (c – d) + (a – b) (c + d) + 2 (ac + bd)$
(v) $(x + y)(2x + y) + (x + 2y)(x – y)$
(vi) $(x + y)(x^2– xy + y^2)$
(vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$
(viii) $(a + b + c)(a + b – c)$

Solution :
i) $(x^2– 5) (x + 5) + 25$
$= x^3 + 5x^2 - 5x - 25 + 25$
$= x^3 + 5x^2 - 5x$

ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 5b^3 + 3a^2 + 20$

iii) $(t + s^2)(t^2 – s)$

$= t(t^2 – s) + s^2(t^2 – s)$

$= t^3 – st + s^2t^2 – s^3$

$= t^3 – s^3 – st + s^2t^2$

iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

$= (a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

$=(ac - ad + bc - bd) + (ac + ad - bc - bd) + (2ac + 2bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= 4ac$

v) $(x + y)(2x + y) + (x + 2y)(x – y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= 3x^2 + 4xy - y^2$

vi) $(x + y)(x^2 – xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3$

vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 - 16y^2$

viii) $(a + b + c)(a + b – c)$

$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$

$= a^2 + b^2 - c^2 + 2ab$

Exercise 9.5 Page No: 151

1. Use a suitable identity to get each of the following products.

(i) (x + 3) (x + 3)
(ii) (2y + 5) (2y + 5)
(iii) (2a - 7) (2a - 7)
(iv) (3a - 1/2)(3a - 1/2)
(v) (1.1m – 0.4) (1.1m + 0.4)
(vi) $(a^2+ b^2) (- a^2+ b^2)$
(vii) (6x - 7) (6x + 7)
(viii) (- a + c) (- a + c)
(ix) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right)$
(x) (7a - 9b) (7a - 9b)

Solution:

i) $(x + 3) (x + 3) = (x + 3)^2$
$= x^2 + 6x + 9$
Using $(a+b)^2 = a^2 + b^2 + 2ab$

ii)
$(2y + 5) (2y + 5) = (2y + 5)^2$
$= 4y^2 + 20y + 25$
Using $(a+b)^2 = a^2 + b^2 + 2ab$

iii) $(2a – 7) (2a – 7) = (2a – 7)^2$
$= 4a^2 – 28a + 49$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

iv) $(3a - 1/2)(3a - 1/2) = (3a - 1/2)^2$

$= (3a-1/2)(3a-1/2)= 9a^2 -3a+(1/4)$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

v) $(1.1m – 0.4) (1.1m + 0.4)$

$= 1.21m^2 – 0.16$

Using $(a - b)(a + b) = a^2 - b^2$

vi) $(a^2+ b^2) (– a^2+ b^2)$
$= (b^2 + a^2) (b^2 – a^2)$
$= -a^4 + b^4$
Using $(a - b)(a + b) = a^2 - b^2$

vii)
$(6x - 7)(6x + 7)$
$= 36x^2 - 49$
Using $(a - b)(a + b) = a^2 - b^2$

viii) $(-a + c)(-a + c) = (-a + c)^2$
$= c^2 + a^2 - 2ac$
Using $(a-b)^2 = a^2 + b^2 - 2ab$

ix) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$

$= (x^2/4) + (9y^2/16) + (3xy/4)$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

x) $(7a - 9b)(7a - 9b) = (7a - 9b)^2$

$= 49a^2 - 126ab + 81b^2$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.
(i) $(x + 3)(x + 7)$
(ii) $(4x + 5)(4x + 1)$
(iii) $(4x - 5)(4x - 1)$
(iv) $(4x + 5)(4x - 1)$
(v) $(2x + 5y)(2x + 3y)$
(vi) $(2a^2 + 9)(2a^2 + 5)$
(vii) $(xyz - 4)(xyz - 2)$

Solution:

i) $(x + 3)(x + 7)$

$= x^2 + (3+7)x + 21$
$= x^2 + 10x + 21$

ii) $(4x + 5)(4x + 1)$
$= 16x^2 + (5 + 1)4x + 5$
$= 16x^2 + 24x + 5$

iii) $(4x - 5)(4x - 1)$
$= 16x^2 + (-5-1)4x + 5$
$= 16x^2 - 20x + 5$

iv) $(4x + 5)(4x - 1)$
$= 16x^2 + (5-1)4x - 5$
$= 16x^2 + 16x - 5$

v) $(2x + 5y)(2x + 3y)$
$= 4x^2 + (5y + 3y)2x + 15y^2$
$= 4x^2 + 16xy + 15y^2$

vi) $(2a^2 + 9)(2a^2 + 5)$
$= 4a^4 + (9+5)2a^2 + 45$
$= 4a^4 + 28a^2 + 45$

vii) $(xyz - 4)(xyz - 2)$
$= x^2y^2z^2 + (-4 -2)xyz + 8$
$= x^2y^2z^2 - 6xyz + 8$

3. Find the following squares by using the identities.
(i) $(b - 7)^2$
(ii) $(xy + 3z)^2$
(iii) $(6x^2 - 5y)^2$
(iv) $[(2m/3) + (3n/2)]^2$
(v) $(0.4p – 0.5q)^2$
(vi) $(2xy + 5y)^2$

Solution:
Using identities:
$(a - b)^2 = a^2 + b^2 - 2ab$
$(a + b)^2 = a^2 + b^2 + 2ab$

(i) $(b - 7)^2 = b^2 - 14b + 49$

(ii) $(xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2 = 36x^4 - 60x^2y + 25y^2$

(iv) $[(2m/3\}) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$

(v) $(0.4p – 0.5q)^2 = 0.16p^2 - 0.4pq + 0.25q^2$

(vi) (2xy + 5y)^2 = $4x^2y^2 + 20xy^2 + 25y^2$

4. Simplify.

(i) $(a^2 – b^2)^2$
(ii) $(2x + 5)^2 - (2x - 5)^2$
(iii) $(7m – 8n)^2 + (7m + 8n)^2$
(iv) $(4m + 5n)^2 + (5m + 4n)^2$
(v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
(vi) $(ab + bc)^2 – 2ab^2c$
(vii) $(m^2 – n^2m)^2 + 2m^3n^2$

Solution:

i) $(a^2 – b^2)^2 = a^4 + b^4 - 2a^2b^2$

ii) $(2x + 5)^2 - (2x - 5)^2$
$= 4x^2 + 20x + 25 - (4x^2 - 20x + 25)$
$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$
$= 40x$

iii) $(7m – 8n)^2 + (7m + 8n)^2$
$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 49n^2$
$= 98m^2 + 128n^2$

iv) $(4m + 5n)^2 + (5m + 4n)^2$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$

v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$
$= 4p^2 - 4q^2$

vi) $(ab + bc)^2 – 2ab^2c$
$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$
$= a^2b^2 + b^2c^2$

vii) $(m^2 – n^2m)^2 + 2m^3n^2$
$= m^4 - 2m^3n^2 + m^2n^4 + 2m^3n^2$
$= m^4 + m^2n^4$

5. Show that.

(i) $(3x + 7)^2 – 84x = (3x – 7)^2$
(ii) $(9p – 5q)^2 + 180pq = (9p + 5q)^2$
(iii) $(\frac{4}{3}m - \frac{3}{4}n)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
(iv) $(4pq + 3q)^2 – (4pq – 3q)^2 = 48pq^2$
(v) $(a – b)(a + b) + (b – c)(b + c) + (c – a)(c + a) = 0$

Solution:

i) LHS = $(3x + 7)^2 - 84x$
$= 9x^2 + 42x + 49 - 84x$
$= 9x^2 - 42x + 49$
= RHS
LHS = RHS

ii) LHS = $(9p - 5q)^2 + 180pq$
$= 81p^2 - 90pq + 25q^2 + 180pq$
$= 81p^2 + 90pq + 25q^2$
RHS = $(9p + 5q)^2$
$= 81p^2 + 90pq + 25q^2$
LHS = RHS

(iii) $LHS = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

= RHS
LHS = RHS

iv) LHS = $(4pq + 3q)^2 - (4pq - 3q)^2$

= $16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$
$= 48pq^2$

RHS = $48pq^2$
LHS = RHS

v) LHS = $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$
= $a^2 - b^2 + b^2 - c^2 + c^2 - a^2$
$= 0$
= RHS

6. Using identities, evaluate.
(i) 71^2
(ii) 99^2
(iii) 102^2
(iv) 998^2
(v) 5.2^2
(vi) 297 x 303
(vii) 78 x 82
(viii) 8.9^2
(ix) 10.5 x 9.5

Solution:

i) $71^2 = (70+1)^2$
$= 70^2 + 140 + 1^2$
$= 4900 + 140 + 1$
$= 5041$

ii) 99^2
$= (100 - 1)^2$
$= 100^2 - 200 + 1^2$
$= 10000 - 200 + 1$
$= 9801$

iii) $102^2 = (100 + 2)^2$
$= 100^2 + 400 + 2^2$
$= 10000 + 400 + 4$
$= 10404$

iv) $998^2 = (1000 - 2)^2$
$= 1000^2 - 4000 + 2^2$
$= 1000000 - 4000 + 4$
$= 996004$

v) $5.2^2 = (5 + 0.2)^2$
$= 5^2 + 2 + 0.2^2$
$= 25 + 2 + 0.4$
$= 27.4$

vi) 297×303
$= (300 - 3)(300 + 3)$
$= 300^2 - 3^2$
$= 90000 - 9$
$= 89991$

vii) 78×82
$= (80 - 2)(80 + 2)$
$= 80^2 - 2^2$
$= 6400 - 4$
$= 6396$

viii) $8.9^2 = (9 - 0.1)^2$
$= 9^2 - 1.8 + 0.1^2$
$= 81 - 1.8 + 0.01$
$= 79.21$

ix) 10.5 x 9.5
$= (10 + 0.5)(10 - 0.5)$
$= 10^2 - 0.5^2$
$= 100 - 0.25$
$= 99.75$

7. Using $a^2 - b^2 = (a + b)(a - b)$, find
(i) $51^2 - 49^2$
(ii) $(1.02)^2 - (0.98)^2$
(iii) $153^2 - 147^2$
(iv) $12.1^2 - 7.9^2$

Solution:

i) $51^2 - 49^2$
$= (51 + 49)(51 - 49)$
$= 100 \times 2$
$= 200$

ii) $(1.02)^2 - (0.98)^2$
$= (1.02 + 0.98)(1.02 - 0.98)$
$= 2 \times 0.04$
$= 0.08$

iii) $153^2 - 147^2$
$= (153 + 147)(153 - 147)$
$= 300 \times 6$
$= 1800$

iv) $12.1^2 - 7.9^2$
$= (12.1 + 7.9)(12.1 - 7.9)$
$= 20 \times 4.2 = 84$

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find
(i) 103 x 104
(ii) 5.1 x 5.2
(iii) 103 x 98
(iv) 9.7 x 9.8

Solution:

i) 103 x 104
$= (100 + 3)(100 + 4)$
$= 100^2 + (3 + 4)100 + 12$
$= 10000 + 700 + 12$
$= 10712$

ii) 5.1 x 5.2
= (5 + 0.1)(5 + 0.2)
= 5^2 + (0.1 + 0.2)5 + 0.1 x 0.2
= 25 + 1.5 + 0.02
= 26.52

iii) 103 x 98
= (100 + 3)(100 - 2)
= 100^2 + (3-2)100 - 6
= 10000 + 100 - 6
= 10094

iv) 9.7 x 9.8
= (9 + 0.7)(9 + 0.8)
= 9^2 + (0.7 + 0.8)9 + 0.56
= 81 + 13.5 + 0.56
= 95.06

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.: 9 ALGEBRAIC EXPRESSIONS AND IDENTITIES

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of various topics. The most probable questions from examination point of view are given below.

TYPE:1 IDENTIFYING TERMS,COEFFICIENTS AND TYPE OF ALGEBRAIC EXPRESSION

1. Identify Terms and their Coefficients:
$(i) 1 + x + xy^2$ $(ii) \frac{x}{2} + \frac{y}{2} - xy$

2. Classify these polynomials as monomials, binomials, trinomials.
$(i) 100$ $(ii) x + y + z$ $(iii) x + x^2 + x^3 + x^4$

TYPE: 2 OPERATIONS ON ALGEBRAIC EXPRESSIONS

3. Add and subtract (second from first) the following algebraic expressions $(i) 2p^2 q^2 - 3pq + 4, 5 + 7p^2q^2 - 3$
$(ii) 3xyz + 5yz - 7zx, 5xy - 2yz + 10xyz$

4. Obtain the product of the following:
$(i) (y^2)(-x^3)(xy^6)$ $(ii) \left[\frac{7}{-3}p^2q\right]\left[\frac{3}{7}q^2p\right]$

5. Find the product of $-5x^2y, -\frac{2}{3}xy^2z, \frac{8}{15}xyz^2 and\ -\frac{1}{4}z$. Verify the result when

$$x = 1, y = 2\ and\ z = 3$$

6. Simplify $(i)(5x-2)(3+x)$ $(ii)\ (p^2-q^2)(2p+q)$
$(iii)(a+b)\ (c-d)+(a-b)(c+d)+6(ac+bd)$

TYPE: 3 USE OF IDENTITIES

7. Use suitable identity to get the products:
$(i)(6x-7)(6x+7)$ $(ii)(-a+c)(-a+c)$ $(iii)\left(3x+\frac{1}{2}\right)\left(3x+\frac{1}{2}\right)$
$(iv)(4x+5)(4x+1)$ $(v)(4x-5)(4x-1)$ $(vi)\ (4x+5)(4x-1)$
8. Find the squares by using identities:
$(i)(6x^2-5y)^2$ $(ii)\ (xy+z)^2$
9. Simplify using identity:
$(i)(2y+3)^2-(2y-3)^2$ $(ii)(ab+bc)^2-2ab^2c$ $(iii)\ (b^2-c^2)^2$
10. Using Identities evaluate:
$(i)(103)^2$ $(ii)\ 78\times 82$ $(iii)\ (1.02)^2-(0.98)^2$ $(iv)\ (5.1)\times(5.2)$

CH-10 VISUALISING SOLID SHAPES

Exercise 10.3 Page No: 166

1. **Can a polyhedron have for its faces:**
   - **(i) 3 Triangles?**
   - **(ii) 4 triangles?**
   - **(iii) A square and four triangles?**

Solution:

i) No, such polyhedrons are not possible. Such figures should have minimum 4 faces.

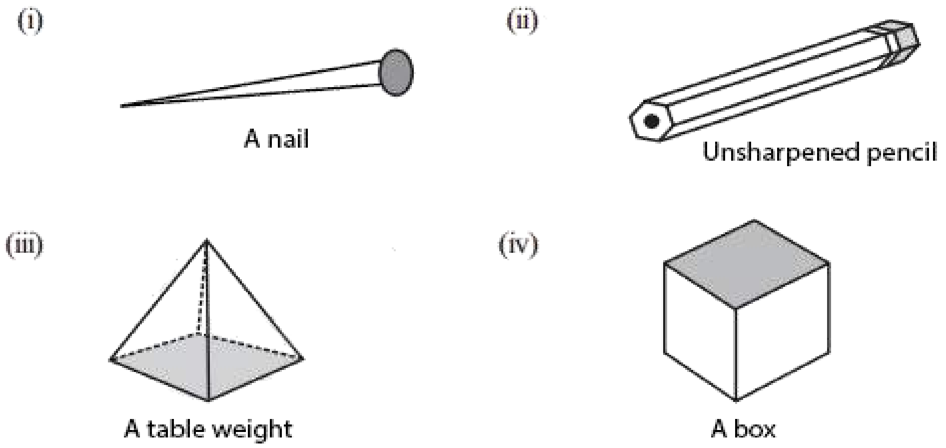
ii) Yes, a triangular pyramid has 4 triangular faces.

iii) Yes, as square pyramid has a square face and 4 triangular faces.

2. Is it possible to have a polyhedron with any given number of faces? (Hint : Think of a pyramid)

Solution: It is possible, only if the number of faces are greater than or equal to 4.

3. Which are prisms among the following:



i) A nail: Not a polyhedron as it has a curved surface. This is not a prism.
ii) Unsharpened Pencil: t is a prism.
iii) A table Weight: It is not a prism.
iv) A Box: It is a prism.

4. (i) How are prisms and cylinders alike ?
(ii) How are pyramids and cones alike ?

Solution:

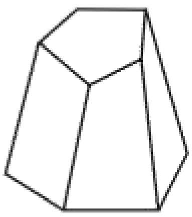
i) A cylinder can be looks like circular prism, a prism with circular base.

ii) A cone can be a circular pyramid, a pyramid with circular base.

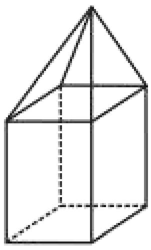
5. Is a square prism same as a cube? Explain.

Solution: yes, a square prism can also be a cube. A square prism has a square as its base. However, its height is not necessarily same as the side of the square. Thus, a square prism can also be a cuboid.

6. Verify Euler's formula for these solids.



(i)



(ii)

Solution:

i) Number of faces, F = 7
Number of edges, E = 15
Number of vertices, V = 10
As per formula**, F + V - E = 2**
Substitute the values, we have
F + V - E = 7 + 10 - 15
= 2
Verified.

ii)
Here F = 9
V = 9
E = 16

Using formula, F+ V- E = 2
F+ V- E = 9+9-16
= 2
Hence, Euler's formula is verified

7. Using Euler's formula, find the unknown:

Faces	?	5	20
Vertices	6	?	12
Edges	12	9	?

Solution:

Euler's formula: F + V - E = 2
Where, F = Faces, V = Vertices and E = Edges

i) F + 6 - 12 = 2

F = 2 + 6

F = 8

iii) 5 + V - 9 = 2
V-4 = 2
V = 6

iv) 20 + 12 - E = 2
32 - E = 2
E = 30

8. Can a polyhedron have 10 faces, 20 edges and 15 vertices?
Solution: From the given data, we have

F = 10
E = 20
V = 15
Every polyhedron satisfies Euler's formula, which is stated as, F + V - E = 2
For the given polygon, F + V - E = 10 + 15 - 20 = 25 - 20 = 5, which is not equal to 2 Therefore, A polyhedron is not possible as Euler's formula is not satisfied.

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH:10 VIZUALISING SOLID SHAPES

SUBJECT: MATHEMATICS **CLASS: VIII**

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 FACES, EDGES & VERTICES; POLYHEDRONS

1. Draw the following
   (i) 3 polyhedrons
   (ii) 3 solids which are not polyhedrons
   (iii) 3 convex polyhedrons
   (iv) 3 regular polyhedrons

2. Which are two important members of polyhedron family? Draw them and the number of their faces, vertices and edges.

3. Explain why (or why not) can a polyhedron have for its faces
   (i) 3 triangles
   (ii) 4 triangles
   (iii) A square and 4 triangles

4. How are (i) prisms and cylinders alike? (ii) pyramids and cones alike?

5. Is a square prism same as a cube? Explain.

TYPE: 2 EULER'S FORMULA (F+V-E=2)

6. Using Euler's formula find the unknown

Faces	?	5	20
Vertices	6	?	12
Edges	12	9	?