Exercise 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Solution:

According to the question,

Total number of balls = 30

Numbers of boundary = 6

Number of time batswoman didn't hit boundary = 30 – 6 = 24

Probability she did not hit a boundary = 24/30 = 4/5

2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl
Also check whether the sum of these probabilities is 1.

Solution:

Total numbers of families = 1500

(i) Numbers of families having 2 girls = 475

Probability = Numbers of families having 2 girls/Total numbers of families

= 475/1500 = 19/60

(ii) Numbers of families having 1 girls = 814

Probability = Numbers of families having 1 girls/Total numbers of families

= 814/1500 = 407/750

(iii) Numbers of families having 2 girls = 211

Probability = Numbers of families having 0 girls/Total numbers of families

= 211/1500

Sum of the probability = (19/60)+(407/750)+(211/1500)

= (475+814+211)/1500

= 1500/1500 = 1

Yes, the sum of these probabilities is 1.

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Solution:

Number of times 2 heads come up = 72

Total number of times the coins were tossed = 200

∴, the probability of 2 heads coming up = 72/200 = 9/25

5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

(i) earning ₹10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning ₹16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than ₹7000 per month and does not own any vehicle.

(iv) earning ₹13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

Solution:

Total number of families = 2400

(i) Numbers of families earning ₹10000 –13000 per month and owning exactly 2 vehicles = 29

∴, the probability that the family chosen is earning ₹10000 – 13000 per month and owning exactly 2 vehicles = 29/2400

(ii) Number of families earning ₹16000 or more per month and owning exactly 1 vehicle = 579

∴, the probability that the family chosen is earning₹16000 or more per month and owning exactly 1 vehicle = 579/2400

(iii) Number of families earning less than ₹7000 per month and does not own any vehicle = 10

∴, the probability that the family chosen is earning less than ₹7000 per month and does not own any vehicle = 10/2400 = 1/240

(iv) Number of families earning ₹13000-16000 per month and owning more than 2 vehicles = 25

∴, the probability that the family chosen is earning ₹13000 – 16000 per month and owning more than 2 vehicles = 25/2400 = 1/96

(v) Number of families owning not more than 1 vehicle = 10+160+0+305+1+535+2+469+1+579

= 2062

∴, the probability that the family chosen owns not more than 1 vehicle = 2062/2400 = 1031/1200

7. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	**Number of students**
like	**135**
dislike	**65**

Find the probability that a student chosen at random
(i) likes statistics, (ii) does not like it.

Solution:

Total number of students = 135+65 = 200

(i) Number of students who like statistics = 135

, the probability that a student likes statistics = 135/200 = 27/40

(ii) Number of students who do not like statistics = 65

∴, the probability that a student does not like statistics = 65/200 = 13/40

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Solution:

Total number of bags present = 11

Number of bags containing more than 5 kg of flour = 7

∴, the probability that any of the bags chosen at random contains more than 5 kg of flour = 7/11

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.15 : PROBABILITY

SUBJECT: MATHEMATICS **CLASS: IX**

This chapter consists of 1 different topics. The most probable questions from the examination point of view are given below.

TYPE:1: EXPERIMENTAL APPROACH TO PROBABILITY.

Q1 In a cricket match, a batsman hits a boundary 8 times out of 40 balls he plays. Find the probability that he didn't hit a boundary.

Q2. A die is thrown. What is the probability of getting a multiple of 3 on the upper face?

Q3. 80 bulbs are selected at random from a lot and their life time in hours is recorded in the form of a frequency table given below:

Life time (In hours)	300	500	700	900	1100
Frequency	10	12	23	25	10

Find the probability that bulbs selected randomly from the lot has life less than 900 hours.

ACHIVER'S SECTION:

Q1. Bulbs are packed in cartons each containing 40 bulbs. Seven hundred cartons were examined for defective bulbs and the results are given in the following table:

Number of defective bulbs	0	1	2	3	4	5	6	More than 6
Frequency	400	180	48	41	18	8	3	2

Q2. A recent survey found that the ages of workers in a factory are distributed as follows:

Distance in km	20-29	30-39	40-49	50-59	60 and above
Number of workers	38	27	56	46	3

If a person is selected at random, find the probability that the person is:

a) 40 years or more

b) Under 40 years

c) Having age from 30 to 39 years

d) Under 60 but over 39 years

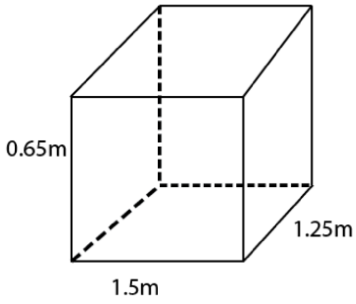
Exercise 13.1

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i)The area of the sheet required for making the box.

(ii)The cost of sheet for it, if a sheet measuring 1m^2 costs Rs. 20.

Solution:



Given: length (l) of box = 1.5m

Breadth (b) of box = 1.25 m

Depth (h) of box = 0.65m

(i) Box is to be open at top

Area of sheet required.

= 2lh+2bh+lb

= [2×1.5×0.65+2×1.25×0.65+1.5×1.25]m^2

= (1.95+1.625+1.875) m^2 = 5.45 m^2

(ii) Cost of sheet per m^2 area = Rs.20.

Cost of sheet of 5.45 m^2 area = Rs (5.45×20)

= Rs.109.

2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and ceiling at the rate of Rs 7.50 per m^2.

Solution:

Length (l) of room = 5m

Breadth (b) of room = 4m

Height (h) of room = 3m

It can be observed that four walls and the ceiling of the room are to be white washed.

Total area to be white washed = Area of walls + Area of ceiling of room

= 2lh+2bh+lb

= [2×5×3+2×4×3+5×4]

= (30+24+20)

= 74

Area = 74 m^2

Also,

Cost of white wash per m^2 area = Rs.7.50 (Given)

Cost of white washing 74 m^2 area = Rs. (74×7.50)

= Rs. 555

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs.15000, find the height of the hall.

[Hint: Area of the four walls = Lateral surface area.]

Solution:

Let length, breadth, and height of the rectangular hall be l, b, and h respectively.

Area of four walls = 2lh+2bh

= 2(l+b)h

Perimeter of the floor of hall = 2(l+b)

= 250 m

Area of four walls = 2(l+b) h = 250h m^2

Cost of painting per square meter area = Rs.10

Cost of painting 250h square meter area = Rs (250h×10) = Rs.2500h

However, it is given that the cost of paining the walls is Rs. 15000.

15000 = 2500h

Or h = 6

Therefore, the height of the hall is 6 m.

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm×10 cm×7.5 cm can be painted out of this container?

Solution:

Total surface area of one brick = 2(lb +bh+lb)

= [2(22.5×10+10×7.5+22.5×7.5)] cm^2

= 2(225+75+168.75) cm^2

= (2×468.75) cm^2

= 937.5 cm^2

Let n bricks can be painted out by the paint of the container

Area of n bricks = (n×937.5) cm^2 = 937.5n cm^2

As per given instructions, area that can be painted by the paint of the container = 9.375 m^2 = 93750 cm^2

So, we have, 93750 = 937.5n

n = 100

Therefore, 100 bricks can be painted out by the paint of the container.

5. A cubical box has each edge 10 cm and another cuboidal box is 12.5cm long, 10 cm wide and 8 cm high

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Solution:

From the question statement, we have

Edge of a cube = 10cm

Length, l = 12.5 cm

Breadth, b = 10cm

Height, h = 8 cm

(i) Find the lateral surface area for both the figures

Lateral surface area of cubical box = 4 $(edge)^2$

= $4(10)^2$

= 400 cm^2 …(1)

Lateral surface area of cuboidal box = 2[lh+bh]

= [2(12.5×8+10×8)]

= (2×180) = 360

Therefore, Lateral surface area of cuboidal box is 360 cm^2. …(2)

From (1) and (2), lateral surface area of the cubical box is more than the lateral surface area of the cuboidal box. The difference between both the lateral surfaces is, 40 cm^2.

(Lateral surface area of cubical box – Lateral surface area of cuboidal box=400cm^2–360cm^2 = 40 cm^2)

(ii) Find the total surface area for both the figures

The total surface area of the cubical box = 6$(edge)^2$ = 6$(10\ cm)^2$ = 600 cm^2…(3)

Total surface area of cuboidal box

= 2[lh+bh+lb]

= [2(12.5×8+10×8+12.5×100)]

= 610

This implies, Total surface area of cuboidal box is 610 cm^2..(4)

From (3) and (4), the total surface area of the cubical box is smaller than that of the cuboidal box. And their difference is 10cm^2.

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2

6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30cm long, 25 cm wide and 25 cm high.

(i)What is the area of the glass?

(ii)How much of tape is needed for all the 12 edges?

Solution:

Length of greenhouse, say l = 30cm

Breadth of greenhouse, say b = 25 cm

Height of greenhouse, say h = 25 cm

(i) Total surface area of greenhouse = Area of the glass = 2[lb+lh+bh]

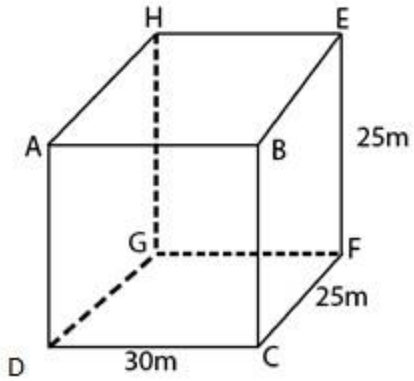
= [2(30×25+30×25+25×25)]

= [2(750+750+625)]

= (2×2125) = 4250

Total surface area of the glass is 4250 cm^2

(ii)



From figure, tape is required along sides AB, BC, CD, DA, EF, FG, GH, HE AH, BE, DG, and CF.

Total length of tape = 4(l+b+h)

= [4(30+25+25)] (after substituting the values)

= 320

Therefore, 320 cm tape is required for all the 12 edges.

7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Solution:

Let l, b and h be the length, breadth and height of the box.

Bigger Box:

l = 25cm

b = 20 cm

h = 5 cm

Total surface area of bigger box = 2(lb+lh+bh)

= [2(25×20+25×5+20×5)]

= [2(500+125+100)]

= 1450 cm^2

Extra area required for overlapping 1450×5/100 cm^2

= 72.5 cm^2

While considering all over laps, total surface area of bigger box

= (1450+72.5) cm^2 = 1522.5 cm^2

Area of cardboard sheet required for 250 such bigger boxes

= (1522.5×250) cm^2 = 380625 cm^2

Smaller Box:

Similarly, total surface area of smaller box = [2(15×12+15×5+12×5)] cm^2

= [2(180+75+60)] cm^2

= (2×315) cm^2

= 630 cm^2

Therefore, extra area required for overlapping 630×5/100 cm^2 = 31.5 cm^2

Total surface area of 1 smaller box while considering all overlaps

= (630+31.5) cm^2 = 661.5 cm^2

Area of cardboard sheet required for 250 smaller boxes = (250×661.5) cm^2 = 165375 cm^2

In Short:

Box	Dimensions (in cm)	Total surface area (in cm^2)	Extra area required for overlapping (in cm^2)	Total surface area for all overlaps (in cm^2)	Area for 250 such boxes (in cm^2)
Bigger Box	l = 25 b = 20 c = 5	1450	1450×5/100 = 72.5	(1450+72.5) = 1522.5	(1522.5×250) = 380625
Smaller Box	l = 15 b = 12 h =5	630	630×5/100 = 31.5	(630+31.5) = 661.5	(250×661.5) = 165375

Now, Total cardboard sheet required = (380625+165375) cm^2

= 546000 cm^2

Given: Cost of 1000 cm^2 cardboard sheet = Rs. 4

Therefore, Cost of 546000 cm^2 cardboard sheet =Rs. (546000×4)/1000 = Rs. 2184

Therefore, the cost of cardboard required for supplying 250 boxes of each kind will be Rs. 2184.

8. Praveen wanted to make a temporary shelter for her car, by making a box – like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?

Solution:

Let l, b and h be the length, breadth and height of the shelter.

Given:

l = 4m

b = 3m

h = 2.5m

Tarpaulin will be required for the top and four wall sides of the shelter.

Using formula, Area of tarpaulin required = 2(lh+bh)+lb

On putting the values of l, b and h, we get

= [2(4×2.5+3×2.5)+4×3] m^2

= [2(10+7.5)+12]m^2

= 47m^2

Therefore, 47 m^2 tarpaulin will be required.

Exercise 13.2

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder. (Assume π =22/7)

Solution:

Height of cylinder, h = 14cm

Let the diameter of the cylinder be d

Curved surface area of cylinder = 88 cm^2

We know that, formula to find Curved surface area of cylinder is 2πrh.

So 2πrh =88 cm^2 (r is the radius of the base of the cylinder)

2×(22/7)×r×14 = 88 cm^2

2r = 2 cm

d =2 cm

Therefore, the diameter of the base of the cylinder is 2 cm.

2. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square meters of the sheet are required for the same? Assume π = 22/7

Solution:

Let h be the height and r be the radius of a cylindrical tank.

Height of cylindrical tank, h = 1m

Radius = half of diameter = (140/2) cm = 70cm = 0.7m

Area of sheet required = Total surface are of tank = 2πr(r+h) unit square

= [2×(22/7)×0.7(0.7+1)]

= 7.48

Therefore, 7.48 square meters of the sheet are required.

3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4cm. (see fig. 13.11). Find its

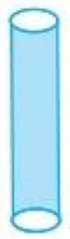


Fig. 13.11

(i) inner curved surface area,

(ii) outer curved surface area

(iii) total surface area

(Assume π=22/7)

Solution:

Let r_1 and r_2 Inner and outer radii of cylindrical pipe

r_1 = 4/2 cm = 2 cm

r_2 = 4.4/2 cm = 2.2 cm

Height of cylindrical pipe, h = length of cylindrical pipe = 77 cm

(i) curved surface area of outer surface of pipe = $2\pi r_1 h$

= 2×(22/7)×2×77 cm^2

= 968 cm^2

(ii) curved surface area of outer surface of pipe = $2\pi r_2 h$

= 2×(22/7)×2.2×77 cm^2

= (22×22×2.2) cm^2

= 1064.8 cm^2

(iii) Total surface area of pipe = inner curved surface area+ outer curved surface area+ Area of both circular ends of pipe.

= $2r_1h+2r_2h+2\pi(r_1^2-r_2^2)$

= 9668+1064.8+2×(22/7)×(2.2^2-2^2)

= 2031.8+5.28

= 2038.08 cm^2

Therefore, the total surface area of the cylindrical pipe is 2038.08 cm^2.

4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to

move once over to level a playground. Find the area of the playground in m^2? (Assume π = 22/7)

Solution:

A roller is shaped like a cylinder.

Let h be the height of the roller and r be the radius.

h = Length of roller = 120 cm

Radius of the circular end of roller = r = (84/2) cm = 42 cm

Now, CSA of roller = 2πrh

= 2×(22/7)×42×120

= 31680 cm^2

Area of field = 500×CSA of roller

= (500×31680) cm^2

= 15840000 cm^2

= 1584 m^2.

Therefore, area of playground is 1584 m^2. Answer!

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.

(Assume π = 22/7)

Solution:

Let h be the height of a cylindrical pillar and r be the radius.

Given:

Height cylindrical pillar = h = 3.5 m

Radius of the circular end of pillar = r = diameter/2 = 50/2 = 25cm = 0.25m

CSA of pillar = 2πrh

= 2×(22/7)×0.25×3.5

= 5.5 m^2

Cost of painting 1 m^2 area = Rs. 12.50

Cost of painting 5.5 m^2 area = Rs (5.5×12.50)

= Rs.68.75

Therefore, the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2 is Rs 68.75.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the base of the cylinder is 0.7 m, find its height. (Assume π = 22/7)

Solution:

Let h be the height of the circular cylinder and r be the radius.

Radius of the base of cylinder, r = 0.7m

CSA of cylinder = 2πrh

CSA of cylinder = 4.4m^2

Equating both the equations, we have

2×(22/7)×0.7×h = 4.4

Or h = 1

Therefore, the height of the cylinder is 1 m.

7. The inner diameter of a circular well is 3.5m. It is 10m deep. Find

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.

(Assume π = 22/7)

Solution:

Inner radius of circular well, r = 3.5/2m = 1.75m

Depth of circular well, say h = 10m

(i) Inner curved surface area = 2πrh

= (2×(22/7)×1.75×10)

= 110

Therefore, the inner curved surface area of the circular well is 110 m^2.

(ii)Cost of plastering 1 m^2 area = Rs.40

Cost of plastering 110 m^2 area = Rs (110×40)

= Rs.4400

Therefore, the cost of plastering the curved surface of the well is Rs. 4400.

8. In a hot water heating system, there is cylindrical pipe of length 28 m and diameter 5 cm. Find

the total radiating surface in the system. (Assume π = 22/7)

Solution:

Height of cylindrical pipe = Length of cylindrical pipe = 28m

Radius of circular end of pipe = diameter/ 2 = 5/2 cm = 2.5cm = 0.025m

Now, CSA of cylindrical pipe = 2πrh, where r = radius and h = height of the cylinder

= 2×(22/7)×0.025×28 m^2

= 4.4m^2

The area of the radiating surface of the system is 4.4m^2.

9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in

diameter and 4.5m high.

(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank. (Assume π = 22/7)

Solution:

Height of cylindrical tank, h = 4.5m

Radius of the circular end , r = (4.2/2)m = 2.1m

(i) the lateral or curved surface area of cylindrical tank is 2πrh

= 2×(22/7)×2.1×4.5 m^2

$= (44 \times 0.3 \times 4.5)\ m^2$

$= 59.4\ m^2$

Therefore, CSA of tank is 59.4 m^2.

(ii) Total surface area of tank = $2\pi r(r+h)$

$= 2\times(22/7)\times2.1\times(2.1+4.5)$

$= 44\times0.3\times6.6$

$= 87.12\ m^2$

Now, Let S m^2 steel sheet be actually used in making the tank.

$S(1 - 1/12) = 87.12\ m^2$

This implies, $S = 95.04\ m^2$

Therefore, 95.04m^2 steel was used in actual while making such a tank.

10. In fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth.

The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade. (Assume π = 22/7)

Fig. 13.12

Solution:

Say h = height of the frame of lampshade, looks like cylindrical shape

r = radius

Total height is h = (2.5+30+2.5) cm = 35cm and

r = (20/2) cm = 10cm

Use curved surface area formula to find the cloth required for covering the lampshade which is $2\pi rh$

$= (2\times(22/7)\times10\times35)\ cm^2$

$= 2200\ cm^2$

Hence, 2200 cm^2 cloth is required for covering the lampshade.

11. The students of Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? (Assume π =22/7)

Solution:

Radius of the circular end of cylindrical penholder, r = 3cm

Height of penholder, h = 10.5cm

Surface area of a penholder = CSA of pen holder + Area of base of penholder

= $2\pi rh+\pi r^2$

= $2\times(22/7)\times3\times10.5+(22/7)\times3^2= 1584/7$

Therefore, Area of cardboard sheet used by one competitor is 1584/7 cm^2

So, Area of cardboard sheet used by 35 competitors = 35×1584/7 = 7920 cm^2

Therefore, 7920 cm^2 cardboard sheet will be needed for the competition.

Exercise 13.3

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area (Assume π=22/7)

Solution:

Radius of the base of cone = diameter/ 2 = (10.5/2)cm = 5.25cm

Slant height of cone, say l = 10 cm

CSA of cone is = πrl

= (22/7)×5.25×10 = 165

Therefore, the curved surface area of the cone is 165 cm^2.

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m. (Assume π = 22/7)

Solution:

Radius of cone, r = 24/2 m = 12m

Slant height, l = 21 m

Formula: Total Surface area of the cone = $\pi r(l+r)$

Total Surface area of the cone = (22/7)×12×(21+12) m^2

= 1244.57m^2

3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

(Assume π = 22/7)

Solution:

Slant height of cone, l = 14 cm

Let the radius of the cone be r.

(i) We know, CSA of cone = πrl

Given: Curved surface area of a cone is 308 cm^2

(308) = (22/7)×r×14

308 = 44 r

r = 308/44 = 7

Radius of a cone base is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base (πr^2)

Total surface area of cone = $308+(22/7)\times7^2$ = 308+154

Therefore, the total surface area of the cone is 462 cm^2.

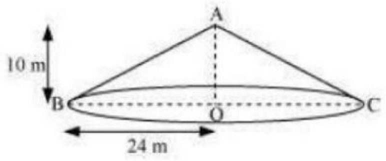
4. A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

(Assume π=22/7)

Solution:



Let ABC be a conical tent

Height of conical tent, h = 10 m

Radius of conical tent, r = 24m

Let the slant height of the tent be l.

(i) In right triangle ABO, we have

$AB^2 = AO^2+BO^2$(using Pythagoras theorem)

$l^2 = h^2+r^2$

$= (10)^2+(24)^2$

= 676

l = 26

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = πrl

$= (22/7)\times24\times26$ m^2

Cost of 1 m^2 canvas = Rs 70

Cost of (13728/7)m^2 canvas is equal to Rs $(13728/7)\times70$ = Rs 137280

Therefore, the cost of the canvas required to make such a tent is Rs 137280.

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14]

Solution:

Height of conical tent, h = 8m

Radius of base of tent, r = 6m

Slant height of tent, $l^2 = (r^2+h^2)$

$l^2 = (6^2+8^2) = (36+64) = (100)$

or l = 10

Again, CSA of conical tent = πrl

= (3.14×6×10) m^2

= 188.4m^2

Let the length of tarpaulin sheet required be L

As 20 cm will be wasted, therefore,

Effective length will be (L-0.2m).

Breadth of tarpaulin = 3m (given)

Area of sheet = CSA of tent

[(L–0.2)×3] = 188.4
L-0.2 = 62.8

L = 63

Therefore, the length of the required tarpaulin sheet will be 63 m.

6. The slant height and base diameter of conical tomb are 25m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2. (Assume π = 22/7)

Solution:

Slant height of conical tomb, l = 25m

Base radius, r = diameter/2 = 14/2 m = 7m

CSA of conical tomb = πrl

= (22/7)×7×25 = 550

CSA of conical tomb= 550m^2

Cost of white-washing 550 m^2 area, which is Rs (210×550)/100

= Rs. 1155

Therefore, cost will be Rs. 1155 while white-washing tomb.

7. A joker's cap is in the form of right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume π =22/7)

Solution:

Radius of conical cap, r = 7 cm

Height of conical cap, h = 24cm

Slant height, $l^2 = (r^2+h^2)$

= (7^2+24^2)

= (49+576)

= (625)

Or l = 25 cm

CSA of 1 conical cap = πrl

= (22/7)×7×24

= 550

CSA of 10 caps = (10×550) cm^2 = 5500 cm^2

Therefore, the area of the sheet required to make 10 such caps is 5500 cm^2.

8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take √(1.04) =1.02)

Solution:

Given:

Radius of cone, r = diameter/2 = 40/2 cm = 20cm = 0.2 m

Height of cone, h = 1m

Slant height of cone is l, and $l^2 = (r^2+h^2)$

Using given values, $l^2 = (0.2^2+1^2)$

= (1.04)

Or l = 1.02

Slant height of the cone is 1.02 m

Now,

CSA of each cone = πrl

= (3.14×0.2×1.02)

= 0.64056

CSA of 50 such cones = (50×0.64056) = 32.028

CSA of 50 such cones = 32.028 m^2

Again,

Cost of painting 1 m^2 area = Rs 12 (given)

Cost of painting 32.028 m^2 area = Rs (32.028×12)

= Rs.384.336

= Rs.384.34 (approximately)

Therefore, the cost of painting all these cones is Rs. 384.34.

Exercise 13.4

1. Find the surface area of a sphere of radius:

(i) 10.5cm (ii) 5.6cm (iii) 14cm

(Assume π=22/7)

Solution:

Formula: Surface area of sphere (SA) = $4\pi r^2$

(i) Radius of sphere, r = 10.5 cm

SA = $4\times(22/7)\times10.5^2 = 1386$

Surface area of sphere is 1386 cm^2

(ii) Radius of sphere, r = 5.6cm

Using formula, SA = $4\times(22/7)\times5.6^2 = 394.24$

Surface area of sphere is 394.24 cm^2

(iii) Radius of sphere, r = 14cm

SA = $4\pi r^2$

$= 4\times(22/7)\times(14)^2$

= 2464

Surface area of sphere is 2464 cm^2

2. Find the surface area of a sphere of diameter:

(i) 14cm (ii) 21cm (iii) 3.5cm

(Assume π = 22/7)

Solution:

(i) Radius of sphere, r = diameter/2 = 14/2 cm = 7 cm

Formula for Surface area of sphere = $4\pi r^2$

$= 4\times(22/7)\times7^2 = 616$

Surface area of a sphere is 616 cm^2

(ii) Radius (r) of sphere = 21/2 = 10.5 cm

Surface area of sphere = $4\pi r^2$

$= 4\times(22/7)\times10.5^2 = 1386$

Surface area of a sphere is 1386 cm^2

Therefore, the surface area of a sphere having diameter 21cm is 1386 cm^2

(iii) Radius(r) of sphere = 3.5/2 = 1.75 cm

Surface area of sphere = $4\pi r^2$

$= 4\times(22/7)\times1.75^2 = 38.5$

Surface area of a sphere is 38.5 cm^2

3. Find the total surface area of a hemisphere of radius 10 cm. [Use π=3.14]

Solution:

Radius of hemisphere, r = 10cm

Formula: Total surface area of hemisphere = $3\pi r^2$

$= 3\times3.14\times10^2 = 942$

The total surface area of given hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Solution:

Let r_1 and r_2 be the radii of spherical balloon and spherical balloon when air is pumped into it respectively. So

$r_1 = 7$cm

$r_2 = 14$ cm

Now, Required ratio = (initial surface area)/(Surface area after pumping air into balloon)

$= 4r_1^2/4r_2^2$

$= (r_1/r_2)^2$

$= (7/14)^2 = (1/2)^2 = ¼$

Therefore, the ratio between the surface areas is 1:4.

5. A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. (Assume π = 22/7)

Solution:

Inner radius of hemispherical bowl, say r = diameter/2 = (10.5)/2 cm = 5.25 cm

Formula for Surface area of hemispherical bowl = $2\pi r^2$

$= 2\times(22/7)\times(5.25)^2 = 173.25$

Surface area of hemispherical bowl is 173.25 cm^2

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 1 cm^2 area = Rs 16 /100

Cost of tin-plating 173.25 cm^2 area = Rs. (16×173.25)/100 = Rs 27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl at the rate of Rs 16 per 100 cm^2 is Rs **27.72.**

6. Find the radius of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let the radius of the sphere be r.

Surface area of sphere = 154 (given)

Now,

$4\pi r^2 = 154$

$r^2 = (154\times7)/(4\times22) = (49/4)$

r = (7/2) = 3.5

The radius of the sphere is 3.5 cm.

7. The diameter of the moon is approximately one fourth of the diameter of the earth.

Find the ratio of their surface areas.

Solution:

If diameter of earth is said d, then the diameter of moon will be d/4 (as per given statement)

Radius of earth = d/2

Radius of moon = ½×d/4 = d/8

Surface area of moon = $4\pi(d/8)^2$

Surface area of earth = $4\pi(d/2)^2$

$$\text{Ratio of their Surface areas} = \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = 4/64 = 1/16$$

The ratio between their surface areas is 1:16.

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume π =22/7)

Solution:

Given:

Inner radius of hemispherical bowl = 5cm

Thickness of the bowl = 0.25 cm

Outer radius of hemispherical bowl = (5+0.25) cm = 5.25 cm

Formula for outer CSA of hemispherical bowl = $2\pi r^2$, where r is radius of hemisphere

= $2\times(22/7)\times(5.25)^2 = 173.25$

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see fig. 13.22). Find

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in(i) and (ii).

Fig. 13.22

Solution:

(i) Surface area of sphere = $4\pi r^2$, where r is the radius of sphere

(ii) Height of cylinder, h = r+r =2r

Radius of cylinder = r

CSA of cylinder formula = $2\pi rh = 2\pi r(2r)$ (using value of h)

= $4\pi r^2$

(iii) Ratio between areas = (Surface area of sphere)/CSA of Cylinder)

= $4r^2/4r^2 = 1/1$

Ratio of the areas obtained in (i) and (ii) is 1:1.

Exercise 13.5

1. A matchbox measures 4 cm×2.5cm×1.5cm. What will be the volume of a packet containing 12 such boxes?

Solution:

Dimensions of a matchbox (a cuboid) are l×b×h = 4 cm×2.5 cm×1.5 cm respectively

Formula to find the volume of matchbox = l×b×h = (4×2.5×1.5) = 15

Volume of matchbox = 15 cm^3

Now, volume of 12 such matchboxes = (15×12) cm^3 = 180 cm^3

Therefore, the volume of 12 matchboxes is 180cm^3.

2. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m^3= 1000 l)

Solution:

Dimensions of a cuboidal water tank are: l = 6 m and b = 5 m and h = 4.5 m

Formula to find volume of tank, V = l×b×h

Put the values, we get

V = (6×5×4.5) = 135

Volume of water tank is 135 m^3

Again,

We are given that, amount of water that 1m^3 volume can hold = 1000 l

Amount of water, 135 m^3volume hold = (135×1000) litres = 135000 litres

Therefore, given cuboidal water tank can hold up to135000 litres of water.

3. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Solution:

Given:

Length of cuboidal vessel, l = 10 m

Width of cuboidal vessel, b = 8m

Volume of cuboidal vessel, V = 380 m^3

Let the height of the given vessel be h.

Formula for Volume of a cuboid, V = l×b×h

Using formula, we have

l×b×h = 380

10×8×h= 380

Or h = 4.75

Therefore, the height of the vessels is 4.75 m.

4. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m^3.

Solution:

The given pit has its length(l) as 8m, width (b)as 6m and depth (h)as 3 m.

Volume of cuboidal pit = l×b×h = (8×6×3) = 144 (using formula)

Required Volume is 144 m^3

Now,

Cost of digging per m^3 volume = Rs 30

Cost of digging 144 m^3 volume = Rs (144×30) = Rs 4320

5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Solution:

Length (l) and depth (h) of tank is 2.5 m and 10 m respectively.

To find: The value of breadth, say b.

Formula to find the volume of a tank = l×b×h = (2.5× b×10) m^3= 25b m^3

Capacity of tank= 25b m^3, which is equal to 25000b litres

Also, capacity of a cuboidal tank is 50000 litres of water (Given)

Therefore, 25000 b = 50000

This implies, b = 2

Therefore, the breadth of the tank is 2 m.

6. A village, having a population of 4000, requires 150 litres of water per head per day.

It has a tank measuring 20 m×15 m×6 m. For how many days will the water of this tank last?

Solution:

Length of the tank = l = 20 m

Breadth of the tank = b = 15 m

Height of the tank = h = 6 m

Total population of a village = 4000

Consumption of the water per head per day = 150 litres

Water consumed by the people in 1 day = (4000×150) litres = 600000 litres …(1)

Formula to find the capacity of tank, C = l×b×h

Using given data, we have

C = (20×15×6) m^3= 1800 m^3

Or C = 1800000 litres

Let water in this tank last for d days.

Water consumed by all people in d days = Capacity of tank (using equation (1))

600000 d =1800000

d = 3

Therefore, the water of this tank will last for 3 days. **Answer**

7. A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates each

measuring 1.5m×1.25 m×0.5 m that can be stored in the godown.

Solution:

From statement, we have

Length of the godown = 40 m

Breadth = 25 m

Height = 15 m

Whereas,

Length of the wooden crate = 1.5 m

Breadth = 1.25 m

Height = 0.5 m

Since godown and wooden crate are in cuboidal shape. Find the volume of each using formula, V = lbh.

Now,

Volume of godown = (40×25×15) m^3 = 15000 m^3

Volume of a wooden crate = (1.5×1.25×0.5) m^3 = 0.9375 m^3

Let us consider that, n wooden crates can be stored in the godown, then

Volume of n wooden crates = Volume of godown

0.9375×n =15000

Or n= 15000/0.9375 = 16000

Hence, the number of wooden crates that can be stored in the godown is 16,000.

8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Solution:

Side of a cube = 12cm (Given)

Find the volume of cube:

Volume of cube = $(Side)^3 = (12)^3 cm^3 = 1728 cm^3$

Surface area of a cube with side 12 cm = $6a^2 = 6(12)^2 cm^2$ …(1)

Cube is cut into eight small cubes of equal volume, say side of each cube is p.

Volume of a small cube = p^3

Surface area = $6p^2$ …(2)

Volume of each small cube = (1728/8) cm^3 = 216 cm^3

Or $(p)^3$ = 216 cm^3

Or p = 6 cm

Now, Surface areas of the cubes ratios = (Surface area of bigger cube)/(Surface area of smaller cubes)

From equation (1) and (2), we get

Surface areas of the cubes ratios = $(6a^2)/(6p^2) = a^2/p^2 = 12^2/6^2 = 4$

Therefore, the required ratio is 4 : 1.

9. A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Solution:

Given:

Depth of river, h = 3 m

Width of river, b = 40 m

Rate of water flow = 2km per hour = 2000m/60min = 100/3 m/min

Now, Volume of water flowed in 1 min = (100/3) × 40 × 3 = 4000m^3

Therefore, 4000 m^3water will fall into the sea in a minute.

Exercise 13.6

1. The circumference of the base of cylindrical vessel is 132cm and its height is 25cm.

How many litres of water can it hold? (1000 cm^3= 1L) (Assume π = 22/7)

Solution:

Circumference of the base of cylindrical vessel = 132 cm

Height of vessel, h = 25 cm

Let r be the radius of the cylindrical vessel.

Step 1: Find the radius of vessel

We know that, circumference of base = 2πr, so

2πr = 132 (given)

r = (132/(2 π))

r = 66×7/22 = 21

Radius is 21 cm

Step 2: Find the volume of vessel

Formula: Volume of cylindrical vessel = $\pi r^2 h$

= $(22/7)\times 21^2\times 25$

= 34650

Therefore, volume is 34650 cm^3

Since, **1000 cm^3 = 1L**

So, Volume = 34650/1000 L= 34.65L

Therefore, vessel can hold 34.65 litres of water.

2. The inner diameter of a cylindrical wooden pipe is 24cm and its outer diameter is 28 cm. The length of the pipe is 35cm.Find the mass of the pipe, if 1cm^3 of wood has a mass of 0.6g. (Assume π = 22/7)

Solution:

Inner radius of cylindrical pipe, say r_1 = diameter$_1$/ 2 = 24/2 cm = 12cm

Outer radius of cylindrical pipe, say r_2 = diameter$_2$/ 2 = 28/2 cm = 14 cm

Height of pipe, h = Length of pipe = 35cm

Now, the Volume of pipe = $\pi(r_2^2 - r_1^2)h$ cm^3

Substitute the values.

Volume of pipe = 110×52 cm^3 = 5720 cm^3

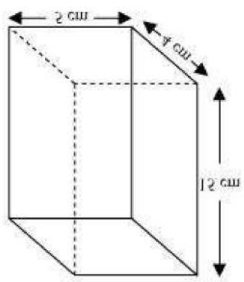
Since**, Mass of 1 cm^3 wood = 0.6 g**

Mass of 5720 cm^3 wood = (5720×0.6) g = 3432 g or 3.432 kg. Answer!

3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5cm and width 4cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7cm and height 10cm. Which container has greater capacity and by how much? (Assume π=22/7)

Solution:

1. tin can will be cuboidal in shape



Dimensions of tin can are

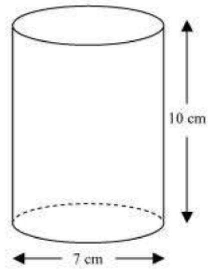
Length, l = 5 cm

Breadth, b = 4 cm

Height, h = 15 cm

Capacity of tin can = l×b×h= (5×4×15) cm^3 = 300 cm^3

1. Plastic cylinder will be cylindrical in shape.



Dimensions of plastic can are:

Radius of circular end of plastic cylinder, r = 3.5cm

Height , H = 10 cm

Capacity of plastic cylinder = $\pi r^2 H$

Capacity of plastic cylinder = $(22/7)\times(3.5)^2\times10 = 385$

Capacity of plastic cylinder is 385 cm^3

From results of (i) and (ii), plastic cylinder has more capacity.

Difference in capacity = (385-300) cm^3 = 85cm^3

4. If the lateral surface of a cylinder is 94.2cm^2 and its height is 5cm, then find

(i) radius of its base (ii) its volume.[Use π= 3.14]

Solution:

CSA of cylinder = 94.2 cm^2

Height of cylinder, h = 5cm

(i) Let radius of cylinder be r.

Using CSA of cylinder, we get

$2\pi rh = 94.2$

$2\times3.14\times r\times5 = 94.2$

r = 3

radius is 3 cm

(ii) Volume of cylinder

Formula for volume of cylinder = $\pi r^2 h$

Now, $\pi r^2 h = (3.14\times(3)^2\times5)$ (using value of r from (i))

= 141.3

Volume is 141.3 cm^3

5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) inner curved surface area of the vessel

(ii) radius of the base

(iii) capacity of the vessel

(Assume π = 22/7)

Solution:

(i) Rs 20 is the cost of painting 1 m^2 area.

Rs 1 is the cost to paint 1/20 m^2 area

So, Rs 2200 is the cost of painting = (1/20×2200) m^2

= 110 m^2 area

The inner surface area of the vessel is 110m^2.

(ii) Radius of the base of the vessel, let us say r.

Height (h) = 10 m and

Surface area formula = 2πrh

Using result of (i)

2πrh = 110 m^2

2×22/7×r×10 = 110

r = 1.75

Radius is 1.75 m.

(iii) Volume of vessel formula = $\pi r^2 h$

Here r = 1.75 and h = 10

Volume = $(22/7)\times(1.75)^2\times 10$ = 96.25

Volume of vessel is 96.25 m^3

Therefore, the capacity of the vessel is 96.25 m^3 or 96250 litres.

6. The capacity of a closed cylindrical vessel of height 1m is15.4 liters. How many square meters of metal sheet would be needed to make it? (Assume π = 22/7)

Solution:

Height of cylindrical vessel, h = 1 m

Capacity of cylindrical vessel = 15.4 litres = 0.0154 m^3

Let r be the radius of the circular end.

Now,

Capacity of cylindrical vessel = $(22/7)\times r^2\times 1$ = 0.0154

After simplifying, we get, r = 0.07 m

Again, total surface area of vessel = 2πr(r+h)

= 2×22/7×0.07(0.07+1)

= 0.44×1.07

= 0.4708

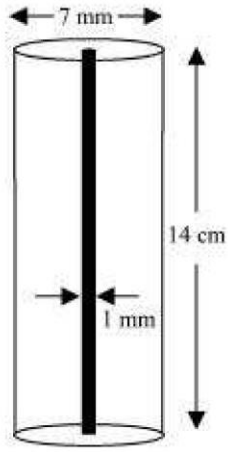
Total surface area of vessel is 0.4708 m^2

Therefore, 0.4708 m^2 of the metal sheet would be required to make the cylindrical vessel.

7. A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the

length of the pencil is 14 cm, find the volume of the wood and that of the graphite. (Assume π = 22/7)

Solution:



Radius of pencil, r_1 = 7/2 mm = 0.7/2 cm = 0.35 cm

Radius of graphite, r_2 = 1/2 mm = 0.1/2 cm = 0.05 cm

Height of pencil, h = 14 cm

Formula to find, volume of wood in pencil = $(r_1^2 - r_2^2)h$ cubic units

Substitute the values, we have

= $[(22/7)\times(0.35^2-0.05^2)\times 14]$

= 44×0.12

= 5.28

This implies, volume of wood in pencil = 5.28 cm^3

Again,

Volume of graphite = $r_2^2 h$ cubic units

Substitute the values, we have

= $(22/7)\times 0.05^2\times 14$

= 44×0.0025

= 0.11

So, the volume of graphite is 0.11 cm^3.

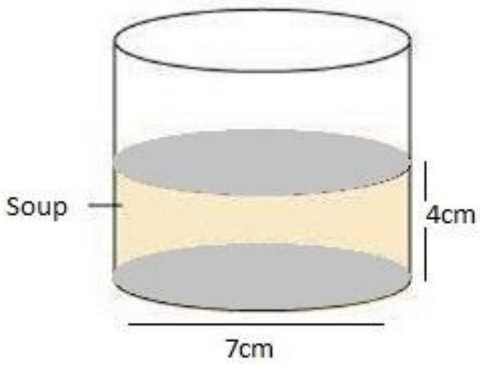
8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? (Assume π = 22/7)

Solution:

Diameter of cylindrical bowl = 7 cm

Radius of cylindrical bowl, r = 7/2 cm = 3.5 cm

Bowl is filled with soup to a height of4cm, so h = 4 cm



Volume of soup in one bowl= $\pi r^2 h$

$(22/7)\times 3.5^2\times 4 = 154$

Volume of soup in one bowl is 154 cm^3

Therefore,

Volume of soup given to 250 patients = (250×154) cm^3= 38500 cm^3

= 38.5litres. Answer!

Exercise 13.7

1. Find the volume of the right circular cone with

(i) radius 6cm, height 7 cm (ii) radius 3.5 cm, height 12 cm (Assume π = 22/7)

Solution:

Volume of cone = $(1/3)\pi r^2 h$ cube units

Where r be radius and h be the height of the cone

(i) Radius of cone, r = 6 cm

Height of cone, h = 7cm

Say, V be the volume of the cone, we have

$V = (1/3)\times(22/7)\times 36\times 7$

$= (12\times 22)$

= 264

The volume of the cone is 264 cm^3.

(ii) Radius of cone, r = 3.5cm

Height of cone, h = 12cm

Volume of cone = $(1/3)\times(22/7)\times 3.5^2\times 7 = 154$

Hence,

The volume of the cone is 154 cm^3.

2. Find the capacity in litres of a conical vessel with

(i) radius 7cm, slant height 25 cm (ii) height 12 cm, slant height 12 cm

(Assume π = 22/7)

Solution:

(i) Radius of cone, r =7 cm

Slant height of cone, l = 25 cm

$$\text{Height of cone, h} = \sqrt{l^2 - r^2}$$
$$\text{h} = \sqrt{25^2 - 7^2}$$
$$\text{h} = \sqrt{625 - 49}$$

or h = 24

Height of the cone is 24 cm

Now,

Volume of cone, V = (1/3) $\pi r^2 h$ (formula)

V = (1/3)×(22/7) ×7^2×24

= (154×8)

= 1232

So, the volume of the vessel is 1232 cm^3

Therefore, capacity of the conical vessel = (1232/1000) liters (because 1L = 1000 cm^3)

= 1.232 Liters.

(ii) Height of cone, h = 12 cm

Slant height of cone, l = 13 cm

$$\text{Radius of cone, r} = \sqrt{l^2 - h^2}$$
$$\text{r} = \sqrt{13^2 - 12^2}$$
$$\text{r} = \sqrt{169 - 144}$$

r = 5

Hence, the radius of cone is 5 cm.

Now, Volume of cone, V = (1/3)$\pi r^2 h$

V = (1/3)×(22/7)×52×12 cm^3

= 2200/7

Volume of cone is 2200/7 cm^3

Now, Capacity of the conical vessel= 2200/7000 litres (1L = 1000 cm^3)

= 11/35 litres

3. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

Solution:

Height of the cone, h = 15 cm

Volume of cone =1570 cm^3

Let r be the radius of the cone

As we know: Volume of cone, V = (1/3) $\pi r^2 h$

So, (1/3) $\pi r^2 h$ = 1570

(1/3)×3.14×r^2 ×15 = 1570

r^2 = 100

r = 10

Radius of the base of cone 10 cm.

4. If the volume of a right circular cone of height 9cm is 48πcm^3, find the diameter of its base.

Solution:

Height of cone, h = 9cm

Volume of cone =48π cm^3

Let r be the radius of the cone.

As we know: Volume of cone, V = (1/3) $\pi r^2 h$

So, 1/3 π r^2(9) = 48 π

r^2 = 16

r = 4

Radius of cone is 4 cm.

So, diameter = 2×Radius = 8

Thus, diameter of base is 8cm.

5. A conical pit of top diameter 3.5m is 12m deep. What is its capacity in kiloliters?

(Assume π = 22/7)

Solution:

Diameter of conical pit = 3.5 m

Radius of conical pit, r = diameter/ 2 = (3.5/2)m = 1.75m

Height of pit, h = Depth of pit = 12m

Volume of cone, V = (1/3) $\pi r^2 h$

V = (1/3)×(22/7) ×$(1.75)^2$×12 = 38.5

Volume of cone is 38.5 m^3

Hence, capacity of the pit = (38.5×1) kiloliters = 38.5 kiloliters.

6. The volume of a right circular cone is 9856cm^3. If the diameter of the base is 28cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

(Assume π = 22/7)

Solution:

Volume of a right circular cone = 9856 cm^3

Diameter of the base = 28 cm

(i) Radius of cone, r = (28/2) cm = 14 cm

Let the height of the cone be h

Volume of cone, V = (1/3) $\pi r^2 h$

(1/3) $\pi r^2 h$ = 9856

(1/3)×(22/7) ×14×14×h = 9856

h = 48

The height of the cone is 48 cm.

(ii) Slant height of cone, $l = \sqrt{r^2 + h^2}$

$l = \sqrt{14^2 + 48^2} = \sqrt{196 + 2304} = 50$

Slant height of the cone is 50 cm.

(iii) curved surface area of cone = πrl

= (22/7)×14×50

= 2200

curved surface area of the cone is 2200 cm^2.

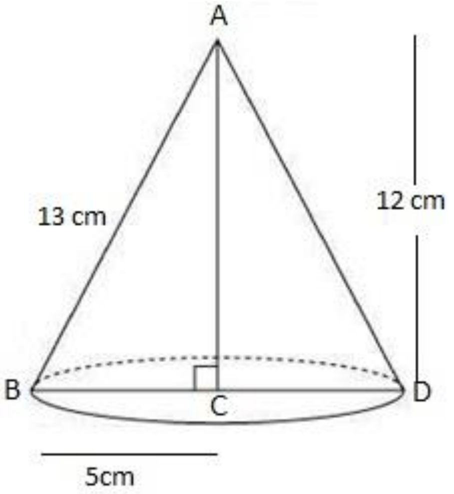
7. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Solution:

Height (h)= 12 cm

Radius (r) = 5 cm, and

Slant height (l) = 13 cm



Volume of cone, V = (1/3) $\pi r^2 h$

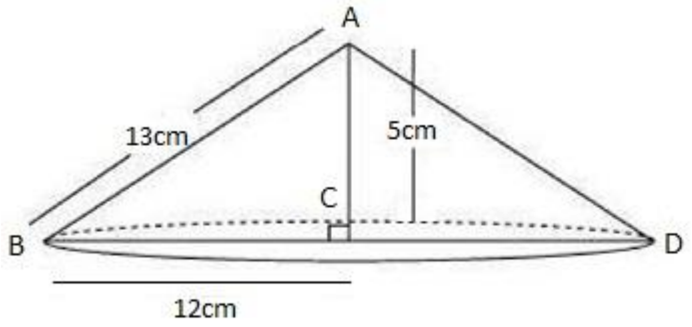
V = (1/3)×π×5^2×12

$= 100\pi$

Volume of the cone so formed is 100π cm^3.

8. If the triangle ABC in the Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Solution:



A right-angled ΔABC is revolved about its side 5cm, a cone will be formed of radius as 12 cm, height as 5 cm, and slant height as 13 cm.

Volume of cone = $(1/3)\ \pi r^2 h$; where r is the radius and h be the height of cone

$= (1/3)\times\pi\times12\times12\times5$

$= 240\ \pi$

The volume of the cones of formed is 240π cm^3.

So, required ratio = (result of question 7) / (result of question 8) = $(100\pi)/(240\pi) = 5/12 = 5:12$.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas.

(Assume π = 22/7)

Solution:

Radius (r) of heap = (10.5/2) m = 5.25

Height (h) of heap = 3m

Volume of heap = $(1/3)\pi r^2 h$

$= (1/3)\times(22/7)\times5.25\times5.25\times3$

$= 86.625$

The volume of the heap of wheat is 86.625 m^3.

Again,

Area of canvas required= CSA of cone = πrl, where $l = \sqrt{r^2 + h^2}$

After substituting the values, we have

CSA of cone = $[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + 3^2}]$

$= (22/7)\times5.25\times6.05$

$= 99.825$

Therefore, the area of the canvas is 99.825 m^2.

Exercise 13.8

1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

(Assume π =22/7)

Solution:

(i) Radius of sphere, r = 7 cm

Using, Volume of sphere = (4/3) πr^3

= (4/3)×(22/7)×7^3

= 4312/3

Hence, volume of the sphere is 4312/3 cm^3

(ii) Radius of sphere, r = 0.63 m

Using, volume of sphere = (4/3) πr^3

= (4/3)×(22/7)×0.63^3

= 1.0478

Hence, volume of the sphere is 1.05 m^3 (approx).

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

(Assume π =22/7)

Solution:

(i) Diameter = 28 cm

Radius, r = 28/2 cm = 14cm

Volume of the solid spherical ball = (4/3) πr^3

Volume of the ball = (4/3)×(22/7)×14^3 = 34496/3

Hence, volume of the ball is 34496/3 cm^3

(ii) Diameter = 0.21 m

Radius of the ball =0.21/2 m= 0.105 m

Volume of the ball = (4/3)πr^3

Volume of the ball = (4/3)× (22/7)×0.105^3 m^3

Hence, volume of the ball = 0.004851 m^3

3.The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7)

Solution:

Given,

Diameter of a metallic ball = 4.2 cm

Radius(r) of the metallic ball, r = 4.2/2 cm = 2.1 cm

Volume formula = 4/3 πr^3

Volume of the metallic ball = (4/3)×(22/7)×2.1 cm^3

Volume of the metallic ball = 38.808 cm^3

Now, using relationship between, density, mass and volume,

Density = Mass/Volume

Mass = Density × volume

= (8.9×38.808) g

= 345.3912 g

Mass of the ball is 345.39 g (approx).

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Let the diameter of earth be "d". Therefore, the radius of earth will be will be d/2

Diameter of moon will be d/4 and the radius of moon will be d/8

Find the volume of the moon :

Volume of the moon = (4/3) πr^3 = (4/3) π $(d/8)^3$ = 4/3π(d^3/512)

Find the volume of the earth :

Volume of the earth = (4/3) πr^3= (4/3) π $(d/2)^3$ = 4/3π(d^3/8)

Fraction of the volume of the earth is the volume of the moon

$$\text{Volume of the moon/ volume of the earth} = \frac{\frac{4}{3}\pi(\frac{d^3}{512})}{\frac{4}{3}\pi(\frac{d^3}{8})} = 8/512 = 1/64$$

Answer: Volume of moon is of the 1/64 volume of earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7)

Solution:

Diameter of hemispherical bowl = 10.5 cm

Radius of hemispherical bowl, r = 10.5/2 cm = 5.25 cm

Formula for volume of the hemispherical bowl = (2/3) πr^3

Volume of the hemispherical bowl = (2/3)×(22/7)×5.25^3 = 303.1875

Volume of the hemispherical bowl is 303.1875 cm^3

Capacity of the bowl = (303.1875)/1000 L = 0.303 litres(approx.)

Therefore, hemispherical bowl can hold 0.303 litres of milk.

6. A hemi spherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7)

Solution:

Inner Radius of the tank, (r) = 1m

Outer Radius (R) = 1.01m

Volume of the iron used in the tank = $(2/3)\ \pi(R^3 - r^3)$

Put values,

Volume of the iron used in the hemispherical tank = $(2/3)\times(22/7)\times(1.01^3 - 1^3) = 0.06348$

So, volume of the iron used in the hemispherical tank is 0.06348 m^3.

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let r be the radius of a sphere.

Surface area of sphere = $4\pi r^2$

$4\pi r^2 = 154\ cm^2$ (given)

$r^2 = (154\times7)/(4\times22)$

r = 7/2

Radius is 7/2 cm

Now,

Volume of the sphere = $(4/3)\ \pi r^3$

Volume of the sphere = $(4/3)\times(22/7)\times(7/2)^3 = 179\frac{2}{3}$

Volume of the sphere is $179\frac{2}{3}\ cm^3$

8. A dome of a building is in the form of a hemi sphere. From inside, it was white-washed at the cost of Rs. 4989.60. If the cost of white-washing isRs20 per square meter, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Solution:

(i) Cost of white-washing the dome from inside = Rs 4989.60

Cost of white-washing 1m^2 area = Rs 20

CSA of the inner side of dome = 498.96/2 m^2 = 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of inner side of dome = 249.48 m^2 (from (i))

Formula to find CSA of a hemi sphere = $2\pi r^2$

$2\pi r^2 = 249.48$

$2\times(22/7)\times r^2 = 249.48$

$r^2 = (249.48\times7)/(2\times22)$

$r^2 = 39.69$

$r = 6.3$

So, radius is 6.3 m

Volume of air inside the dome = Volume of hemispherical dome

Using formula, volume of the hemisphere = $2/3\ \pi r^3$

$= (2/3)\times(22/7)\times6.3\times6.3\times6.3$

$= 523.908$

$= 523.9$(approx.)

Answer: Volume of air inside the dome is 523.9 m^3.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

Solution:

Volume of the solid sphere = $(4/3)\pi r^3$

Volume of twenty seven solid sphere = $27\times(4/3)\pi r^3 = 36\ \pi\ r^3$

(i) New solid iron sphere radius = r'

Volume of this new sphere = $(4/3)\pi(r')^3$

$(4/3)\pi(r')^3 = 36\ \pi\ r^3$

$(r')^3 = 27r^3$

$r' = 3r$

Radius of new sphere will be 3r (thrice the radius of original sphere)

(ii) Surface area of iron sphere of radius r, S $=4\pi r^2$

Surface area of iron sphere of radius r'= $4\pi\ (r')^2$

Now

$S/S' = (4\pi r^2)/(\ 4\pi\ (r')^2)$

$S/S' = r^2/(3r')^2 = 1/9$

The ratio of S and S' is 1: 9.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume $\pi = 22/7$)

Solution:

Diameter of capsule = 3.5 mm

Radius of capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm

Volume of spherical capsule = $4/3\ \pi r^3$

Volume of spherical capsule = $(4/3)\times(22/7)\times(1.75)^3 = 22.458$

Answer: The volume of the spherical capsule is 22.46 mm^3.

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.9 : AREAS OF PARALLELOGRAMS AND TRIANGLES

SUBJECT: MATHEMATICS **CLASS: IX**

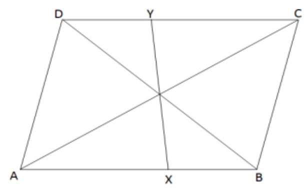
This chapter consists of 2 different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 PARALLELOGRAMS ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

Q.1 Prove that parallelograms on the same base and between the same parallels are equal in area.

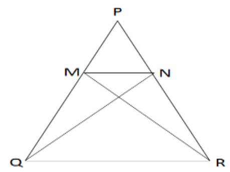
Q.2. A rectangle and a parallelogram are on the same base and between the same parallels. If height of parallelogram is 4 cm and length of base of rectangle is 8 cm, find the area of parallelogram.

Q3. The diagonals of a parallelogram ABCD intersect at O. A line through O meets AB at X and CD at Y. Show that area of quadrilateral AXYD is half the area of parallelogram ABCD.

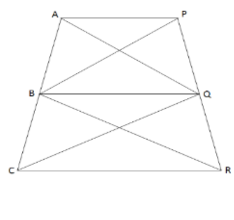


TYPE: 2 TRIANGLES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

Q.1 In the adjoining figure, M and N are poits on sides PQ and PR respectively of ΔPQR such that $ar(\Delta QRN) = ar(\Delta QRM)$. Show that $MN \parallel QR$.



Q.2. In the adjoining figure, $AP \parallel BQ \parallel CR$. Prove that $ar(\Delta AQC) = ar(\Delta PBR)$.



ACHIEVER'S SECTION:

Q.1. If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram. Show that a median of a triangle divides it into two triangles of equal areas.

Q.2. Show that a median of a triangle divides it into two triangles of equal areas.

Ch. 13 Surface Areas and Volumes

Surface Area Formulas

Shapes	Surface Areas
Cuboid	2(lb + bh + hl)
Cube	$6a^2$
Right Circular Cylinder	2πr(r + h)
Right Circular Cone	πr(l + r), ($l^2 = h^2 + r^2$)
Sphere	$4 \pi r^2$

Volume and Capacity

The **volume** of an object is the measure of the space it occupies and the **capacity** of an object is the volume of substance its interior can accommodate. The unit of measurement of either volume or capacity is a cubic unit.

Volume Formulas

Shapes	Volumes
Cuboid	length × breadth × height
Cube	a^3
Right Circular Cylinder	$\pi r^2 h$
Right Circular Cone	⅓ $\pi r^2 h$

Sphere	$4/3\ \pi\ r^3$

Exercise 13.1 Page No: 213

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i)The area of the sheet required for making the box.

(ii)The cost of sheet for it, if a sheet measuring 1m² costs Rs. 20.

Solution:

Given: length (l) of box = 1.5m

Breadth (b) of box = 1.25 m

Depth (h) of box = 0.65m

(i) Box is to be open at top

Area of sheet required.

= 2lh+2bh+lb

= $[2\times1.5\times0.65+2\times1.25\times0.65+1.5\times1.25]m^2$

= $(1.95+1.625+1.875)\ m^2 = 5.45\ m^2$

(ii) Cost of sheet per m^2 area = Rs.20.

Cost of sheet of 5.45 m^2 area = Rs (5.45×20)

= Rs.109.

2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and ceiling at the rate of Rs 7.50 per m^2.

Solution:

Length (l) of room = 5m

Breadth (b) of room = 4m

Height (h) of room = 3m

It can be observed that four walls and the ceiling of the room are to be white washed.

Total area to be white washed = Area of walls + Area of ceiling of room

= 2lh+2bh+lb

= $[2\times5\times3+2\times4\times3+5\times4]$

= (30+24+20)

= 74

Area = 74 m^2

Also,

Cost of white wash per m^2 area = Rs.7.50 (Given)

Cost of white washing 74 m^2 area = Rs. (74×7.50)

= Rs. 555

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs.15000, find the height of the hall.

[Hint: Area of the four walls = Lateral surface area.]

Solution:

Let length, breadth, and height of the rectangular hall be l, b, and h respectively.

Area of four walls = 2lh+2bh

= 2(l+b)h

Perimeter of the floor of hall = 2(l+b)

= 250 m

Area of four walls = 2(l+b) h = 250h m^2

Cost of painting per square meter area = Rs.10

Cost of painting 250h square meter area = Rs (250h×10) = Rs.2500h

However, it is given that the cost of paining the walls is Rs. 15000.

15000 = 2500h

Or h = 6

Therefore, the height of the hall is 6 m.

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm×10 cm×7.5 cm can be painted out of this container?

Solution:

Total surface area of one brick = 2(lb +bh+lb)

= [2(22.5×10+10×7.5+22.5×7.5)] cm^2

= 2(225+75+168.75) cm^2

= (2×468.75) cm^2

= 937.5 cm^2

Let n bricks can be painted out by the paint of the container

Area of n bricks = (n×937.5) cm^2 = 937.5n cm^2

As per given instructions, area that can be painted by the paint of the container = 9.375 m^2 = 93750 cm^2

So, we have, 93750 = 937.5n

n = 100

Therefore, 100 bricks can be painted out by the paint of the container.

5. A cubical box has each edge 10 cm and another cuboidal box is 12.5cm long, 10 cm wide and 8 cm high

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Solution:

From the question statement, we have

Edge of a cube = 10cm

Length, l = 12.5 cm

Breadth, b = 10cm

Height, h = 8 cm

(i) Find the lateral surface area for both the figures

Lateral surface area of cubical box = 4 $(edge)^2$

= $4(10)^2$

= 400 cm^2 …(1)

Lateral surface area of cuboidal box = 2[lh+bh]

= [2(12.5×8+10×8)]

= (2×180) = 360

Therefore, Lateral surface area of cuboidal box is 360 cm^2. …(2)

From (1) and (2), lateral surface area of the cubical box is more than the lateral surface area of the cuboidal box. The difference between both the lateral surfaces is, 40 cm^2.

(Lateral surface area of cubical box – Lateral surface area of cuboidal box=400cm^2–360cm^2 = 40 cm^2)

(ii) Find the total surface area for both the figures

The total surface area of the cubical box = $6(edge)^2$ = $6(10\ cm)^2$ = 600 cm^2…(3)

Total surface area of cuboidal box

= 2[lh+bh+lb]

= [2(12.5×8+10×8+12.5×100)]

= 610

This implies, Total surface area of cuboidal box is 610 cm^2..(4)

From (3) and (4), the total surface area of the cubical box is smaller than that of the cuboidal box. And their difference is 10cm^2.

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2

6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30cm long, 25 cm wide and 25 cm high.

(i)What is the area of the glass?

(ii)How much of tape is needed for all the 12 edges?

Solution:

Length of greenhouse, say l = 30cm

Breadth of greenhouse, say b = 25 cm

Height of greenhouse, say h = 25 cm

(i) Total surface area of greenhouse = Area of the glass = 2[lb+lh+bh]

= [2(30×25+30×25+25×25)]

= [2(750+750+625)]

= (2×2125) = 4250

Total surface area of the glass is 4250 cm^2

(ii)

Total length of tape = 4(l+b+h)

= [4(30+25+25)] (after substituting the values)

= 320

Therefore, 320 cm tape is required for all the 12 edges.

7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Solution:

Let l, b and h be the length, breadth and height of the box.

Bigger Box:

l = 25cm

b = 20 cm

h = 5 cm

Total surface area of bigger box = 2(lb+lh+bh)

= [2(25×20+25×5+20×5)]

= [2(500+125+100)]

= 1450 cm^2

Extra area required for overlapping 1450×5/100 cm^2

= 72.5 cm^2

While considering all over laps, total surface area of bigger box

= (1450+72.5) cm^2 = 1522.5 cm^2

Area of cardboard sheet required for 250 such bigger boxes

= (1522.5×250) cm^2 = 380625 cm^2

Smaller Box:

Similarly, total surface area of smaller box = [2(15×12+15×5+12×5)] cm^2

= [2(180+75+60)] cm^2

= (2×315) cm^2

= 630 cm^2

Therefore, extra area required for overlapping 630×5/100 cm^2 = 31.5 cm^2

Total surface area of 1 smaller box while considering all overlaps

= (630+31.5) cm^2 = 661.5 cm^2

Area of cardboard sheet required for 250 smaller boxes = (250×661.5) cm^2 = 165375 cm^2

Now, Total cardboard sheet required = (380625+165375) cm^2

= 546000 cm^2

Given: Cost of 1000 cm^2 cardboard sheet = Rs. 4

Therefore, Cost of 546000 cm^2 cardboard sheet =Rs. (546000×4)/1000 = Rs. 2184

Therefore, the cost of cardboard required for supplying 250 boxes of each kind will be Rs. 2184.

8. Praveen wanted to make a temporary shelter for her car, by making a box – like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?

Solution:

Let l, b and h be the length, breadth and height of the shelter.

Given:

l = 4m

b = 3m

h = 2.5m

Tarpaulin will be required for the top and four wall sides of the shelter.

Using formula, Area of tarpaulin required = 2(lh+bh)+lb

On putting the values of l, b and h, we get

= [2(4×2.5+3×2.5)+4×3] m^2

= [2(10+7.5)+12]m^2

= 47m^2

Therefore, 47 m^2 tarpaulin will be required.

Exercise 13.2

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder. (Assume π =22/7)

Solution:

Height of cylinder, h = 14cm

Let the diameter of the cylinder be d

Curved surface area of cylinder = 88 cm^2

We know that, formula to find Curved surface area of cylinder is 2πrh.

So 2πrh =88 cm^2 (r is the radius of the base of the cylinder)

2×(22/7)×r×14 = 88 cm^2

2r = 2 cm

d =2 cm

Therefore, the diameter of the base of the cylinder is 2 cm.

2. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square meters of the sheet are required for the same? Assume π = 22/7

Solution:

Let h be the height and r be the radius of a cylindrical tank.

Height of cylindrical tank, h = 1m

Radius = half of diameter = (140/2) cm = 70cm = 0.7m

Area of sheet required = Total surface are of tank = 2πr(r+h) unit square

= [2×(22/7)×0.7(0.7+1)]

= 7.48

Therefore, 7.48 square meters of the sheet are required.

3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4cm. (see fig. 13.11). Find its

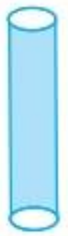


Fig. 13.11

(i) inner curved surface area,

(ii) outer curved surface area

(iii) total surface area

(Assume π=22/7)

Solution:

Let r_1 and r_2 Inner and outer radii of cylindrical pipe

r_1 = 4/2 cm = 2 cm

r_2 = 4.4/2 cm = 2.2 cm

Height of cylindrical pipe, h = length of cylindrical pipe = 77 cm

(i) curved surface area of outer surface of pipe = $2\pi r_1h$

= 2×(22/7)×2×77 cm^2

= 968 cm^2

(ii) curved surface area of outer surface of pipe = $2\pi r_2h$

= 2×(22/7)×2.2×77 cm^2

$= (22×22×2.2)\ cm^2$

$= 1064.8\ cm^2$

(iii) Total surface area of pipe = inner curved surface area+ outer curved surface area+ Area of both circular ends of pipe.

$= 2r_1h+2r_2h+2π(r_1^2-r_2^2)$

$= 9668+1064.8+2×(22/7)×(2.2^2-2^2)$

= 2031.8+5.28

$= 2038.08\ cm^2$

Therefore, the total surface area of the cylindrical pipe is $2038.08\ cm^2$.

4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to

move once over to level a playground. Find the area of the playground in m^2? (Assume π = 22/7)

Solution:

A roller is shaped like a cylinder.

Let h be the height of the roller and r be the radius.

h = Length of roller = 120 cm

Radius of the circular end of roller = r = (84/2) cm = 42 cm

Now, CSA of roller = 2πrh

= 2×(22/7)×42×120

$= 31680\ cm^2$

Area of field = 500×CSA of roller

$= (500×31680)\ cm^2$

$= 15840000\ cm^2$

$= 1584\ m^2$.

Therefore, area of playground is $1584\ m^2$. Answer!

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.

(Assume π = 22/7)

Solution:

Let h be the height of a cylindrical pillar and r be the radius.

Given:

Height cylindrical pillar = h = 3.5 m

Radius of the circular end of pillar = r = diameter/2 = 50/2 = 25cm = 0.25m

CSA of pillar = 2πrh

= 2×(22/7)×0.25×3.5

$= 5.5\ m^2$

Cost of painting 1 m^2 area = Rs. 12.50

Cost of painting 5.5 m^2 area = Rs (5.5×12.50)

= Rs.68.75

Therefore, the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2 is Rs 68.75.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the base of the cylinder is 0.7 m, find its height. (Assume π = 22/7)

Solution:

Let h be the height of the circular cylinder and r be the radius.

Radius of the base of cylinder, r = 0.7m

CSA of cylinder = 2πrh

CSA of cylinder = 4.4m^2

Equating both the equations, we have

2×(22/7)×0.7×h = 4.4

Or h = 1

Therefore, the height of the cylinder is 1 m.

7. The inner diameter of a circular well is 3.5m. It is 10m deep. Find

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.

(Assume π = 22/7)

Solution:

Inner radius of circular well, r = 3.5/2m = 1.75m

Depth of circular well, say h = 10m

(i) Inner curved surface area = 2πrh

= (2×(22/7)×1.75×10)

= 110

Therefore, the inner curved surface area of the circular well is 110 m^2.

(ii)Cost of plastering 1 m^2 area = Rs.40

Cost of plastering 110 m^2 area = Rs (110×40)

= Rs.4400

Therefore, the cost of plastering the curved surface of the well is Rs. 4400.

8. In a hot water heating system, there is cylindrical pipe of length 28 m and diameter 5 cm. Find

the total radiating surface in the system. (Assume π = 22/7)

Solution:

Height of cylindrical pipe = Length of cylindrical pipe = 28m

Radius of circular end of pipe = diameter/ 2 = 5/2 cm = 2.5cm = 0.025m

Now, CSA of cylindrical pipe = 2πrh, where r = radius and h = height of the cylinder

= 2×(22/7)×0.025×28 m^2

= 4.4m^2

The area of the radiating surface of the system is 4.4m^2.

9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in

diameter and 4.5m high.

(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank. (Assume π = 22/7)

Solution:

Height of cylindrical tank, h = 4.5m

Radius of the circular end , r = (4.2/2)m = 2.1m

(i) the lateral or curved surface area of cylindrical tank is 2πrh

= 2×(22/7)×2.1×4.5 m^2

= (44×0.3×4.5) m^2

= 59.4 m^2

Therefore, CSA of tank is 59.4 m^2.

(ii) Total surface area of tank = 2πr(r+h)

= 2×(22/7)×2.1×(2.1+4.5)

= 44×0.3×6.6

= 87.12 m^2

Now, Let S m^2 steel sheet be actually used in making the tank.

S(1 -1/12) = 87.12 m^2

This implies, S = 95.04 m^2

Therefore, 95.04m^2 steel was used in actual while making such a tank.

10. In fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth.

The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade. (Assume π = 22/7)

Fig. 13.12

Solution:

Say h = height of the frame of lampshade, looks like cylindrical shape r = radius

Total height is h = (2.5+30+2.5) cm = 35cm and

r = (20/2) cm = 10cm

Use curved surface area formula to find the cloth required for covering the lampshade which is 2πrh

= (2×(22/7)×10×35) cm^2

= 2200 cm^2

Hence, 2200 cm^2 cloth is required for covering the lampshade.

11. The students of Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? (Assume π =22/7)

Solution:

Radius of the circular end of cylindrical penholder, r = 3cm

Height of penholder, h = 10.5cm

Surface area of a penholder = CSA of pen holder + Area of base of penholder

= $2\pi rh+\pi r^2$

= $2\times(22/7)\times3\times10.5+(22/7)\times3^2$= 1584/7

Therefore, Area of cardboard sheet used by one competitor is 1584/7 cm^2

So, Area of cardboard sheet used by 35 competitors = 35×1584/7 = 7920 cm^2

Therefore, 7920 cm^2 cardboard sheet will be needed for the competition.

Exercise 13.3

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area (Assume π=22/7)

Solution:

Radius of the base of cone = diameter/ 2 = (10.5/2)cm = 5.25cm

Slant height of cone, say l = 10 cm

CSA of cone is = πrl

= (22/7)×5.25×10 = 165

Therefore, the curved surface area of the cone is 165 cm^2.

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m. (Assume π = 22/7)

Solution:

Radius of cone, r = 24/2 m = 12m

Slant height, l = 21 m

Formula: Total Surface area of the cone = $\pi r(l+r)$

Total Surface area of the cone = $(22/7)\times12\times(21+12)\ m^2$

= $1244.57m^2$

3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

(Assume π = 22/7)

Solution:

Slant height of cone, l = 14 cm

Let the radius of the cone be r.

(i) We know, CSA of cone = πrl

Given: Curved surface area of a cone is 308 cm^2

(308) = $(22/7)\times r\times14$

308 = 44 r

r = 308/44 = 7

Radius of a cone base is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base (πr^2)

Total surface area of cone = $308+(22/7)\times7^2$ = 308+154

Therefore, the total surface area of the cone is 462 cm^2.

4. A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

(Assume π=22/7)

Solution:

Let ABC be a conical tent

Height of conical tent, h = 10 m

Radius of conical tent, r = 24m

Let the slant height of the tent be l.

(i) In right triangle ABO, we have

$AB^2 = AO^2+BO^2$(using Pythagoras theorem)

$l^2 = h^2+r^2$

= $(10)^2+(24)^2$

= 676

l = 26

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = πrl

= $(22/7)\times24\times26\ m^2$

Cost of 1 m^2 canvas = Rs 70

Cost of (13728/7)m^2 canvas is equal to Rs (13728/7)×70 = Rs 137280

Therefore, the cost of the canvas required to make such a tent is Rs 137280.

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14]

Solution:

Height of conical tent, h = 8m

Radius of base of tent, r = 6m

Slant height of tent, $l^2 = (r^2+h^2)$

$l^2 = (6^2+8^2) = (36+64) = (100)$

or l = 10

Again, CSA of conical tent = πrl

= (3.14×6×10) m^2

= 188.4m^2

Let the length of tarpaulin sheet required be L

As 20 cm will be wasted, therefore,

Effective length will be (L-0.2m).

Breadth of tarpaulin = 3m (given)

Area of sheet = CSA of tent

[(L–0.2)×3] = 188.4

L-0.2 = 62.8

L = 63

Therefore, the length of the required tarpaulin sheet will be 63 m.

6. The slant height and base diameter of conical tomb are 25m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2. (Assume π = 22/7)

Solution:

Slant height of conical tomb, l = 25m

Base radius, r = diameter/2 = 14/2 m = 7m

CSA of conical tomb = πrl

= (22/7)×7×25 = 550

CSA of conical tomb= 550m^2

Cost of white-washing 550 m^2 area, which is Rs (210×550)/100

= Rs. 1155

Therefore, cost will be Rs. 1155 while white-washing tomb.

7. A joker's cap is in the form of right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume π =22/7)

Solution:

Radius of conical cap, r = 7 cm

Height of conical cap, h = 24cm

Slant height, $l^2 = (r^2+h^2)$

$= (7^2+24^2)$

= (49+576)

= (625)

Or l = 25 cm

CSA of 1 conical cap = πrl

= (22/7)×7×25

= 550

CSA of 10 caps = (10×550) cm^2 = 5500 cm^2

Therefore, the area of the sheet required to make 10 such caps is 5500 cm^2.

8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take √(1.04) =1.02)

Solution:

Given:

Radius of cone, r = diameter/2 = 40/2 cm = 20cm = 0.2 m

Height of cone, h = 1m

Slant height of cone is l, and $l^2 = (r^2+h^2)$

Using given values, $l^2 = (0.2^2+1^2)$

= (1.04)

Or l = 1.02

Slant height of the cone is 1.02 m

Now,

CSA of each cone = πrl

= (3.14×0.2×1.02)

= 0.64056

CSA of 50 such cones = (50×0.64056) = 32.028

CSA of 50 such cones = 32.028 m^2

Again,

Cost of painting 1 m^2 area = Rs 12 (given)

Cost of painting 32.028 m^2 area = Rs (32.028×12)

= Rs.384.336

= Rs.384.34 (approximately)

Therefore, the cost of painting all these cones is Rs. 384.34.

Exercise 13.4

1. Find the surface area of a sphere of radius:

(i) 10.5cm (ii) 5.6cm (iii) 14cm

(Assume π=22/7)

Solution:

Formula: Surface area of sphere (SA) = $4\pi r^2$

(i) Radius of sphere, r = 10.5 cm

SA = $4\times(22/7)\times10.5^2 = 1386$

Surface area of sphere is 1386 cm^2

(ii) Radius of sphere, r = 5.6cm

Using formula, SA = $4\times(22/7)\times5.6^2 = 394.24$

Surface area of sphere is 394.24 cm^2

(iii) Radius of sphere, r = 14cm

SA = $4\pi r^2$

= $4\times(22/7)\times(14)^2$

= 2464

Surface area of sphere is 2464 cm^2

2. Find the surface area of a sphere of diameter:

(i) 14cm (ii) 21cm (iii) 3.5cm

(Assume π = 22/7)

Solution:

(i) Radius of sphere, r = diameter/2 = 14/2 cm = 7 cm

Formula for Surface area of sphere = $4\pi r^2$

= $4\times(22/7)\times7^2 = 616$

Surface area of a sphere is 616 cm^2

(ii) Radius (r) of sphere = 21/2 = 10.5 cm

Surface area of sphere = $4\pi r^2$

= $4\times(22/7)\times10.5^2 = 1386$

Surface area of a sphere is 1386 cm^2

Therefore, the surface area of a sphere having diameter 21cm is 1386 cm^2

(iii) Radius(r) of sphere = 3.5/2 = 1.75 cm

Surface area of sphere = $4\pi r^2$

= $4\times(22/7)\times1.75^2$ = 38.5

Surface area of a sphere is 38.5 cm^2

3. Find the total surface area of a hemisphere of radius 10 cm. [Use π=3.14]

Solution:

Radius of hemisphere, r = 10cm

Formula: Total surface area of hemisphere = $3\pi r^2$

= $3\times3.14\times10^2$ = 942

The total surface area of given hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Solution:

Let r_1 and r_2 be the radii of spherical balloon and spherical balloon when air is pumped into it respectively. So

r_1 = 7cm

r_2 = 14 cm

Now, Required ratio = (initial surface area)/(Surface area after pumping air into balloon)

= $4r_1^2/4r_2^2$

= $(r_1/r_2)^2$

= $(7/14)^2 = (1/2)^2$ = ¼

Therefore, the ratio between the surface areas is 1:4.

5. A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. (Assume π = 22/7)

Solution:

Inner radius of hemispherical bowl, say r = diameter/2 = (10.5)/2 cm = 5.25 cm

Formula for Surface area of hemispherical bowl = $2\pi r^2$

= $2\times(22/7)\times(5.25)^2$ = 173.25

Surface area of hemispherical bowl is 173.25 cm^2

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 1 cm^2 area = Rs 16 /100

Cost of tin-plating 173.25 cm^2 area = Rs. (16×173.25)/100 = Rs 27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl at the rate of Rs 16 per 100 cm^2 is Rs **27.72.**

6. Find the radius of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let the radius of the sphere be r.

Surface area of sphere = 154 (given)

Now,

$4\pi r^2 = 154$

$r^2 = (154\times7)/(4\times22) = (49/4)$

$r = (7/2) = 3.5$

The radius of the sphere is 3.5 cm.

7. The diameter of the moon is approximately one fourth of the diameter of the earth.

Find the ratio of their surface areas.

Solution:

If diameter of earth is said d, then the diameter of moon will be d/4 (as per given statement)

Radius of earth = d/2

Radius of moon = ½×d/4 = d/8

Surface area of moon = $4\pi(d/8)^2$

Surface area of earth = $4\pi(d/2)^2$

$$\text{Ratio of their Surface areas} = \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = 4/64 = 1/16$$

The ratio between their surface areas is 1:16.

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume π =22/7)

Solution:

Given:

Inner radius of hemispherical bowl = 5cm

Thickness of the bowl = 0.25 cm

Outer radius of hemispherical bowl = (5+0.25) cm = 5.25 cm

Formula for outer CSA of hemispherical bowl = $2\pi r^2$, where r is radius of hemisphere

$= 2\times(22/7)\times(5.25)^2 = 173.25$

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see fig. 13.22). Find

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in(i) and (ii).

Fig. 13.22

Solution:

(i) Surface area of sphere = $4\pi r^2$, where r is the radius of sphere

(ii) Height of cylinder, h = r+r =2r

Radius of cylinder = r

CSA of cylinder formula = $2\pi rh = 2\pi r(2r)$ (using value of h)

= $4\pi r^2$

(iii) Ratio between areas = (Surface area of sphere)/CSA of Cylinder)

= $4r^2/4r^2$ = 1/1

Ratio of the areas obtained in (i) and (ii) is 1:1.

Exercise 13.5

1. A matchbox measures 4 cm×2.5cm×1.5cm. What will be the volume of a packet containing 12 such boxes?

Solution:

Dimensions of a matchbox (a cuboid) are l×b×h = 4 cm×2.5 cm×1.5 cm respectively

Formula to find the volume of matchbox = l×b×h = (4×2.5×1.5) = 15

Volume of matchbox = 15 cm^3

Now, volume of 12 such matchboxes = (15×12) cm^3 = 180 cm^3

Therefore, the volume of 12 matchboxes is 180cm^3.

2. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m^3= 1000 l)

Solution:

Dimensions of a cuboidal water tank are: l = 6 m and b = 5 m and h = 4.5 m

Formula to find volume of tank, V = l×b×h

Put the values, we get

V = (6×5×4.5) = 135

Volume of water tank is 135 m^3

Again,

We are given that, amount of water that 1m^3 volume can hold = 1000 l

Amount of water, 135 m^3volume hold = (135×1000) litres = 135000 litres

Therefore, given cuboidal water tank can hold up to135000 litres of water.

3. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Solution:

Given:

Length of cuboidal vessel, l = 10 m

Width of cuboidal vessel, b = 8m

Volume of cuboidal vessel, V = 380 m^3

Let the height of the given vessel be h.

Formula for Volume of a cuboid, V = l×b×h

Using formula, we have

l×b×h = 380

10×8×h= 380

Or h = 4.75

Therefore, the height of the vessels is 4.75 m.

4. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m^3.

Solution:

The given pit has its length(l) as 8m, width (b)as 6m and depth (h)as 3 m.

Volume of cuboidal pit = l×b×h = (8×6×3) = 144 (using formula)

Required Volume is 144 m^3

Now,

Cost of digging per m^3 volume = Rs 30

Cost of digging 144 m^3 volume = Rs (144×30) = Rs 4320

5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Solution:

Length (l) and depth (h) of tank is 2.5 m and 10 m respectively.

To find: The value of breadth, say b.

Formula to find the volume of a tank = l×b×h = (2.5× b×10) m^3= 25b m^3

Capacity of tank= 25b m^3, which is equal to 25000b litres

Also, capacity of a cuboidal tank is 50000 litres of water (Given)

Therefore, 25000 b = 50000

This implies, b = 2

Therefore, the breadth of the tank is 2 m.

6. A village, having a population of 4000, requires 150 litres of water per head per day.

It has a tank measuring 20 m×15 m×6 m. For how many days will the water of this tank last?

Solution:

Length of the tank = l = 20 m

Breadth of the tank = b = 15 m

Height of the tank = h = 6 m

Total population of a village = 4000

Consumption of the water per head per day = 150 litres

Water consumed by the people in 1 day = (4000×150) litres = 600000 litres …(1)

Formula to find the capacity of tank, C = l×b×h

Using given data, we have

C = (20×15×6) m^3= 1800 m^3

Or C = 1800000 litres

Let water in this tank last for d days.

Water consumed by all people in d days = Capacity of tank (using equation (1))

600000 d =1800000

d = 3

Therefore, the water of this tank will last for 3 days. **Answer**

7. A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates each

measuring 1.5m×1.25 m×0.5 m that can be stored in the godown.

Solution:

From statement, we have

Length of the godown = 40 m

Breadth = 25 m

Height = 15 m

Whereas,

Length of the wooden crate = 1.5 m

Breadth = 1.25 m

Height = 0.5 m

Since godown and wooden crate are in cuboidal shape. Find the volume of each using formula, V = lbh.

Now,

Volume of godown = (40×25×15) m^3 = 15000 m^3

Volume of a wooden crate = (1.5×1.25×0.5) m^3 = 0.9375 m^3

Let us consider that, n wooden crates can be stored in the godown, then

Volume of n wooden crates = Volume of godown

0.9375×n =15000

Or n= 15000/0.9375 = 16000

Hence, the number of wooden crates that can be stored in the godown is 16,000.

8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Solution:

Side of a cube = 12cm (Given)

Find the volume of cube:

Volume of cube = $(\text{Side})^3 = (12)^3 \text{cm}^3 = 1728 \text{cm}^3$

Surface area of a cube with side 12 cm = $6a^2 = 6(12)^2 \text{cm}^2$ …(1)

Cube is cut into eight small cubes of equal volume, say side of each cube is p.

Volume of a small cube = p^3

Surface area = $6p^2$ …(2)

Volume of each small cube = $(1728/8)$ cm^3 = 216 cm^3

Or $(p)^3$ = 216 cm^3

Or p = 6 cm

Now, Surface areas of the cubes ratios = (Surface area of bigger cube)/(Surface area of smaller cubes)

From equation (1) and (2), we get

Surface areas of the cubes ratios = $(6a^2)/(6p^2) = a^2/p^2 = 12^2/6^2 = 4$

Therefore, the required ratio is 4 : 1.

9. A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Solution:

Given:

Depth of river, h = 3 m

Width of river, b = 40 m

Rate of water flow = 2km per hour = 2000m/60min = 100/3 m/min

Now, Volume of water flowed in 1 min = (100/3) × 40 × 3 = 4000m^3

Therefore, 4000 m^3 water will fall into the sea in a minute.

Exercise 13.6

1. The circumference of the base of cylindrical vessel is 132cm and its height is 25cm.

How many litres of water can it hold? (1000 cm^3= 1L) (Assume π = 22/7)

Solution:

Circumference of the base of cylindrical vessel = 132 cm

Height of vessel, h = 25 cm

Let r be the radius of the cylindrical vessel.

Step 1: Find the radius of vessel

We know that, circumference of base = $2\pi r$, so

$2\pi r = 132$ (given)

$r = (132/(2\pi))$

$r = 66\times7/22 = 21$

Radius is 21 cm

Step 2: Find the volume of vessel

Formula: Volume of cylindrical vessel = $\pi r^2 h$

$= (22/7)\times21^2\times25$

$= 34650$

Therefore, volume is 34650 cm^3

Since, **1000 cm^3 = 1L**

So, Volume = 34650/1000 L= 34.65L

Therefore, vessel can hold 34.65 litres of water.

2. The inner diameter of a cylindrical wooden pipe is 24cm and its outer diameter is 28 cm. The length of the pipe is 35cm.Find the mass of the pipe, if 1cm^3 of wood has a mass of 0.6g. (Assume π = 22/7)

Solution:

Inner radius of cylindrical pipe, say r_1 = $diameter_1$/ 2 = 24/2 cm = 12cm

Outer radius of cylindrical pipe, say r_2 = $diameter_2$/ 2 = 28/2 cm = 14 cm

Height of pipe, h = Length of pipe = 35cm

Now, the Volume of pipe = $\pi(r_2^2-r_1^2)h$ cm^3

Substitute the values.

Volume of pipe = 110×52 cm^3 = 5720 cm^3

Since**, Mass of 1 cm^3 wood = 0.6 g**

Mass of 5720 cm^3 wood = (5720×0.6) g = 3432 g or 3.432 kg. Answer!

3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5cm and width 4cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7cm and height 10cm. Which container has greater capacity and by how much? (Assume π=22/7)

Solution:

tin can will be cuboidal in shape

Dimensions of tin can are

Length, l = 5 cm

Breadth, b = 4 cm

Height, h = 15 cm

Capacity of tin can = l×b×h= (5×4×15) cm^3 = 300 cm^3

1.Plastic cylinder will be cylindrical in shape.

Dimensions of plastic can are:

Radius of circular end of plastic cylinder, r = 3.5cm

Height , H = 10 cm

Capacity of plastic cylinder = $\pi r^2 H$

Capacity of plastic cylinder = $(22/7)\times(3.5)^2\times10 = 385$

Capacity of plastic cylinder is 385 cm^3

From results of (i) and (ii), plastic cylinder has more capacity.

Difference in capacity = (385-300) cm^3 = 85cm^3

4. If the lateral surface of a cylinder is 94.2cm^2 and its height is 5cm, then find

(i) radius of its base (ii) its volume.[Use π= 3.14]

Solution:

CSA of cylinder = 94.2 cm^2

Height of cylinder, h = 5cm

(i) Let radius of cylinder be r.

Using CSA of cylinder, we get

$2\pi rh = 94.2$

$2\times3.14\times r\times5 = 94.2$

r = 3

radius is 3 cm

(ii) Volume of cylinder

Formula for volume of cylinder = $\pi r^2 h$

Now, $\pi r^2 h = (3.14\times(3)^2\times5)$ (using value of r from (i))

= 141.3

Volume is 141.3 cm^3

5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) inner curved surface area of the vessel

(ii) radius of the base

(iii) capacity of the vessel

(Assume π = 22/7)

Solution:

(i) Rs 20 is the cost of painting 1 m^2 area.

Rs 1 is the cost to paint 1/20 m^2 area

So, Rs 2200 is the cost of painting = $(1/20\times2200)$ m^2

= 110 m^2 area

The inner surface area of the vessel is $110 m^2$.

(ii) Radius of the base of the vessel, let us say r.

Height (h) = 10 m and

Surface area formula = $2\pi rh$

Using result of (i)

$2\pi rh = 110 m^2$

$2 \times 22/7 \times r \times 10 = 110$

r = 1.75

Radius is 1.75 m.

(iii) Volume of vessel formula = $\pi r^2 h$

Here r = 1.75 and h = 10

Volume = $(22/7)\times(1.75)^2\times10 = 96.25$

Volume of vessel is $96.25 m^3$

Therefore, the capacity of the vessel is $96.25 m^3$ or 96250 litres.

6. The capacity of a closed cylindrical vessel of height 1m is15.4 liters. How many square meters of metal sheet would be needed to make it? (Assume π = 22/7)

Solution:

Height of cylindrical vessel, h = 1 m

Capacity of cylindrical vessel = 15.4 litres = $0.0154 m^3$

Let r be the radius of the circular end.

Now,

Capacity of cylindrical vessel = $(22/7)\times r^2\times1 = 0.0154$

After simplifying, we get, r = 0.07 m

Again, total surface area of vessel = $2\pi r(r+h)$

$= 2\times22/7\times0.07(0.07+1)$

$= 0.44\times1.07$

= 0.4708

Total surface area of vessel is $0.4708 m^2$

Therefore, $0.4708 m^2$ of the metal sheet would be required to make the cylindrical vessel.

7. A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. (Assume π = 22/7)

Solution:

Radius of pencil, r_1 = 7/2 mm = 0.7/2 cm = 0.35 cm

Radius of graphite, r_2 = 1/2 mm = 0.1/2 cm = 0.05 cm

Height of pencil, h = 14 cm

Formula to find, volume of wood in pencil = $(r_1^2-r_2^2)h$ cubic units

Substitute the values, we have

= $[(22/7)\times(0.35^2-0.05^2)\times14]$

= 44×0.12

= 5.28

This implies, volume of wood in pencil = 5.28 cm^3

Again,

Volume of graphite = r_2^2h cubic units

Substitute the values, we have

= $(22/7)\times0.05^2\times14$

= 44×0.0025

= 0.11

So, the volume of graphite is 0.11 cm^3.

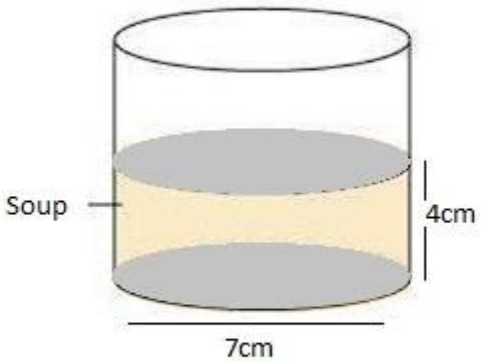
8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? (Assume π = 22/7)

Solution:

Diameter of cylindrical bowl = 7 cm

Radius of cylindrical bowl, r = 7/2 cm = 3.5 cm

Bowl is filled with soup to a height of4cm, so h = 4 cm



Volume of soup in one bowl= πr^2h

$(22/7)\times3.5^2\times4 = 154$

Volume of soup in one bowl is 154 cm^3

Therefore,

Volume of soup given to 250 patients = (250×154) cm^3= 38500 cm^3

= 38.5litres.

Exercise 13.7

1. Find the volume of the right circular cone with

(i) radius 6cm, height 7 cm (ii) radius 3.5 cm, height 12 cm (Assume π = 22/7)

Solution:

Volume of cone = $(1/3)\ \pi r^2 h$ cube units

Where r be radius and h be the height of the cone

(i) Radius of cone, r = 6 cm

Height of cone, h = 7cm

Say, V be the volume of the cone, we have

$V = (1/3)\times(22/7)\times36\times7$

$= (12\times22)$

$= 264$

The volume of the cone is 264 cm^3.

(ii) Radius of cone, r = 3.5cm

Height of cone, h = 12cm

Volume of cone = $(1/3)\times(22/7)\times3.5^2\times7 = 154$

Hence,

The volume of the cone is 154 cm^3.

2. Find the capacity in litres of a conical vessel with

(i) radius 7cm, slant height 25 cm (ii) height 12 cm, slant height 12 cm

(Assume π = 22/7)

Solution:

(i) Radius of cone, r =7 cm

Slant height of cone, l = 25 cm

$$\text{Height of cone, } h = \sqrt{l^2 - r^2}$$
$$h = \sqrt{25^2 - 7^2}$$
$$h = \sqrt{625 - 49}$$

or h = 24

Height of the cone is 24 cm

Now,

Volume of cone, V = $(1/3)\ \pi r^2 h$ (formula)

$V = (1/3)\times(22/7)\times7^2\times24$

$= (154\times8)$

$= 1232$

So, the volume of the vessel is 1232 cm^3

Therefore, capacity of the conical vessel = (1232/1000) liters (because 1L = 1000 cm^3)

= 1.232 Liters.

(ii) Height of cone, h = 12 cm

Slant height of cone, l = 13 cm

Radius of cone, $r = \sqrt{l^2 - h^2}$

$r = \sqrt{13^2 - 12^2}$

$r = \sqrt{169 - 144}$

r = 5

Hence, the radius of cone is 5 cm.

Now, Volume of cone, V = (1/3)πr^2h

V = (1/3)×(22/7)×52×12 cm^3

= 2200/7

Volume of cone is 2200/7 cm^3

Now, Capacity of the conical vessel= 2200/7000 litres (1L = 1000 cm^3)

= 11/35 litres

3. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

Solution:

Height of the cone, h = 15 cm

Volume of cone =1570 cm^3

Let r be the radius of the cone

As we know: Volume of cone, V = (1/3) πr^2h

So, (1/3) πr^2h = 1570

(1/3)×3.14×r^2 ×15 = 1570

r^2 = 100

r = 10

Radius of the base of cone 10 cm.

4. If the volume of a right circular cone of height 9cm is 48πcm^3, find the diameter of its base.

Solution:

Height of cone, h = 9cm

Volume of cone =48π cm^3

Let r be the radius of the cone.

As we know: Volume of cone, V = (1/3) πr^2h

So, $1/3\ \pi\ r^2(9) = 48\ \pi$

$r^2 = 16$

$r = 4$

Radius of cone is 4 cm.

So, diameter = 2×Radius = 8

Thus, diameter of base is 8cm.

5. A conical pit of top diameter 3.5m is 12m deep. What is its capacity in kiloliters?

(Assume π = 22/7)

Solution:

Diameter of conical pit = 3.5 m

Radius of conical pit, r = diameter/ 2 = (3.5/2)m = 1.75m

Height of pit, h = Depth of pit = 12m

Volume of cone, $V = (1/3)\ \pi r^2 h$

$V = (1/3)\times(22/7)\times(1.75)^2\times 12 = 38.5$

Volume of cone is 38.5 m^3

Hence, capacity of the pit = (38.5×1) kiloliters = 38.5 kiloliters.

6. The volume of a right circular cone is 9856cm³. If the diameter of the base is 28cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

(Assume π = 22/7)

Solution:

Volume of a right circular cone = 9856 cm^3

Diameter of the base = 28 cm

(i) Radius of cone, r = (28/2) cm = 14 cm

Let the height of the cone be h

Volume of cone, $V = (1/3)\ \pi r^2 h$

$(1/3)\ \pi r^2 h = 9856$

$(1/3)\times(22/7)\times 14\times 14\times h = 9856$

$h = 48$

The height of the cone is 48 cm.

(ii) Slant height of cone, $l = \sqrt{r^2 + h^2}$

$$l = \sqrt{14^2 + 48^2} = \sqrt{196 + 2304} = 50$$

Slant height of the cone is 50 cm.

(iii) curved surface area of cone = πrl

= (22/7)×14×50

= 2200

curved surface area of the cone is 2200 cm^2.

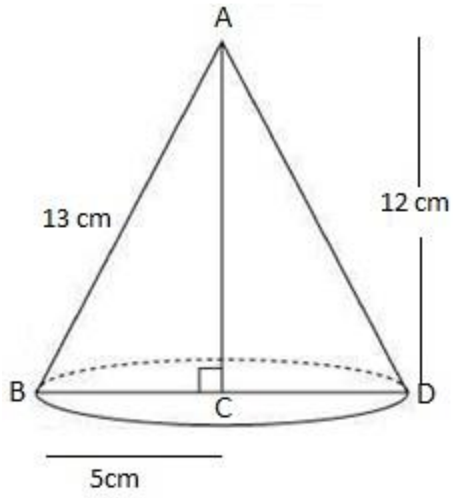
7. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Solution:

Height (h)= 12 cm

Radius (r) = 5 cm, and

Slant height (l) = 13 cm



Volume of cone, V = (1/3) πr^{2}h

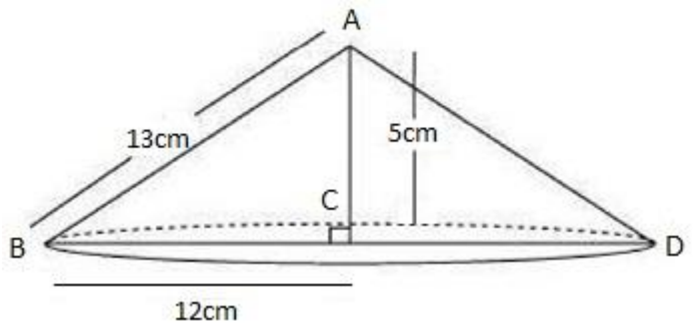
V = (1/3)×π×5^2×12

= 100π

Volume of the cone so formed is 100π cm^3.

8. If the triangle ABC in the Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Solution:



A right-angled ΔABC is revolved about its side 5cm, a cone will be formed of radius as 12 cm, height as 5 cm, and slant height as 13 cm.

Volume of cone = (1/3) πr^{2}h; where r is the radius and h be the height of cone

= (1/3)×π×12×12×5

= 240 π

The volume of the cones of formed is 240π cm^3.

So, required ratio = (result of question 7) / (result of question 8) = (100π)/(240π) = 5/12 = 5:12.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas.

(Assume π = 22/7)

Solution:

Radius (r) of heap = (10.5/2) m = 5.25

Height (h) of heap = 3m

Volume of heap = $(1/3)\pi r^2 h$

= (1/3)×(22/7)×5.25×5.25×3

= 86.625

The volume of the heap of wheat is 86.625 m^3.

Again,

Area of canvas required= CSA of cone = πrl, where $l = \sqrt{r^2 + h^2}$

After substituting the values, we have

CSA of cone = $[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + 3^2}]$

= (22/7)×5.25×6.05

= 99.825

Therefore, the area of the canvas is 99.825 m^2.

Exercise 13.8

1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

(Assume π =22/7)

Solution:

(i) Radius of sphere, r = 7 cm

Using, Volume of sphere = $(4/3)\pi r^3$

= $(4/3)\times(22/7)\times7^3$

= 4312/3

Hence, volume of the sphere is 4312/3 cm^3

(ii) Radius of sphere, r = 0.63 m

Using, volume of sphere = (4/3) πr^3

= $(4/3)\times(22/7)\times 0.63^3$

= 1.0478

Hence, volume of the sphere is 1.05 m^3 (approx).

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

(Assume π =22/7)

Solution:

(i) Diameter = 28 cm

Radius, r = 28/2 cm = 14cm

Volume of the solid spherical ball = (4/3) πr^3

Volume of the ball = $(4/3)\times(22/7)\times 14^3$ = 34496/3

Hence, volume of the ball is 34496/3 cm^3

(ii) Diameter = 0.21 m

Radius of the ball =0.21/2 m= 0.105 m

Volume of the ball = (4/3)πr^3

Volume of the ball = $(4/3)\times (22/7)\times 0.105^3$ m^3

Hence, volume of the ball = 0.004851 m^3

3.The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7)

Solution:

Given,

Diameter of a metallic ball = 4.2 cm

Radius(r) of the metallic ball, r = 4.2/2 cm = 2.1 cm

Volume formula = 4/3 πr^3

Volume of the metallic ball = $(4/3)\times(22/7)\times 2.1$ cm^3

Volume of the metallic ball = 38.808 cm^3

Now, using relationship between, density, mass and volume,

Density = Mass/Volume

Mass = Density × volume

= (8.9×38.808) g

= 345.3912 g

Mass of the ball is 345.39 g (approx).

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Let the diameter of earth be “d”. Therefore, the radius of earth will be will be d/2

Diameter of moon will be d/4 and the radius of moon will be d/8

Find the volume of the moon :

Volume of the moon = (4/3) πr^3 = (4/3) π $(d/8)^3$ = 4/3π(d^3/512)

Find the volume of the earth :

Volume of the earth = (4/3) πr^3= (4/3) π $(d/2)^3$ = 4/3π(d^3/8)

Fraction of the volume of the earth is the volume of the moon

$$\text{Volume of the moon/ volume of the earth} = \frac{\frac{4}{3}\pi(\frac{d^3}{512})}{\frac{4}{3}\pi(\frac{d^3}{8})} = 8/512 = 1/64$$

Answer: Volume of moon is of the 1/64 volume of earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7)

Solution:

Diameter of hemispherical bowl = 10.5 cm

Radius of hemispherical bowl, r = 10.5/2 cm = 5.25 cm

Formula for volume of the hemispherical bowl = (2/3) πr^3

Volume of the hemispherical bowl = (2/3)×(22/7)×5.25^3 = 303.1875

Volume of the hemispherical bowl is 303.1875 cm^3

Capacity of the bowl = (303.1875)/1000 L = 0.303 litres(approx.)

Therefore, hemispherical bowl can hold 0.303 litres of milk.

6. A hemi spherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7)

Solution:

Inner Radius of the tank, (r) = 1m

Outer Radius (R) = 1.01m

Volume of the iron used in the tank = (2/3) π(R^3– r^3)

Put values,

Volume of the iron used in the hemispherical tank = (2/3)×(22/7)×(1.01^3– 1^3) = 0.06348

So, volume of the iron used in the hemispherical tank is 0.06348 m^3.

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Solution:

Let r be the radius of a sphere.

Surface area of sphere = $4\pi r^2$

$4\pi r^2$ = 154 cm^2 (given)

$r^2 = (154×7)/(4 ×22)$

$r = 7/2$

Radius is 7/2 cm

Now,

Volume of the sphere = $(4/3)\ \pi r^3$

Volume of the sphere = $(4/3)×(22/7)×(7/2)^3 = 179\frac{2}{3}$

Volume of the sphere is $179\frac{2}{3}$ cm^3

8. A dome of a building is in the form of a hemi sphere. From inside, it was white-washed at the cost of Rs. 4989.60. If the cost of white-washing isRs20 per square meter, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Solution:

(i) Cost of white-washing the dome from inside = Rs 4989.60

Cost of white-washing 1m^2 area = Rs 20

CSA of the inner side of dome = 498.96/2 m^2 = 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of inner side of dome = 249.48 m^2 (from (i))

Formula to find CSA of a hemi sphere = $2\pi r^2$

$2\pi r^2 = 249.48$

$2×(22/7)×r^2 = 249.48$

$r^2 = (249.48×7)/(2×22)$

$r^2 = 39.69$

$r = 6.3$

So, radius is 6.3 m

Volume of air inside the dome = Volume of hemispherical dome

Using formula, volume of the hemisphere = $2/3\ \pi r^3$

$= (2/3)×(22/7)×6.3×6.3×6.3$

= 523.908

= 523.9(approx.)

Answer: Volume of air inside the dome is 523.9 m^3.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

Solution:

Volume of the solid sphere = $(4/3)\pi r^3$

Volume of twenty seven solid sphere = $27\times(4/3)\pi r^3 = 36\pi r^3$

(i) New solid iron sphere radius = r'

Volume of this new sphere = $(4/3)\pi(r')^3$

$(4/3)\pi(r')^3 = 36\pi r^3$

$(r')^3 = 27r^3$

r'= 3r

Radius of new sphere will be 3r (thrice the radius of original sphere)

(ii) Surface area of iron sphere of radius r, S $=4\pi r^2$

Surface area of iron sphere of radius r'= $4\pi (r')^2$

Now

$S/S' = (4\pi r^2)/(4\pi (r')^2)$

$S/S' = r^2/(3r')^2 = 1/9$

The ratio of S and S' is 1: 9.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume π = 22/7)

Solution:

Diameter of capsule = 3.5 mm

Radius of capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm

Volume of spherical capsule = $4/3\ \pi r^3$

Volume of spherical capsule = $(4/3)\times(22/7)\times(1.75)^3 = 22.458$

Answer: The volume of the spherical capsule is 22.46 mm^3.

Exercise 13.9 Page No: 236

1. A wooden bookshelf has external dimensions as follows: Height = 110cm, Depth = 25cm,

Breadth = 85cm (see fig. 13.31). The thickness of the plank is 5cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

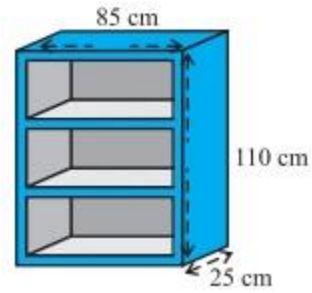


Fig. 13.31

Solution:

External dimensions of book self,

Length, l = 85cm

Breadth, b = 25 cm

Height, h = 110 cm

External surface area of shelf while leaving out the front face of the shelf

= lh+2(lb+bh)

= [85×110+2(85×25+25×110)] = (9350+9750) = 19100

External surface area of shelf is 19100 cm^2

Area of front face = [85×110-75×100+2(75×5)] = 1850+750

So, area is 2600 cm^2

Area to be polished = (19100+2600) cm^2 = 21700 cm^2.

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs. (21700×0.20) = Rs 4340

Dimensions of row of the book shelf

Length(l) = 75 cm

Breadth (b) = 20 cm and

Height(h) = 30 cm

Area to be painted in one row= 2(l+h)b+lh = [2(75+30)× 20+75×30] = (4200+2250) = 6450

So, area is 6450 cm^2.

Area to be painted in 3 rows = (3×6450)cm^2 = 19350 cm^2.

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs (19350 x 0.1) = Rs 1935

Total expense required for polishing and painting = Rs. (4340+1935) = Rs. 6275

Answer: The cost for polishing and painting the surface of the book shelf is Rs. 6275.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in fig. 13.32. Eight such spheres are used forth is purpose, and are to be painted silver. Each support is a cylinder of radius 1.5cm and height 7cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

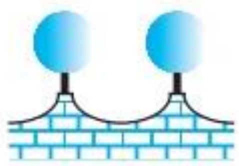


Fig. 13.32

Solution:

Diameter of wooden sphere = 21 cm

Radius of wooden sphere, r = diameter/ 2 = (21/2) cm = 10.5 cm

Formula: Surface area of wooden sphere = $4\pi r^2$

= $4\times(22/7)\times(10.5)^2$ = 1386

So, surface area is 1386 cm^3

Radius of the circular end of cylindrical support = 1.5 cm

Height of cylindrical support = 7 cm

Curved surface area = $2\pi rh$

= 2×(22/7)×1.5×7 = 66

So, CSA is 66 cm^2

Now,

Area of the circular end of cylindrical support = πr^2

= $(22/7)\times1.5^2$

= 7.07

Area of the circular end is 7.07 cm^2

Again,

Area to be painted silver = [8 ×(1386-7.07)] = 8×1378.93 = 11031.44

Area to be painted is 11031.44 cm^2

Cost for painting with silver colour = Rs(11031.44×0.25) =Rs 2757.86

Area to be painted black = (8×66) cm^2 = 528 cm^2

Cost for painting with black colour =Rs (528×0.05) = Rs26.40

Therefore, the total painting cost is:

= Rs(2757.86 +26.40)

= Rs 2784.26

3. The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Solution:

Let the diameter of the sphere be “d”.

Radius of sphere, r_1 = d/2

New radius of sphere, say r_2 = (d/2)×(1-25/100) = 3d/8

Curved surface area of sphere, $(CSA)_1 = 4\pi r_1^2 = 4\pi\times(d/2)^2 = \pi d^2$ …(1)

Curved surface area of sphere when radius is decreased $(CSA)_2 = 4\pi r_2^2 = 4\pi\times(3d/8)^2 = (9/16)\pi d^2$ …(2)

From equation (1) and (2), we have

Decrease in surface area of sphere = $(CSA)_1 - (CSA)_2$

$= \pi d^2 - (9/16)\pi d^2$

$= (7/16)\pi d^2$

$$\text{Percentage decrease in surface area of sphere} = \frac{(CSA)_1 - (CSA)_2}{(CSA)_1} \times 100$$

$= (7d^2/16d^2)\times 100 = 700/16 = 43.75\%$.

Therefore, the percentage decrease in the surface area of the sphere is 43.75% .

Ch. 11 Constructions

Exercise 11.1

1. Construct an angle of 90° at the initial point of a given ray and justify the construction.

Construction Procedure:

To construct an angle 90°, follow the given steps:

1. Draw a ray OA

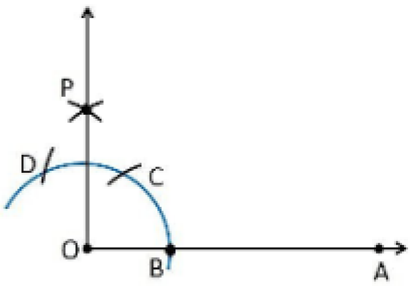
2. Take O as a centre with any radius, draw an arc DCB is that cuts OA at B.

3. With B as a centre with the same radius, mark a point C on the arc DCB.

4. With C as a centre and the same radius, mark a point D on the arc DCB.

5. Take C and D as centre, draw two arcs which intersect each other with the same radius at P.

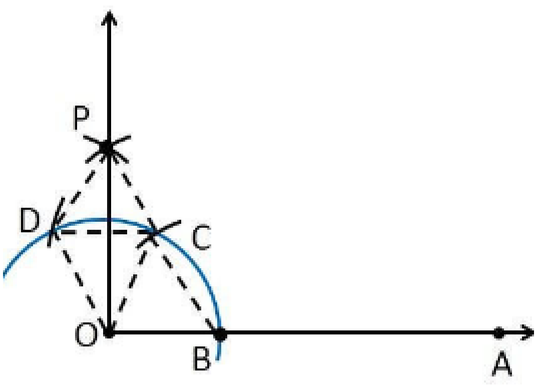
6. Finally, the ray OP is joined which makes an angle 90° with OP is formed.



Justification

To prove ∠POA = 90°

In order to prove this, draw a dotted line from the point O to C and O to D and the angles formed are:



From the construction, it is observed that

OB = BC = OC

Therefore, OBC is an equilateral triangle

So that, ∠BOC = 60°.

Similarly,

OD = DC = OC

Therefore, DOC is an equilateral triangle

So that, ∠DOC = 60°.

From SSS triangle congruence rule

△OBC ≅ OCD

So, ∠BOC = ∠DOC [By C.P.C.T]

Therefore, ∠COP = ½ ∠DOC = ½ (60°).

∠COP = 30°

To find the ∠POA = 90°:

∠POA = ∠BOC+∠COP

∠POA = 60°+30°

∠POA = 90°

Hence, justified.

2. Construct an angle of 45° at the initial point of a given ray and justify the construction.

Construction Procedure:

1. Draw a ray OA

2. Take O as a centre with any radius, draw an arc DCB is that cuts OA at B.

3. With B as a centre with the same radius, mark a point C on the arc DCB.

4. With C as a centre and the same radius, mark a point D on the arc DCB.

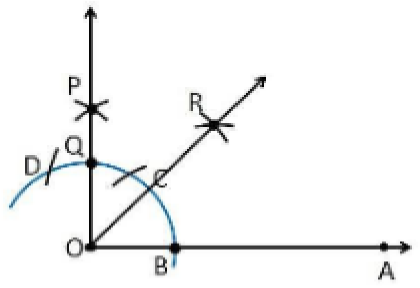
5. Take C and D as centre, draw two arcs which intersect each other with the same radius at P.

6. Finally, the ray OP is joined which makes an angle 90° with OP is formed.

7. Take B and Q as centre draw the perpendicular bisector which intersects at the point R

8. Draw a line that joins the point O and R

9. So, the angle formed ∠ROA = 45°



Justification

From the construction,

∠POA = 90°

From the perpendicular bisector from the point B and Q, which divides the ∠POA into two halves. So it becomes

∠ROA = ½ ∠POA

∠ROA = (½)×90° = 45°

Hence, justified

3. Construct the angles of the following measurements:

(i) 30° (ii) $22\frac{1}{2}^{\circ}$ (iii) 15°

Solution:

(i) 30°

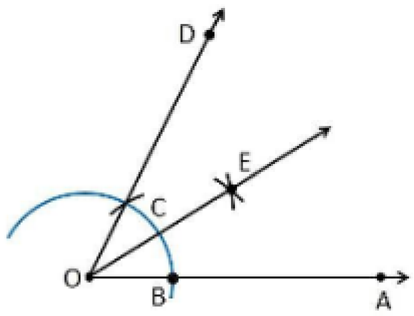
Construction Procedure:

1. Draw a ray OA

2. Take O as a centre with any radius, draw an arc BC which cuts OA at B.

3. With B and C as centres, draw two arcs which intersect each other at the point E and the perpendicular bisector is drawn.

4. Thus, ∠EOA is the required angle making 30° with OA.



(ii) $22\frac{1}{2}^{\circ}$

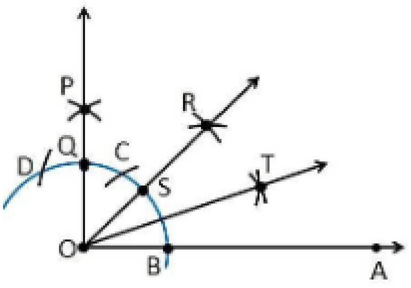
Construction Procedure:

1. Draw an angle ∠POA = 90°

2. Take O as a centre with any radius, draw an arc BC which cuts OA at B and OP at Q

3. Now, draw the bisector from the point B and Q where it intersects at the point R such that it makes an angle ∠ROA = 45°.

4. Again, ∠ROA is bisected such that ∠TOA is formed which makes an angle of 22.5° with OA



(iii) 15°

Construction Procedure:

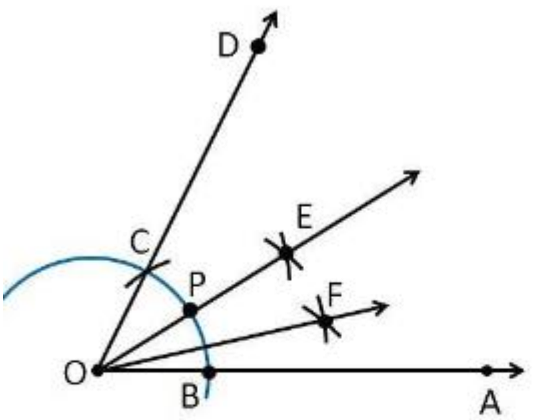
1. An angle ∠DOA = 60° is drawn.

2. Take O as centre with any radius, draw an arc BC which cuts OA at B and OD at C

3. Now, draw the bisector from the point B and C where it intersects at the point E such that it makes an angle ∠EOA = 30°.

4. Again, ∠EOA is bisected such that ∠FOA is formed which makes an angle of 15° with OA.

5. Thus, ∠FOA is the required angle making 15° with OA.



4. Construct the following angles and verify by measuring them by a protractor:

(i) 75° (ii) 105° (iii) 135°

Solution:

(i) 75°

Construction Procedure:

1. A ray OA is drawn.

2. With O as centre draw an arc of any radius and intersect at the point B on the ray OA.

3. With B as centre draw an arc C and C as centre draw an arc D.

4. With D and C as centre draw an arc, that intersect at the point P.

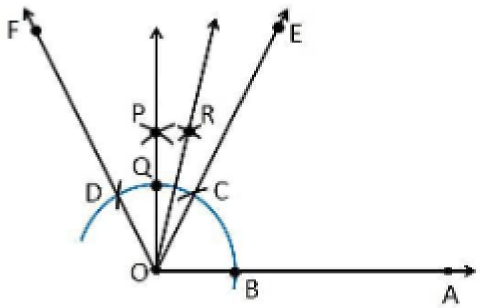
5. Join the points O and P

6. The point that arc intersect the ray OP is taken as Q.

7. With Q and C as centre draw an arc, that intersect at the point R.

8. Join the points O and R

9. Thus, ∠AOE is the required angle making 75° with OA.



(ii) 105°

Construction Procedure:

1. A ray OA is drawn.

2. With O as centre draw an arc of any radius and intersect at the point B on the ray OA.

3. With B as centre draw an arc C and C as centre draw an arc D.

4. With D and C as centre draw an arc, that intersect at the point P.

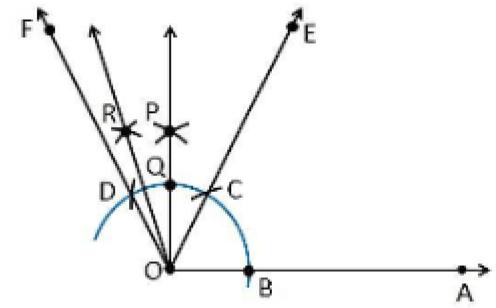
5. Join the points O and P

6. The point that arc intersect the ray OP is taken as Q.

7. With Q and Q as centre draw an arc, that intersect at the point R.

8. Join the points O and R

9. Thus, ∠AOR is the required angle making 105° with OA.



(iii) 135°

Construction Procedure:

1. Draw a line AOA‘

2. Draw an arc of any radius that cuts the line AOA‘at the point B and B‘

3. With B as centre, draw an arc of same radius at the point C.

4. With C as centre, draw an arc of same radius at the point D

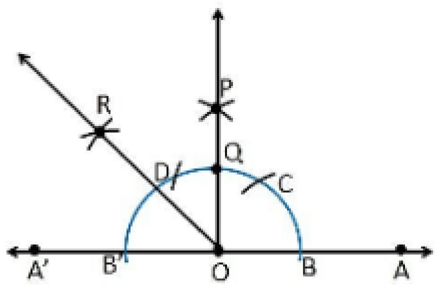
5. With D and C as centre, draw an arc that intersect at the point O

6. Join OP

7. The point that arc intersect the ray OP is taken as Q and it forms an angle 90°

8. With B‘ and Q as centre, draw an arc that intersects at the point R

9. Thus, ∠AOR is the required angle making 135° with OA.



5. Construct an equilateral triangle, given its side and justify the construction.

Construction Procedure:

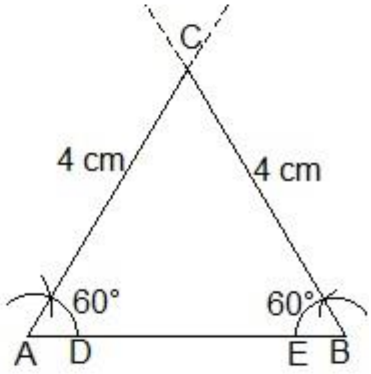
1. Let draw a line segment AB = 4 cm .

2. With A and B as centres, draw two arcs on the line segment AB and note the point as D and E.

3. With D and E as centres, draw the arcs that cuts the previous arc respectively that forms an angle of 60° each.

4. Now, draw the lines from A and B that are extended to meet each other at the point C.

5. Therefore, ABC is the required triangle.



Justification:

From construction, it is observed that

AB = 4 cm, ∠A = 60° and ∠B = 60°

We know that, the sum of the interior angles of a triangle is equal to 180°

∠A+∠B+∠C = 180°

Substitute the values

⇒ 60°+60°+∠C = 180°

⇒ 120°+∠C = 180°

⇒∠C = 60°

While measuring the sides, we get

BC = CA = 4 cm (Sides opposite to equal angles are equal)

AB = BC = CA = 4 cm

∠A = ∠B = ∠C = 60°

Hence, justified.

Exercise 11.2

1. Construct a triangle ABC in which BC = 7cm, ∠B = 75° and AB+AC = 13 cm.

Construction Procedure:

The steps to draw the triangle of given measurement is as follows:

1. Draw a line segment of base BC = 7 cm

2. Measure and draw ∠B = 75° and draw the ray BX

3. Take a compass and measure AB+AC = 13 cm.

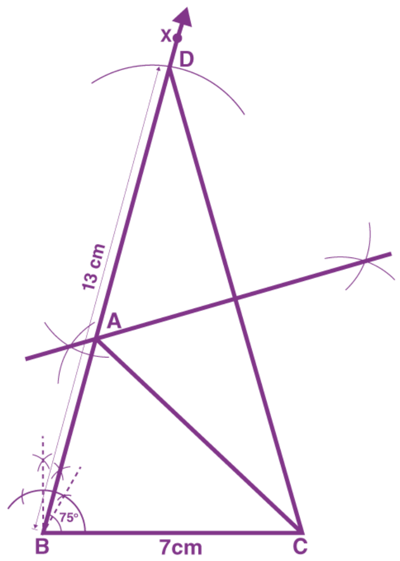
4. With B as the centre, draw an arc at the point be D

5. Join DC

6. Now draw the perpendicular bisector of the line DC and the intersection point is taken as A.

7. Now join AC

8. Therefore, ABC is the required triangle.



2. Construct a triangle ABC in which BC = 8cm, ∠B = 45° and AB–AC = 3.5 cm.

Construction Procedure:

The steps to draw the triangle of given measurement is as follows:

1. Draw a line segment of base BC = 8 cm

2. Measure and draw ∠B = 45° and draw the ray BX

3. Take a compass and measure AB-AC = 3.5 cm.

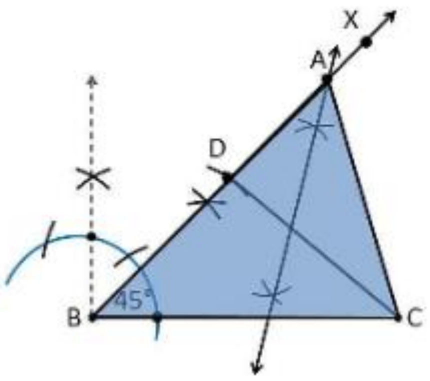
4. With B as centre and draw an arc at the point be D on the ray BX

5. Join DC

6. Now draw the perpendicular bisector of the line CD and the intersection point is taken as A.

7. Now join AC

8. Therefore, ABC is the required triangle.



3. Construct a triangle PQR in which QR = 6cm, ∠Q = 60° and PR–PQ = 2cm.

Construction Procedure:

The steps to draw the triangle of given measurement is as follows:

1. Draw a line segment of base QR = 6 cm

2. Measure and draw ∠Q = 60° and let the ray be QX

3. Take a compass and measure PR–PQ = 2cm.

4. Since PR–PQ is negative, QD will below the line QR.

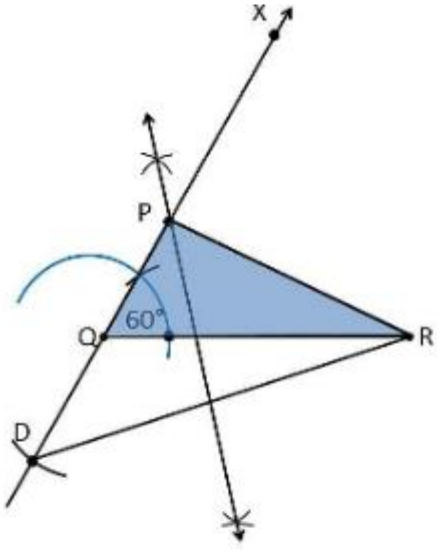
5. With Q as centre and draw an arc at the point be D on the ray QX

6. Join DR

7. Now draw the perpendicular bisector of the line DR and the intersection point is taken as P.

8. Now join PR

9. Therefore, PQR is the required triangle.



4. Construct a triangle XYZ in which ∠Y = 30°, ∠Z = 90° and XY+YZ+ZX = 11 cm.

Construction Procedure:

The steps to draw the triangle of given measurement is as follows:

1. Draw a line segment AB which is equal to XY+YZ+ZX = 11 cm**.**

2. Make an angle ∠Y = 30° from the point A and the angle be∠LAB

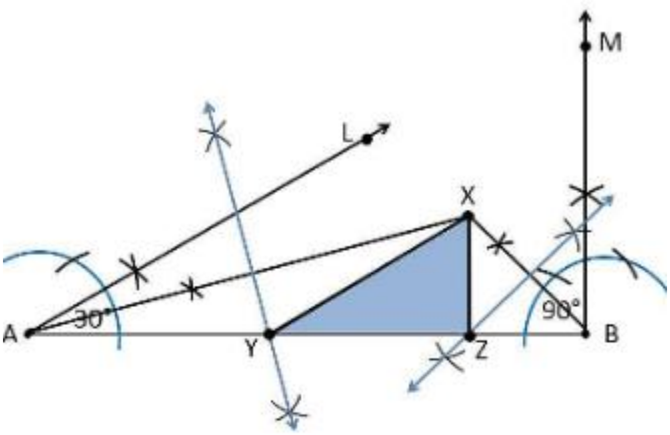
3. Make an angle ∠Z = 90° from the point B and the angle be ∠MAB

4. Bisect ∠LAB and ∠MAB at the point X.

5. Now take the perpendicular bisector of the line XA and XB and the intersection point be Y and Z, respectively.

6. Join XY and XZ

7. Therefore, XYZ is the required triangle



5. Construct a right triangle whose base is 12cm and sum of its hypotenuse and other side is 18 cm.

Construction Procedure:

The steps to draw the triangle of given measurement is as follows:

1. Draw a line segment of base BC = 12 cm

2. Measure and draw ∠B = 90° and draw the ray BX

3. Take a compass and measure AB+AC = 18 cm.

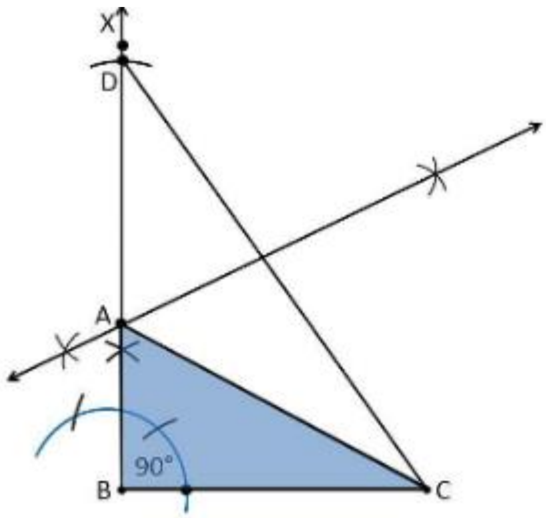
4. With B as centre and draw an arc at the point be D on the ray BX

5. Join DC

6. Now draw the perpendicular bisector of the line CD and the intersection point is taken as A.

7. Now join AC

8. Therefore, ABC is the required triangle.


X
D
A
B
C
90°

Ch. 10 Circles

- The **set of all the points** in a plane that is at a **fixed distance** from a **fixed point** makes a circle.
- A **Fixed point** from which the set of points are at fixed distance is called the **centre** of the circle.
- A circle divides the plane into 3 parts: **interior** (inside the circle), the **circle** itself and **exterior** (outside the circle)

Radius

– The **distance** between the **centre** of the circle and any **point on its edge** is called the **radius.**

To know more about Circles, visit here.

Chord

-The **line segment** within the circle joining any 2 points on the circle is called the chord.

Diameter

– A **Chord** passing through the centre of the circle is called the **diameter.** – The **Diameter is 2 times the radius** and it is the **longest chord**.

Arc

– The **portion** of a circle(curve) **between 2 points** is called an **arc**. – Among the two pieces made by an arc, the **longer** one is called a **major arc** and the **shorter** one is called a **minor arc.**

Circumference

The **perimeter** of a circle is the **distance** covered by going around its **boundary once**. The perimeter of a circle has a special name: **Circumference**, which is π times the diameter which is given by the formula $2\pi r$

Segment and Sector

– A circular **segment** is a region of a circle which is "**cut off**" from the rest of the circle by a secant or a chord. – **Smaller region** cut off by a chord is called **minor segment** and the **bigger region** is called **major segment**. –

-A **sector** is the portion of a circle **enclosed by two radii and an arc**, where the **smaller area** is known as the **minor sector** and the **larger** being the **major sector**.
To know more about Sector of a Circle.

– For **2 equa**l arcs or for semicircles – both the segment and sector is called the **semicircular region.**

Exercise: 10.1

1. Fill in the blanks:

(i) The centre of a circle lies in ____________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a _____________ of the circle.

(iv) An arc is a ___________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and _____________ of the circle.

(vi) A circle divides the plane, on which it lies, in _____________ parts.

Solution:

(i) The centre of a circle lies in **interior** of the circle.

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle.

(iii) The longest chord of a circle is a **diameter** of the circle.

(iv) An arc is a **semicircle** when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and **chord** of the circle.

(vi) A circle divides the plane, on which it lies, in **3 (three)** parts.

2. Write True or False: Give reasons for your Solutions.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Solution:

(i) **True.** Any line segment drawn from the centre of the circle to any point on it is the radius of the circle and will be of equal length.

(ii) **False.** There can be infinite numbers of equal chords of a circle.

(iii) **False.** For unequal arcs, there can be major and minor arcs. So, equal arcs on a circle cannot be said as a major arc or a minor arc.

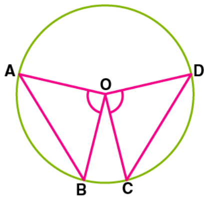
(iv) **True.** Any chord whose length is twice as long as the radius of the circle always passes through the centre of the circle and thus, it is known as the diameter of the circle.

(v) **False.** A sector is a region of a circle between the arc and the two radii of the circle.

(vi) **True.** A circle is a 2d figure and it can be drawn on a plane.

Theorem of equal chords subtending angles at the centre.

– Equal **chords** subtend equal **angles at the centre**.



Proof: AB and CD are the 2 equal chords.

In Δ AOB and Δ COD

OB = OC [Radii]

OA = OD [Radii]

AB = CD [Given]

ΔAOB ≅ ΔCOD (SSS rule)

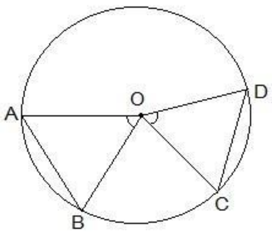
Hence, ∠AOB = ∠COD [CPCT]

Exercise: 10.2

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution:

To recall, a circle is a collection of points whose every point is equidistant from its centre. So, two circles can be congruent only when the distance of every point of both the circles are equal from the centre.



For the second part of the question, it is given that AB = CD i.e. two equal chords.

Now, it is to be proven that angle AOB is equal to angle COD.

Proof:

Consider the triangles ΔAOB and ΔCOD,

OA = OC and OB = OD (Since they are the radii of the circle)

AB = CD (As given in the question)

So, by SSS congruency, ΔAOB ≅ ΔCOD

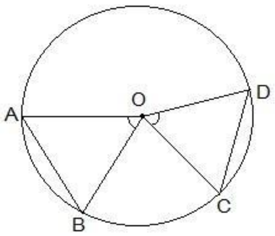
∴ By CPCT we have,

∠AOB = ∠COD. (Hence proved).

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution:

Consider the following diagram-



Here, it is given that ∠AOB = ∠COD i.e. they are equal angles.

Now, we will have to prove that the line segments AB and CD are equal i.e. AB = CD.

Proof:

In triangles AOB and COD,

∠AOB = ∠COD (as given in the question)

OA = OC and OB = OD (these are the radii of the circle)

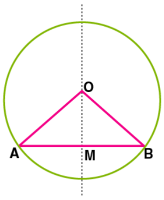
So, by SAS congruency, ΔAOB ≅ ΔCOD.

∴ By the rule of CPCT, we have

AB = CD. (Hence proved).

Perpendicular from the centre to a chord bisects the chord.

Perpendicular from the **centre** of a circle to a **chord bisects the chord**.



Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA,

∠OMA=∠OMB=90^0 OA = OB (radii)

OM = OM (common)

Hence, ΔOMB ≅ ΔOMA (RHS rule)

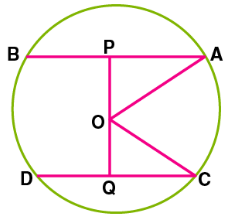
Therefore AM = MB [CPCT]

Circle through 3 points

– There is **one** and **only** one **circle** passing through **three given noncollinear points.** – A unique circle passes through 3 vertices of a triangle ABC called as the **circumcircle.** The **centre** and **radius** are called the **circumcenter** and **circumradius** of this triangle, respectively.

Equal chords are at equal distances from the centre.

Equal chords of a circle(or of congruent circles) are **equidistant from the centre** (or centres).



Proof: Given, AB = CD, O is the centre. Join OA and OC.

Draw, OP⊥AB, OQ⊥CD

In ΔOAP and ΔOCQ,

OA=OC (Radii)

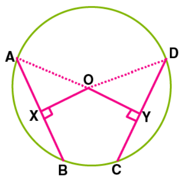
AP=CQ (AB = CD ⇒(1/2)AB = (1/2)CD since OP and OQ bisects the chords AB and CD.)

ΔOAP ≅ ΔOCQ (RHS rule)

Hence, OP=OQ (C.P.C.T.C)

Chords equidistant from the centre are equal

Chords **equidistant** from the centre of a circle are **equal in length.**



Proof: Given OX = OY (The chords AB and CD are at equidistant) OX⊥AB, OY⊥CD

In ΔAOX and ΔDOY

∠OXA =∠OYD (Both 90^0)

OA = OD (Radii)

OX = OY (Given)

ΔAOX ≅ ΔDOY (RHS rule)

Therefore AX = DY (CPCT)

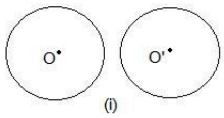
Similarly XB = YC

So, AB = CD

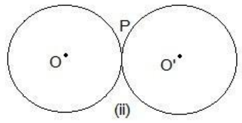
Exercise: 10.3

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

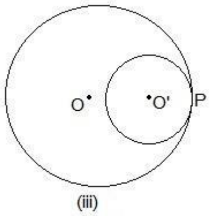
Solution:



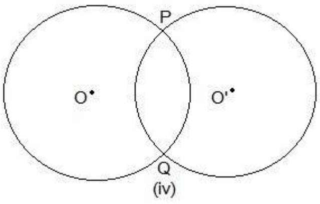
In these two circles, no point is common.



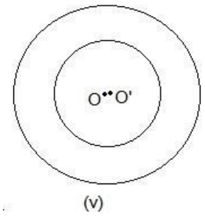
Here, only one point “P” is common.



Even here, P is the common point.



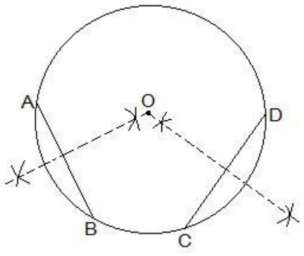
Here, two points are common which are P and Q.



No point is common in the above circle.

2. Suppose you are given a circle. Give a construction to find its centre.

Solution:



The construction steps to find the center of the circle are:

Step I: Draw a circle first.

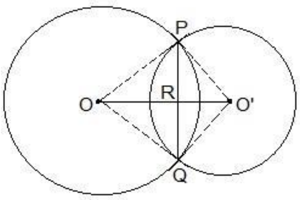
Step II: Draw 2 chords AB and CD in the circle.

Step III: Draw the perpendicular bisectors of AB and CD.

Step IV: Connect the two perpendicular bisectors at a point. This intersection point of the two perpendicular bisectors is the centre of the circle.

3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Solution:



It is given that two circles intersect each other at P and Q.

To prove:

OO' is perpendicular bisector of PQ.

Proof:

Triangle ΔPOO' and ΔQOO' are similar by SSS congruency since

OP = OQ and O'P = OQ (Since they are also the radii)

OO' = OO' (It is the common side)

So, It can be said that ΔPOO' ≅ ΔQOO'

∴ ∠POO' = ∠QOO' — (i)

Even triangles ΔPOR and ΔQOR are similar by SAS congruency as

OP = OQ (Radii)

∠POR = ∠QOR (As ∠POO' = ∠QOO')

OR = OR (Common arm)

So, ΔPOR ≅ ΔQOR

∴ ∠PRO = ∠QRO

Also, we know that

∠PRO+∠QRO = 180°

Hence, ∠PRO = ∠QRO = 180°/2 = 90°

So, OO' is the perpendicular bisector of PQ.

Exercise: 10.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution:

Given parameters are:

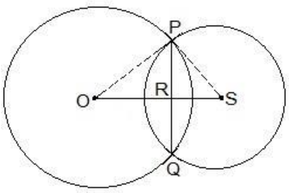
OP = 5cm

OS = 4cm and

PS = 3cm

Also, PQ = 2PR

Now, suppose RS = x. The diagram for the same is shown below.



Consider the ΔPOR,

$OP^2 = OR^2+PR^2$

$\Rightarrow 5^2 = (4-x)^2+PR^2$

$\Rightarrow 25 = 16+x^2-8x+PR^2$

$\therefore PR^2 = 9-x^2+8x$ — (i)

Now consider ΔPRS,

$PS^2 = PR^2+RS^2$

$\Rightarrow 3^2 = PR^2+x^2$

$\therefore PR^2 = 9-x^2$ — (ii)

By equating equation (i) and equation (ii) we get,

$9 -x^2+8x = 9-x^2$

$\Rightarrow 8x = 0$

$\Rightarrow x = 0$

Now, put the value of x in equation (i)

$PR^2 = 9-0^2$

$\Rightarrow PR = 3cm$

∴ The length of the cord i.e. PQ = 2PR

So, PQ = 2×3 = 6cm

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution:

Let AB and CD be two equal cords (i.e. AB = CD). In the above question, it is given that AB and CD intersect at a point, say, E.

It is now to be proven that the line segments AE = DE and CE = BE

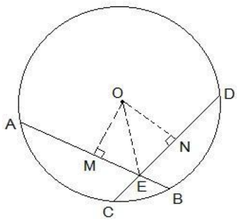
Construction Steps:

Step 1: From the center of the circle, draw a perpendicular to AB i.e. OM ⊥ AB

Step 2: Similarly, draw ON ⊥ CD.

Step 3: Join OE.

Now, the diagram is as follows-



Proof:

From the diagram, it is seen that OM bisects AB and so, OM ⊥ AB

Similarly, ON bisects CD and so, ON ⊥ CD

It is known that AB = CD. So,

AM = ND — (i)

and MB = CN — (ii)

Now, triangles ΔOME and ΔONE are similar by RHS congruency since

∠OME = ∠ONE (They are perpendiculars)

OE = OE (It is the common side)

OM = ON (AB and CD are equal and so, they are equidistant from the centre)

∴ ΔOME ≅ ΔONE

ME = EN (by CPCT) — (iii)

Now, from equations (i) and (ii) we get,

AM+ME = ND+EN

So, AE = ED

Now from equations (ii) and (iii) we get,

MB-ME = CN-EN

So, EB = CE (Hence proved).

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution:

From the question we know the following:

(i) AB and CD are 2 chords which are intersecting at point E.

(ii) PQ is the diameter of the circle.

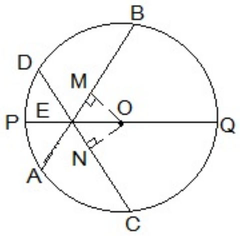
(iii) AB = CD.

Now, we will have to prove that **∠BEQ = ∠CEQ**

For this, the following construction has to be done:

Construction:

Draw two perpendiculars are drawn as OM ⊥ AB and ON ⊥ D. Now, join OE. The constructed diagram will look as follows:



Now, consider the triangles ΔOEM and ΔOEN.

Here,

(i) OM = ON [Since the equal chords are always equidistant from the centre]

(ii) OE = OE [It is the common side]

(iii) ∠OME = ∠ONE [These are the perpendiculars]

So, by RHS congruency criterion, ΔOEM ≅ ΔOEN.

Hence, by CPCT rule, ∠MEO = ∠NEO

∴ ∠BEQ = ∠CEQ (Hence proved).

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 10.25).

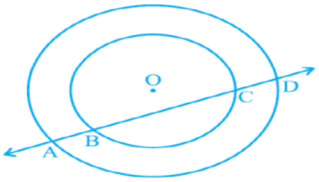


Fig. 10.25

Solution:

The given image is as follows:

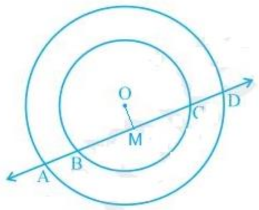


Fig. 10.25

First, draw a line segment from O to AD such that OM ⊥ AD.

So, now OM is bisecting AD since OM ⊥ AD.

Therefore, AM = MD — (i)

Also, since OM ⊥ BC, OM bisects BC.

Therefore, BM = MC — (ii)

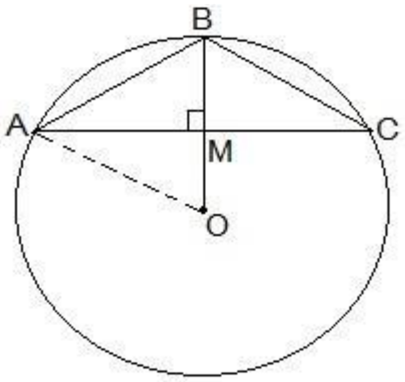
From equation (i) and equation (ii),

AM-BM = MD-MC

∴ AB = CD

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution:



Let the positions of Reshma, Salma and Mandip be represented as A, B and C respectively.

From the question, we know that AB = BC = 6cm.

So, the radius of the circle i.e. OA = 5cm

Now, draw a perpendicular BM ⊥ AC.

Since AB = BC, ABC can be considered as an isosceles triangle. M is mid-point of AC. BM is the perpendicular bisector of AC and thus it passes through the centre of the circle.

Now,

let AM = y and

OM = x

So, BM will be = (5-x).

By applying Pythagorean theorem in ΔOAM we get,

$OA^2 = OM^2 + AM^2$

⇒ $5^2 = x^2 + y^2$ — (i)

Again, by applying Pythagorean theorem in ΔAMB,

$AB^2 = BM^2 + AM^2$

⇒ $6^2 = (5-x)^2 + y^2$ — (ii)

Subtracting equation (i) from equation (ii), we get

$36-25 = (5-x)^2 + y^2 - x^2 - y^2$

Now, solving this equation we get the value of x as

x = 7/5

Substituting the value of x in equation (i), we get

$y^2 + (49/25) = 25$

⇒ $y^2 = 25 - (49/25)$

Solving it we get the value of y as

y = 24/5

Thus,

AC = 2×AM

= 2×y

= 2×(24/5) m

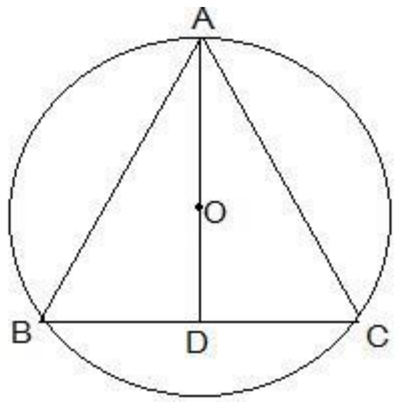
AC = 9.6 m

So, the distance between Reshma and Mandip is 9.6 m.

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution:

First, draw a diagram according to the given statements. The diagram will look as follows.



Here the positions of Ankur, Syed and David are represented as A, B and C respectively. Since they are sitting at equal distances, the triangle ABC will form an equilateral triangle.

AD ⊥ BC is drawn. Now, AD is median of ΔABC and it passes through the centre O.

Also, O is the centroid of the ΔABC. OA is the radius of the triangle.

OA = 2/3 AD

Let the side of a triangle a metres then BD = a/2 m.

Applying Pythagoras theorem in ΔABD,

$AB^2 = BD^2+AD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow AD^2 = a^2 - (a/2)^2$

$\Rightarrow AD^2 = 3a^2/4$

$\Rightarrow AD = \sqrt{3}a/2$

OA = 2/3 AD

$20 \text{ m} = 2/3 \times \sqrt{3}a/2$

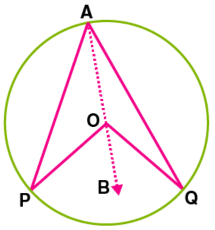
$a = 20\sqrt{3}$ m

So, the length of the string of the toy is $20\sqrt{3}$ m.

Circles and Quadrilaterals

The angle subtended by an arc of a circle on the circle and at the centre

The **angle** subtended by **an arc** at the **centre is double** the angle subtended by it on any **part of the circle**.



Here PQ is the arc of a circle with centre O, that subtends ∠POQ at the centre.

Join AO and extend it to B.

In ΔOAQ OA = OQ….. [Radii]

Hence, ∠OAQ = ∠OQA ……..[Property of isosceles triangle]

Implies ∠BOQ = 2∠OAQ …..[Exterior angle of triangle = Sum of 2 interior angles]

Similarly, ∠BOP = 2∠OAP

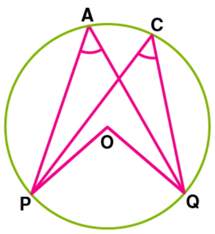
⇒∠BOQ + ∠BOP = 2∠OAQ + 2∠OAP

⇒∠POQ = 2∠PAQ

Hence proved.

Angles in the same segment of a circle.

–**Angles** in the **same segment** of a circle are **equal.**



Consider a circle with centre O.

∠PAQ and ∠PCQ are the angles formed in the major segment PACQ with respect to the arc PQ.

Join OP and OQ

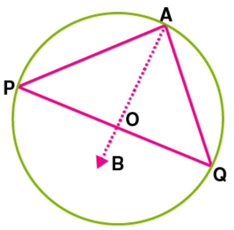
∠POQ = 2∠PAQ = 2∠PCQ ……….[Angle subtended by an arc at the centre is double the angle subtended by it in any part of the circle]

⇒∠PCQ = ∠PAQ

Hence proved

The angle subtended by diameter on the circle

– **Angle** subtended by **diameter** on a circle is a **right angle**. (Angle in a semicircle is a right angle)



Consider a circle with centre O, POQ is the diameter of the circle.

∠PAQ is the angle subtended by diameter PQ at the circumference.

∠POQ is the angle subtended by diameter PQ at the centre.

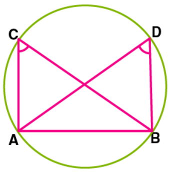
∠PAQ = (1/2)∠POQ…….[Angle subtended by arc at the centre is double the angle at any other part]

∠PAQ = (1/2) × 180^0 = 90^0

Hence proved

Line segment that subtends equal angles at two other points

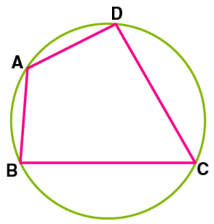
– If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e they are concyclic).



Here ∠ACB=∠ADB and all 4 points A, B, C, D are concyclic.

Cyclic Quadrilateral

– A **Quadrilateral** is called a **cyclic quadrilateral** if all the **four vertices lie on a circle**.



In a circle, if all **four points** A, B, C and D lie **on the circle**, then quadrilateral ABCD is a **cyclic quadrilateral.**

To know more about Cyclic Quadrilateral, visit here.

Sum of opposite angles of a cyclic quadrilateral

– If the sum of a pair of opposite angles of a quadrilateral is 180 degree, the quadrilateral is cyclic.

Sum of pair of opposite angles in a quadrilateral

– The sum of either pair of opposite angles of a cyclic quadrilateral is 180 degree.

Exercise: 10.5

1. In Fig. 10.36, A,B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.

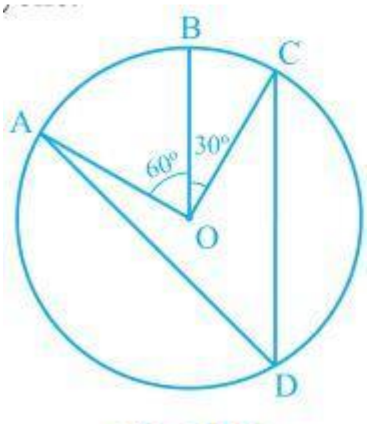


Fig. 10.36

Solution:

It is given that,

∠AOC = ∠AOB+∠BOC

So, ∠AOC = 60°+30°

∴ ∠AOC = 90°

It is known that an angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

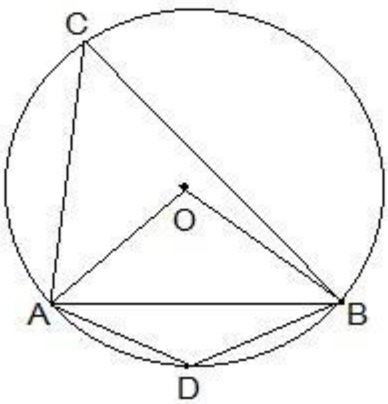
So,

∠ADC = (½)∠AOC

= (½)× 90° = 45°

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



Here, the chord AB is equal to the radius of the circle. In the above diagram, OA and OB are the two radii of the circle.

Now, consider the ΔOAB. Here,

AB = OA = OB = radius of the circle.

So, it can be said that ΔOAB has all equal sides and thus, it is an equilateral triangle.

∴ ∠AOC = 60°

And, ∠ACB = ½ ∠AOB

So, ∠ACB = ½ × 60° = 30°

Now, since ACBD is a cyclic quadrilateral,

∠ADB +∠ACB = 180° (Since they are the opposite angles of a cyclic quadrilateral)

So, ∠ADB = 180°-30° = 150°

So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc are 150° and 30° respectively.

3. In Fig. 10.37, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.

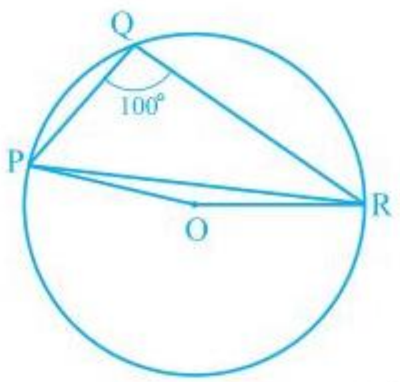


Fig. 10.37

Solution:

Since angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

So, the reflex ∠POR = 2×∠PQR

We know the values of angle PQR as 100°

So, ∠POR = 2×100° = 200°

∴ ∠POR = 360°-200° = 160°

Now, in ΔOPR,

OP and OR are the radii of the circle

So, OP = OR

Also, ∠OPR = ∠ORP

Now, we know sum of the angles in a triangle is equal to 180 degrees

So,

∠POR+∠OPR+∠ORP = 180°

∠OPR+∠OPR = 180°-160°

As ∠OPR = ∠ORP

2∠OPR = 20°

Thus, ∠OPR = 10°

4. In Fig. 10.38, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.

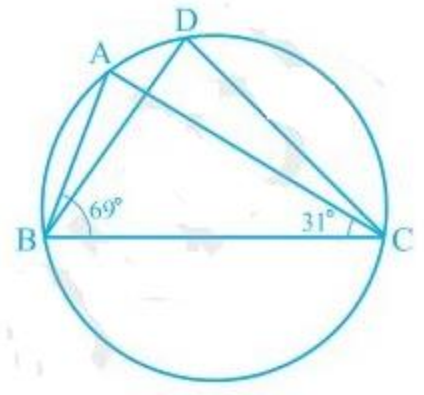


Fig. 10.38

Solution:

We know that angles in the segment of the circle are equal so,

∠BAC = ∠BDC

Now in the in ΔABC, the sum of all the interior angles will be 180°

So, ∠ABC+∠BAC+∠ACB = 180°

Now, by putting the values,

∠BAC = 180°-69°-31°

So, ∠BAC = 80°

∴ ∠BDC = 80°

5. In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find BAC.

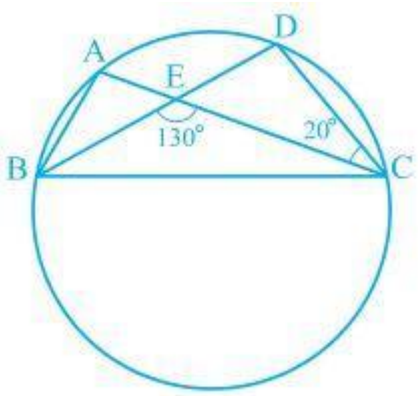


Fig. 10.39

Solution:

We know that the angles in the segment of the circle are equal.

So,

∠ BAC = ∠ CDE

Now, by using the exterior angles property of the triangle

In ΔCDE we get,

∠ CEB = ∠ CDE+∠ DCE

We know that ∠ DCE is equal to 20°

So, ∠ CDE = 110°

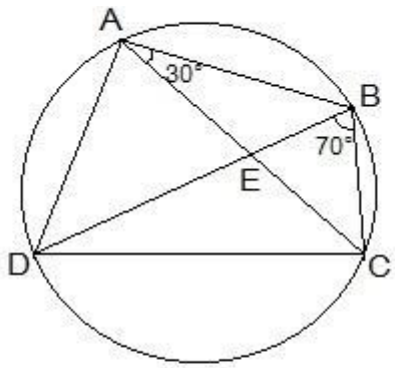
∠ BAC and ∠ CDE are equal

∴ ∠ BAC = 110°

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.

Solution:

Consider the following diagram.



Consider the chord CD,

We know that angles in the same segment are equal.

So, ∠ CBD = ∠ CAD

∴ ∠ CAD = 70°

Now, ∠ BAD will be equal to the sum of angles BAC and CAD.

So, ∠ BAD = ∠ BAC+∠ CAD

= 30°+70°

∴ ∠ BAD = 100°

We know that the opposite angles of a cyclic quadrilateral sums up to 180 degrees.

So,

∠ BCD+∠ BAD = 180°

It is known that ∠ BAD = 100°

So, ∠ BCD = 80°

Now consider the ΔABC.

Here, it is given that AB = BC

Also, ∠ BCA = ∠ CAB (They are the angles opposite to equal sides of a triangle)

∠ BCA = 30°

also, ∠ BCD = 80°

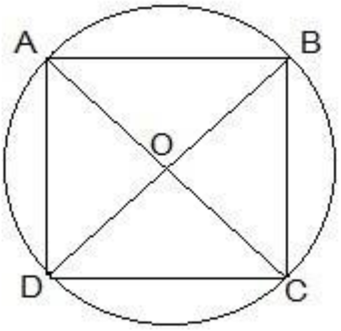
∠ BCA +∠ ACD = 80°

Thus, ∠ ACD = 50° and ∠ ECD = 50°

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Draw a cyclic quadrilateral ABCD inside a circle with center O such that its diagonal AC and BD are two diameters of the circle.



We know that the angles in the semi-circle are equal.

So, ∠ ABC = ∠ BCD = ∠ CDA = ∠ DAB = 90°

So, as each internal angle is 90°, it can be said that the quadrilateral ABCD is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:

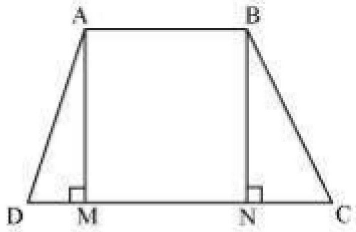
Construction:

Consider a trapezium ABCD with AB | |CD and BC = AD.

Draw AM ⊥ CD and BN ⊥ CD

In ΔAMD and ΔBNC,

The diagram will look as follows:



In ΔAMD and ΔBNC,

AD = BC (Given)

∠AMD = ∠BNC (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

△AMD ≅ △BNC (RHS congruence rule)

∠ADC = ∠BCD (CPCT) ... (1)

∠BAD and ∠ADC are on the same side of transversal AD.

$\angle BAD + \angle ADC = 180^0$ ... (2)

$\angle BAD + \angle BCD = 180^0$ [Using equation (1)]

This equation shows that the opposite angles are supplementary.

Therefore, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that ∠ ACP = ∠ QCD.

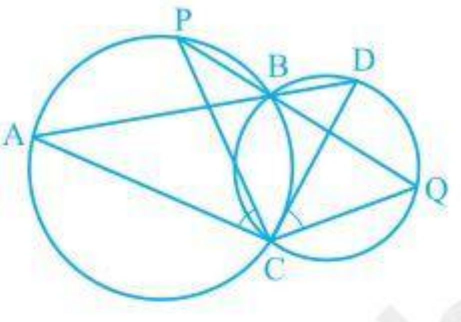


Fig. 10.40

Solution:

Construction:

Join the chords AP and DQ.

For chord AP, we know that angles in the same segment are equal.

So, ∠ PBA = ∠ ACP — (i)

Similarly for chord DQ,

∠ DBQ = ∠ QCD — (ii)

It is known that ABD and PBQ are two line segments which are intersecting at B.

At B, the vertically opposite angles will be equal.

∴ ∠ PBA = ∠ DBQ — (iii)

From equation (i), equation (ii) and equation (iii) we get,

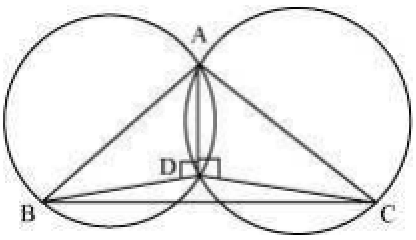
∠ ACP = ∠ QCD

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution:

First draw a triangle ABC and then two circles having diameter as AB and AC respectively.

We will have to now prove that D lies on BC and BDC is a straight line.



Proof:

We know that angle in the semi-circle are equal

So, ∠ ADB = ∠ ADC = 90°

Hence, ∠ ADB+∠ ADC = 180°

∴ ∠ BDC is straight line.

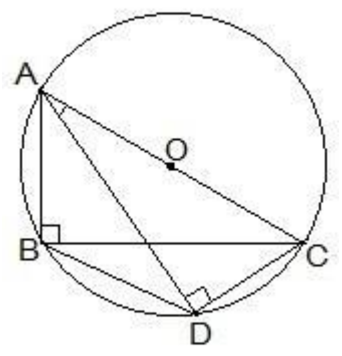
So, it can be said that D lies on the line BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠ CAD = ∠CBD.

Solution:

We know that AC is the common hypotenuse and ∠ B = ∠ D = 90°.

Now, it has to be proven that ∠ CAD = ∠ CBD



Since, ∠ ABC and ∠ ADC are 90°, it can be said that They lie in the semi-circle.

So, triangles ABC and ADC are in the semi-circle and the points A, B, C and D are concyclic.

Hence, CD is the chord of the circle with center O.

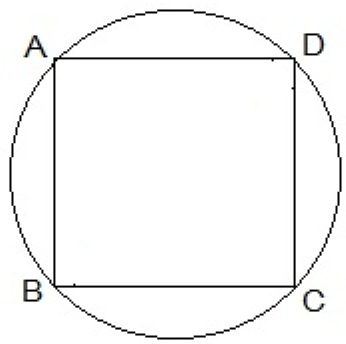
We know that the angles which are in the same segment of the circle are equal.

∴ ∠ CAD = ∠ CBD

12. Prove that a cyclic parallelogram is a rectangle.

Solution:

It is given that ABCD is a cyclic parallelogram and we will have to prove that ABCD is a rectangle.



Proof:

Let ABCD be a cyclic parallelogram.

$\angle A + \angle C = 180^o$ (Opposite angle of cyclic quadrilateral)   ... (1)

We know that opposite angles of a parallelogram are equal

$\angle A = \angle C$ and $\angle B = \angle D$

From equation (1)

$\angle A + \angle C = 180^o$

$\angle A + \angle A = 180^o$

$2 \angle A = 180^o$

$\angle A = 90^o$

Parallelogram ABCD has one of its interior angles as 90°.

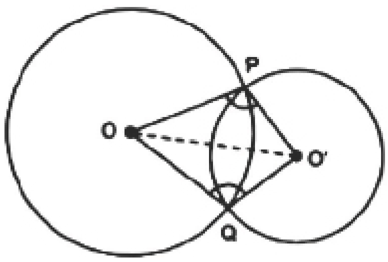
Thus, ABCD is a rectangle.

Exercise: 10.6 (Page No: 186)

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution:

Consider the following diagram



In ΔPOO' and ΔQOO'

OP = OQ (Radius of circle 1)

O'P = O'Q (Radius of circle 2)

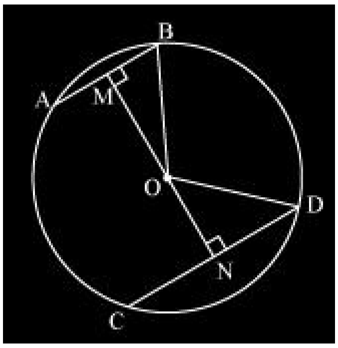
OO' = OO' (Common arm)

So, by SSS congruency, ΔPOO' ≅ ΔQOO'

Thus, ∠OPO' = ∠OQO' (proved).

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 , find the radius of the circle.

Solution:



Here, OM ⊥ AB and ON ⊥ CD. is drawn and OB and OD are joined.

We know that AB bisects BM as the perpendicular from the centre bisects chord.

Since AB = 5 so,

BM = AB/2 = 5/2

Similarly, ND = CD/2 = 11/2

Now, let ON be x.

So, OM = 6−x.

Consider ΔMOB,

$OB^2 = OM^2 + MB^2$

Or,

$$OB^2 = 36 + x^2 - 12x + \frac{25}{4} \qquad \dots (1)$$

Consider ΔNOD,

$OD^2 = ON^2 + ND^2$

Or

$$OD^2 = x^2 + \frac{121}{4} \qquad \dots (2)$$

We know, OB = OD (radii)

From equation 1 and equation 2 we get

$$36+x^2-12x+\frac{25}{4}=x^2+\frac{121}{4}$$

$$12x=36+\frac{25}{4}-\frac{121}{4}$$

$$=\frac{144+25-121}{4}$$

$$12x=\frac{48}{4}=12$$

$$x=1$$

Now, from equation (2) we have,

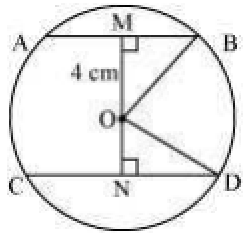
$OD^2= 1^2 +(121/4)$

Or OD = (5/2)×√5 cm

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution:

Consider the following diagram



Here AB and CD are 2 parallel chords. Now, join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

So, OM = 4 cm

MB = AB/2 = 3 cm

Consider ΔOMB

$OB^2 = OM^2+MB^2$

Or, OB = 5cm

Now, consider ΔOND,

OB = OD = 5 (since they are the radii)

ND = CD/2 = 4 cm

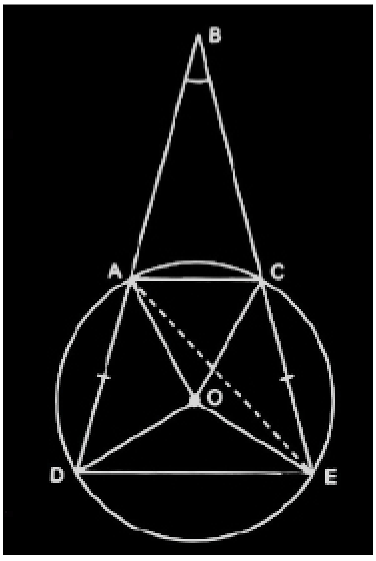
Now, $OD^2= ON^2+ND^2$

Or, ON = 3 cm.

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution:

Consider the diagram



Here AD = CE

We know, any exterior angle of a triangle is equal to the sum of interior opposite angles.

So,

∠DAE = ∠ABC+∠AEC (in ΔBAE) ——————-(i)

DE subtends ∠DOE at the centre and ∠DAE in the remaining part of the circle.

So,

∠DAE = (½)∠DOE ——————-(ii)

Similarly, ∠AEC = (½)∠AOC ——————-(iii)

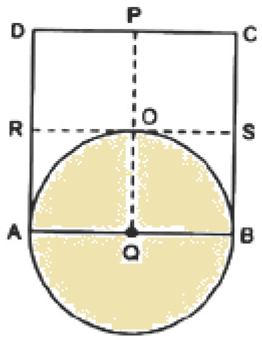
Now, from equation (i), (ii), and (iii) we get,

(½)∠DOE = ∠ABC+(½)∠AOC

Or, ∠ABC = (½)[∠DOE-∠AOC] (hence proved).

5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Solution:



To prove: A circle drawn with Q as centre, will pass through A, B and O (i.e. QA = QB = QO)

Since all sides of a rhombus are equal,

AB = DC

Now, multiply (½) on both sides

(½)AB = (½)DC

So, AQ = DP

BQ = DP

Since Q is the midpoint of AB,

AQ= BQ

Similarly,

RA = SB

Again, as PQ is drawn parallel to AD,

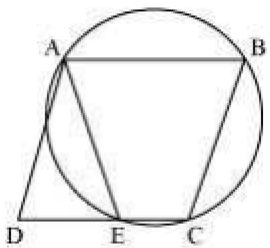
RA = QO

Now, as AQ = BQ and RA = QO we get,

QA = QB = QO (hence proved).

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE, = AD.

Solution:



Here, ABCE is a cyclic quadrilateral. In a cyclic quadrilateral, the sum of the opposite angles is 180°.

So, ∠AEC+∠CBA = 180°

As ∠AEC and ∠AED are linear pair,

∠AEC+∠AED = 180°

Or, ∠AED = ∠CBA … (1)

We know in a parallelogram; opposite angles are equal.

So, ∠ADE = ∠CBA … (2)

Now, from equations (1) and (2) we get,

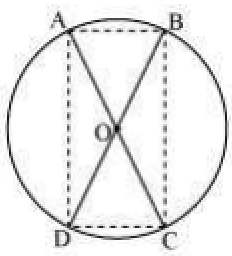
∠AED = ∠ADE

Now, AD and AE are angles opposite to equal sides of a triangle,

∴ AD = AE (proved).

7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Solution:



Here chords AB and CD intersect each other at O.

Consider ΔAOB and ΔCOD,

∠AOB = ∠COD (They are vertically opposite angles)

OB = OD (Given in the question)

OA = OC (Given in the question)

So, by SAS congruency, ΔAOB ≅ ΔCOD

Also, AB = CD (By CPCT)

Similarly, ΔAOD ≅ ΔCOB

Or, AD = CB (By CPCT)

In quadrilateral ACBD, opposite sides are equal.

So, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

So, ∠A = ∠C

Also, as ABCD is a cyclic quadrilateral,

∠A+∠C = 180°

⇒∠A+∠A = 180°

Or, ∠A = 90°

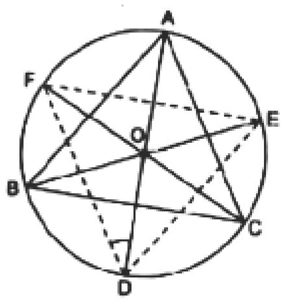
As ACBD is a parallelogram and one of its interior angles is 90°, so, it is a rectangle.

∠A is the angle subtended by chord BD. And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90°–(½)A, 90°–(½)B and 90°–(½)C.

Solution:

Consider the following diagram



Here, ABC is inscribed in a circle with center O and the bisectors of ∠A, ∠B and ∠C intersect the circumcircle at D, E and F respectively.

Now, join DE, EF and FD

As angles in the same segment are equal, so,

∠EDA = ∠FCA —————(i)

∠FDA = ∠EBA —————(i)

By adding equations (i) and (ii) we get,

∠FDA+∠EDA = ∠FCA+∠EBA

Or, ∠FDE = ∠FCA+∠EBA = (½)∠C+(½)∠B

We know, ∠A +∠B+∠C = 180°

So, ∠FDE = (½)[∠C+∠B] = (½)[180°-∠A]

∠FDE = [90-(∠A/2)]

In a similar way,

∠FED = [90° -(∠B/2)] °

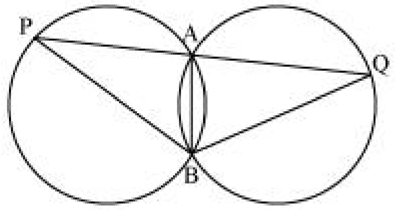
And,

∠EFD = [90° -(∠C/2)] °

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution:

The diagram will be



Here, ∠APB = ∠AQB (as AB is the common chord in both the congruent circles.)

Now, consider ΔBPQ,

∠APB = ∠AQB

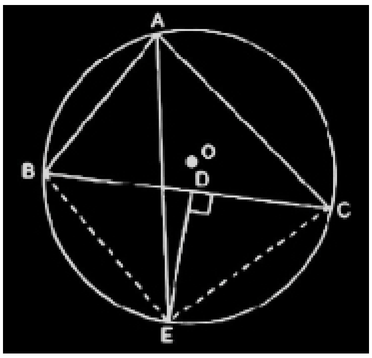
So, the angles opposite to equal sides of a triangle.

∴ BQ = BP

10. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Solution:

Consider this diagram



Here, join BE and CE.

Now, since AE is the bisector of ∠BAC,

∠BAE = ∠CAE

Also,

∴arc BE = arc EC

This implies, chord BE = chord EC

Now, consider triangles ΔBDE and ΔCDE,

DE = DE (It is the common side)

BD = CD (It is given in the question)

BE = CE (Already proved)

So, by SSS congruency, ΔBDE ≅ ΔCDE.

Thus, ∴∠BDE = ∠CDE

We know, $\angle BDE = \angle CDE = 180°$

Or, $\angle BDE = \angle CDE = 90°$

$\therefore DE \perp BC$ (hence proved).

CHAPTER-6

LINES AND ANGLES

Introduction

Point: It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has sposition i.e., it has no magnitude. It is denoted by capital letters A, B, C, O etc.

Line Segment: The straight path joining two points A and B is called a line segment AB. It has end points and a definite length. (No breadth or thickness)

Ray: A line segment which can be extended in only one direction is called a ray.

Line: When a line segment is extended indefinitely in both directions it forms a line.

Collinear Point: If two or more points lie on the same line, then they are called collinear points.

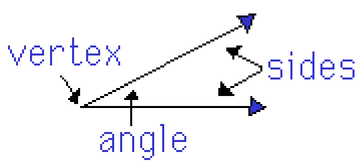
Non-collinear Points: Points which do not lie on the same line are called non-collinear points.

Intersecting Lines: Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.

Concurrent Lines: If two or more lines intersect at the same point, then they are known as concurrent lines.

Angles: An angle is formed by two rays with a common endpoint.

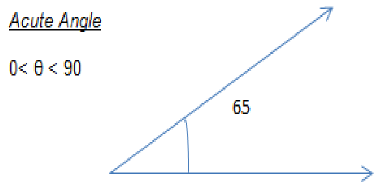
The Common end point is known as the vertex of the angle and the rays as the sides, sometimes as the legs and sometimes the arms of the angle.



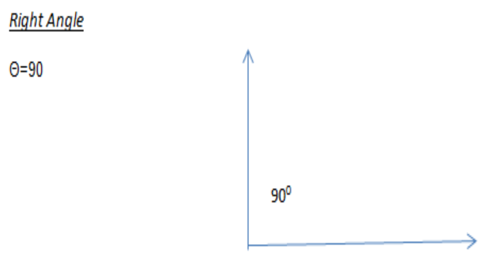
We denote the angle by symbol ∠.

Types of Angle

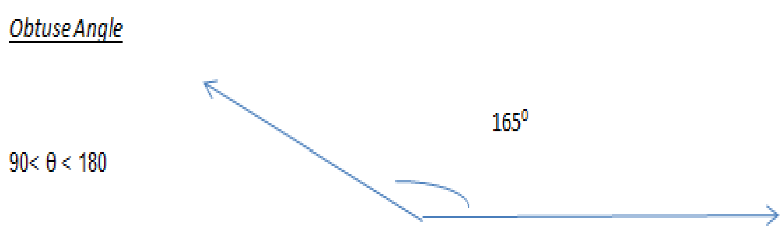
Acute Angle: An angle whose measure is less than one right angle (i.e., less than 90), is called an acute angle.



Right Angle: An angle whose measure is 90 is called a right angle.

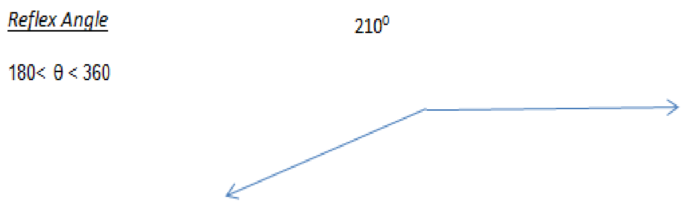


Obtuse Angle: An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180o and more than 90) is called an obtuse angle.



Straight Angle: An angle whose measure is 180o is called a straight angle.

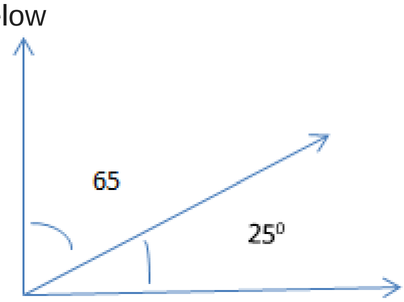
Reflex Angle: An angle whose measure is more than 180o and less than 360o is called a reflex angle.



Complete Angle: An angle whose measure is 360o is called a complete angle.

Adjacent Angles

- Two angles are called adjacent angles if they share the same vertex,they have a common arm. The second arm of the one angle is one side and second arm of other angle is on the other side
- Example given below



Complimentary Angles

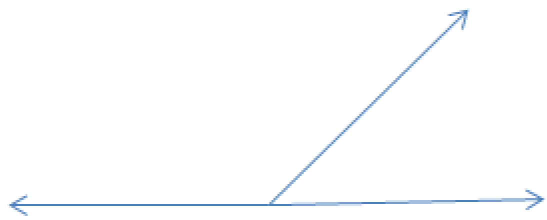
- Two angles, the sum of whose measures is 90 is called Complimentary Angles. Each of these Complimentary Angles are called the complement of each other.

Supplementary Angles

- Two angles , the sum of whose measures is 180 is called Supplementary angles. Each of these Supplementary Angles are called the supplement of each other.

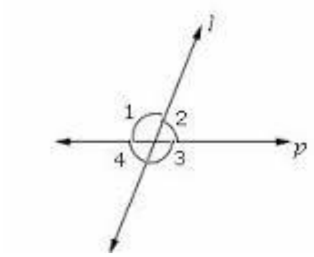
Linear Pair Axioms

- If a ray stands on a line, then the sum of the adjacent angles so formed is $180_0$1800
- And If the sum of the adjacent angles is $180_0$1800,then the non common arms of the angles form a line



Theorem Based on Linear Pair Axiom

The sum of all the angles around a point is 360



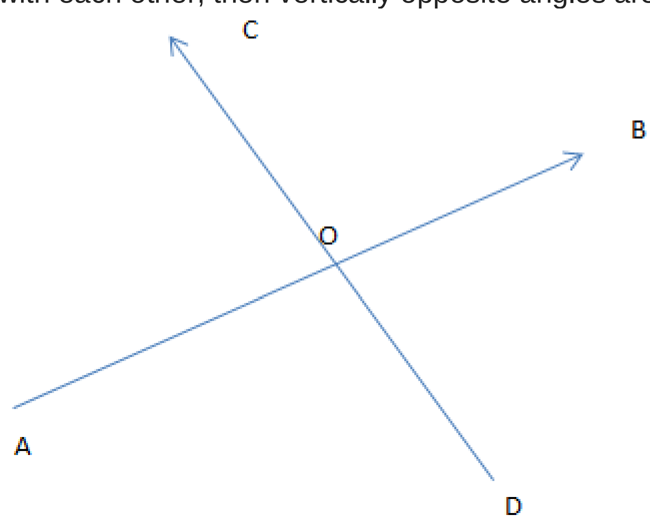
$\angle 1+\angle 2+\angle 3+\angle 4=360$

Vertically Opposite angles

If two lines intersect with each other, then vertically opposite angles are equal

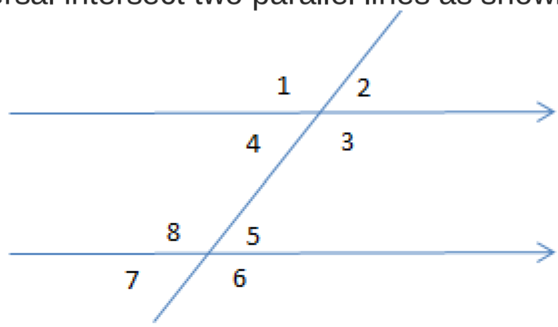
$\angle AOD = \angle COB$

$\angle AOC = \angle BOD$



Transversal across the parallel Lines

If the transversal intersect two parallel lines as shown in below figure



Important Take away from the figure

1. We can see following angles as depicted in the figure above
   $\angle 1, \angle 2, \angle 3, \angle 4$ on the first parallel line
   and $\angle 5, \angle 6, \angle 7, \angle 8$ on the second parallel line.

2. The angles 1,2,6,7 are called exterior angles while the angles 4,3,5,8 are called interior angles
   exterior angles = $\angle 1, \angle 2, \angle 6, \angle 7$
   interior angles = $\angle 4, \angle 3, \angle 5, \angle 8$

3. <u>Corresponding Angles</u> The angles on the same side of the Transversal are known as Corresponding angles
   And Corresponding Angles axiom states that
   $\angle 1 = \angle 8 \quad \angle 2 = \angle 5 \quad \angle 4 = \angle 7 \quad \angle 3 = \angle 6$

4. Each pair of alternate interior angles are equal
   $\angle 3 = \angle 8 \quad \angle 4 = \angle 5$

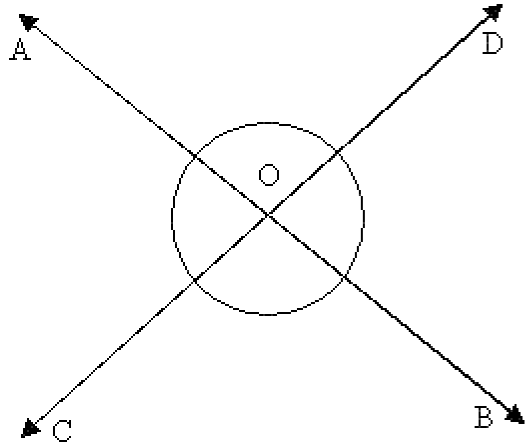
5. Each pair of interior angles on the same side of the transversal is supplementary
   $\angle 4 + \angle 8 = 180 \quad \angle 3 + \angle 5 = 180$

Axiom 1 If a ray stands on a line, then the sum of two adjacent angles so formed is 180º.

Conversely if the sum of two adjacent angles is 180º, then a ray stands on a line (i.e., the non-common arms form a line).

Axiom 2 If the sum of two adjacent angles is 180º, then the non-common arms of the angles form a line. It is called Linear Pair Axiom.

Theorem 1: If two lines intersect each other, then the vertically opposite angles are equal.
Solution: Given: Two lines AB and CD intersect each other at O.



To Prove:
Ray OA stands on line CD.
Hence, ∠AOC + ∠AOD = 180° ....equation (i) [Linear pair axiom]
Again ray OD stands on line AB
Hence, ∠AOD + ∠BOD = 180° ....equation (ii)
From equation (i) and (ii)
∠AOC + ∠AOD = ∠AOD + ∠BOD
Or, ∠AOC + ∠AOD - ∠AOD = ∠BOD
Or, ∠AOC = ∠BOD
Now, again;
Ray OB stands on line CD
So, ∠BOC + ∠BOD = 180° ....equation (iii) (Linear pair axiom)
Again ray OD stands on line AB
So, ∠AOD + ∠BOD = 180° ....equation (iv)
From equation (iii) and (iv);
∠BOC + ∠BOD = ∠AOD + ∠BOD
Or, ∠BOC + ∠BOD - ∠BOD = ∠AOD
Or, ∠BOC = ∠AOD Proved

Axiom 3: If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

Axiom 4 If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines is parallel to each other.

Converse of Transversal across the parallel Lines

If a transversal intersect two lines such that either

1. Any one pair of corresponding angles are equal
2. any one pair of alternate interior angles are equal
3. any one pair of interior angles on the same side of the transversal is supplementary

Then the two lines are parallel

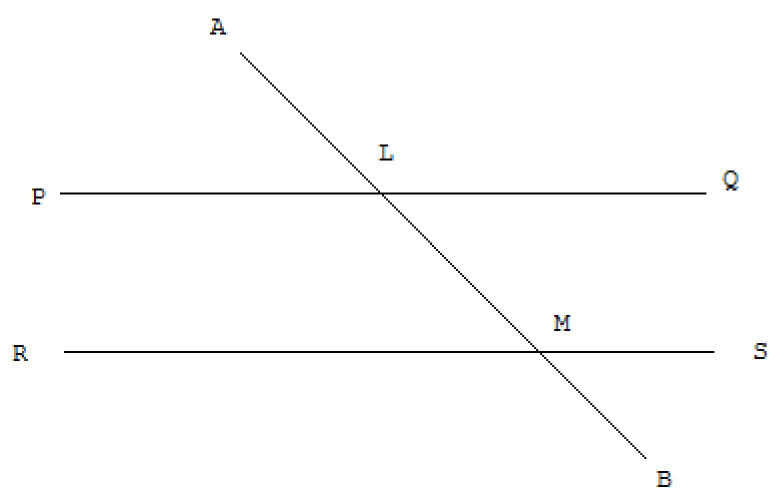
Parallel lines Note

Lines which are parallel to a given line are parallel with each other.

Theorem 2 If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Solution: Given: Let PQ and RS are two parallel lines and AB be the transversal which intersects them on L and M respectively.

To Prove: ∠PLM = ∠SML
And ∠LMR = ∠MLQ



Proof: ∠PLM = ∠RMB ………….equation (i) (Corresponding angles)
∠RMB = ∠SML ………….equation (ii) (vertically opposite angles)

From equation (i) and (ii)
∠PLM = ∠SML

Similarly, ∠LMR = ∠ALP ....equation (iii) (corresponding angles)
∠ALP = ∠MLQ ....equation (iv) (vertically opposite angles)
From equation (iii) and (iv)
∠LMR = ∠MLQ Proved

Theorem 3: If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

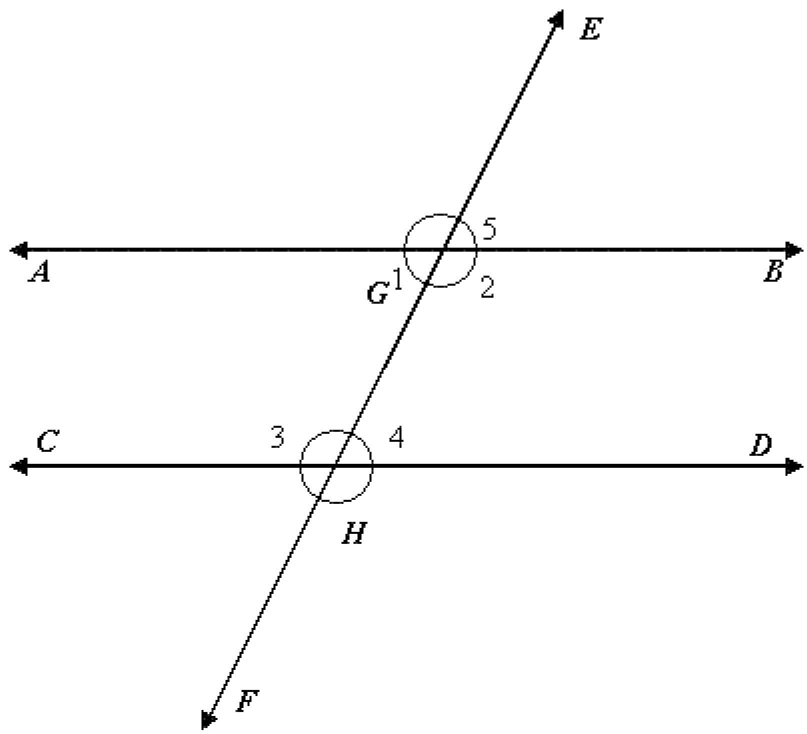
Solution: Given: - A transversal AB intersects two lines PQ and RS such that
∠PLM = ∠SML

To Prove: PQ ||RS
Use same figure as in Theorem 2.
Proof: ∠PLM = ∠SML ....equation (i) (Given)
∠SML = ∠RMB ....equation (ii) (vertically opposite angles)
From equations (i) and (ii);
∠PLM = ∠RMB

But these are corresponding angles.
We know that if a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines ate parallel to each other.
Hence, PQ∥RS Proved.

Theorem 4: If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Solution:

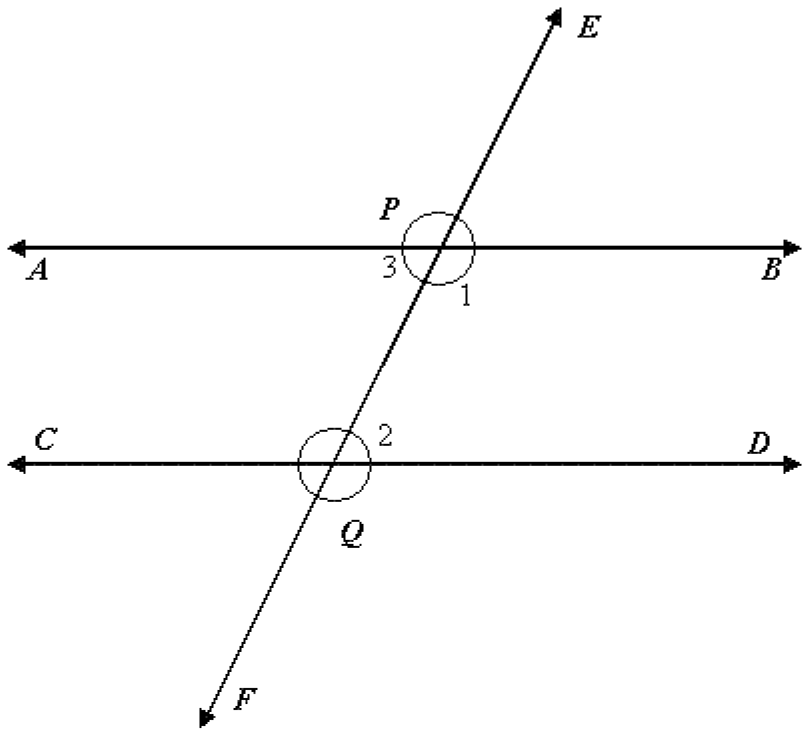


Given: Transversal EF intersects two parallel lines AB and CD at G and H respectively.
To Prove: ∠1 + ∠4 = 180° and ∠2 + ∠3 = 180°

Proof: ∠2 + ∠5 = 180° ....equation (i) (Linear pair of angles)
But ∠5 = ∠3 ....equation (ii) (corresponding angles)
From equations (i) and (ii),
∠2 + ∠3 = 180°
Also, ∠3 + ∠4 = 180° ....equation (iii) (Linear pair)
But ∠3 = ∠1 ....equation (iv) (Alternate interior angles)
From equations (iii) and (iv)
∠1 + ∠4 = 180° and ∠2 + ∠3 = 180° Proved

Theorem 5: If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

Solution:



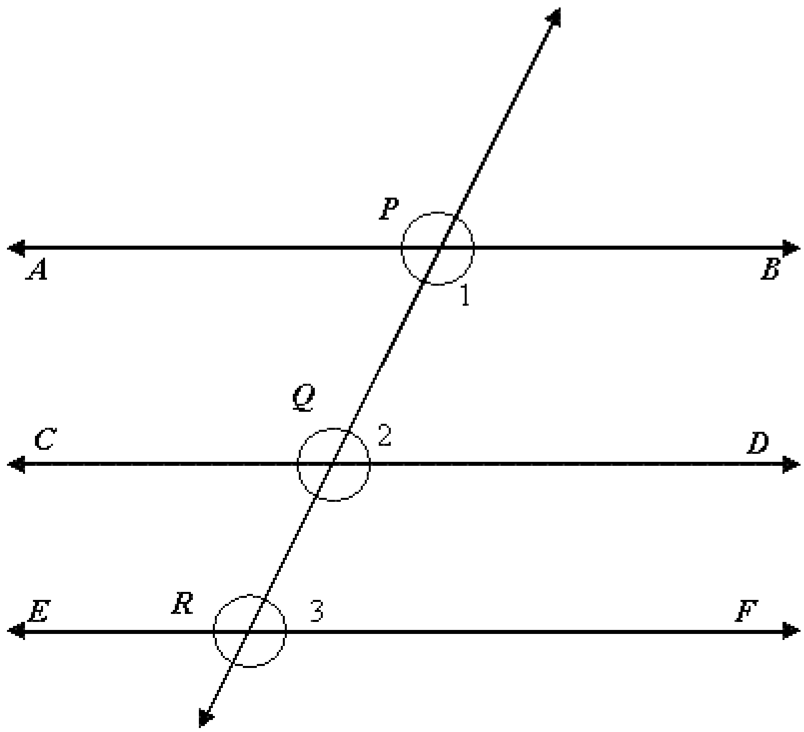
Given: A transversal EF intersects two lines AB and CD at P and Q respectively.
To Prove: AB ||CD

Proof: ∠1 + ∠2 = 180° ....equation (i) (Given)
∠1 + ∠3 = 180° ....equation (ii) (Linear Pair)
From equations (i) and (ii)
∠1 + ∠2 = ∠1 + ∠3
Or, ∠1 + ∠2 - ∠1 = ∠3
Or, ∠2 = ∠3

But these are alternate interior angles. We know that if a transversal intersects two lines such that the pair of alternate interior angles are equal, then the lines are parallel.
Hence, AB║CD Proved.

Theorem 6: Lines which are parallel to the same line are parallel to each other.

Solution:



Given: Three lines AB, CD and EF are such that AB║CD, CD║EF.
To Prove: AB║EF.
Construction: Let us draw a transversal GH which intersects the lines AB, CD and EF at P, Q and R respectively.
Proof: Since, AB║CD and GH is the transversal. Therefore,

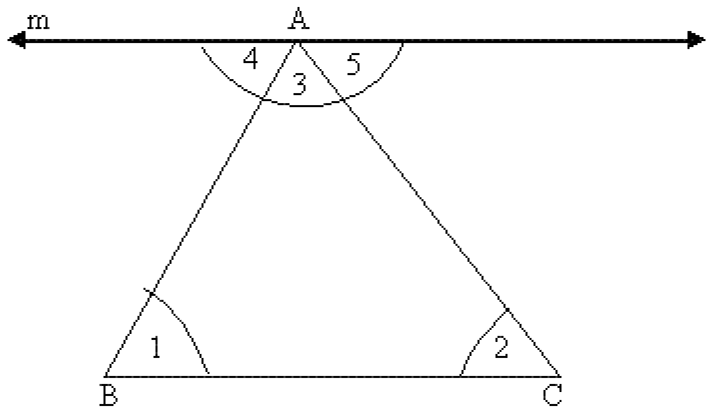
∠1 = ∠2 ....equation (i) (corresponding angles)
Similarly, CD ||EF and GH is transversal. Therefore;
∠2 = ∠3 ....equation (ii) (corresponding angles)
From equations (i) and (ii)
∠1 = ∠3

But these are corresponding angles.
We know that if a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines ate parallel to each other.
Hence, AB║ EF Proved.

Angle Sum Property of Triangle:

Theorem 7: The sum of the angles of a triangle is 180º.

Solution:

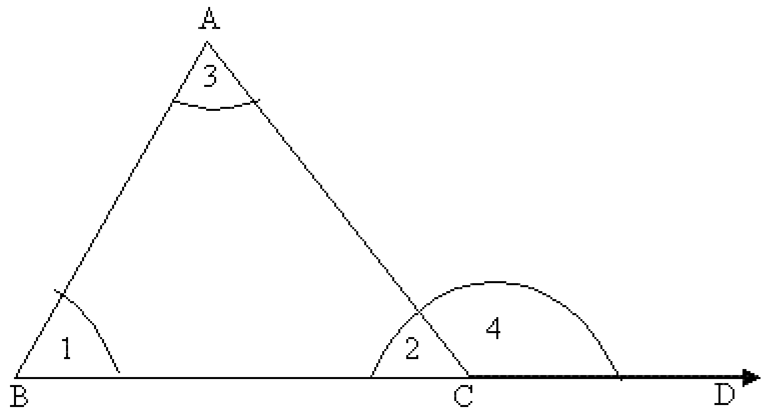


Given: Δ ABC.
To Prove: ∠1 + ∠2 + ∠3 = 180°
Construction: Let us draw a line m though A, parallel to BC.

Proof: BC ||m and AB and AC are its transversal.
Hence, ∠1 = ∠4 ....equation (i) (alternate interior angles)
∠2 = ∠5 ....equation (ii) (alternate interior angles)
By adding equation (i) and (ii)
∠1 + ∠2 = ∠4 + ∠5 ....equation (iii)
Now, by adding ∠3 to both sides of equation (iii), we get
∠1 + ∠2 + ∠3 = ∠4 + ∠5 + ∠3
Since, ∠4 + ∠5 + ∠ = 180° (Linear group of angle)
Hence, ∠1 + ∠2 + ∠3 = 180°
Hence Proved.

Theorem 8: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Solution:



Given: ΔABDC in which side BC is produced to D forming exterior angle ∠ACD of ΔABC.
To Prove: ∠4 = ∠1 + ∠2

Proof: Since, ∠1 + ∠2 + ∠3 = 180°............equation (i) (angle sum of triangle)
∠2 + ∠4 = 180° ....equation (ii) (Linear pair)
From equations (i) and (ii)
∠1 + ∠2 + ∠3 = ∠3 + ∠4
Or, ∠1 + ∠2 + ∠3 - ∠3 = ∠4
Or, ∠1 + ∠2 = ∠4
Hence, ∠4 = ∠1 + ∠2 Proved

Exercise: 6.1 (Page No: 96)

1. In Fig. 6.13, lines AB and CD intersect at O. If AOC +BOE = 70° and BOD = 40°, find BOE and reflex COE.

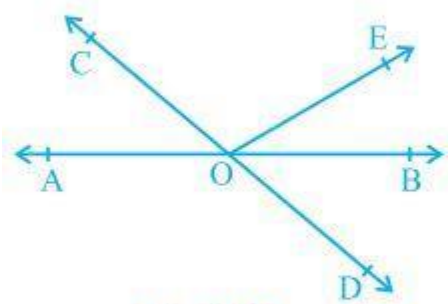


Fig. 6.13

Solution:

From the diagram, we have

(∠AOC +∠BOE +∠COE) and (∠COE +∠BOD +∠BOE) forms a straight line.

So, ∠AOC+∠BOE +∠COE = ∠COE +∠BOD+∠BOE = 180°

Now, by putting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get

∠COE = 110° and ∠BOE = 30°

So, reflex ∠COE = $360^{\circ} – 110^{\circ} = 250^{\circ}$

2. In Fig. 6.14, lines XY and MN intersect at O. If POY = 90° and a : b = 2 : 3, find c.

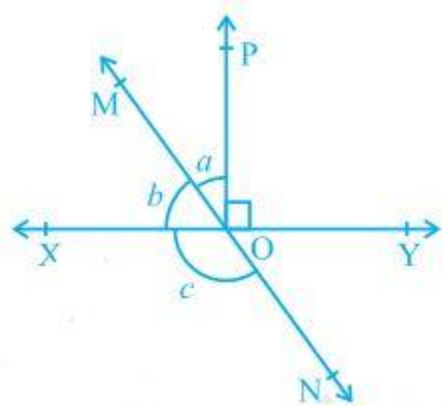


Fig. 6.14

Solution:

We know that the sum of linear pair are always equal to 180°

So,

POY +a +b = 180°

Putting the value of POY = 90° (as given in the question) we get,

a+b = 90°

Now, it is given that a : b = 2 : 3 so,

Let a be 2x and b be 3x

∴ 2x+3x = 90°

Solving this we get

5x = 90°

So, x = 18°

∴ a = 2×18° = 36°

Similarly, b can be calculated and the value will be

b = 3×18° = 54°

From the diagram, b+c also forms a straight angle so,

b+c = 180°

c+54° = 180°

∴ c = 126°

3. In Fig. 6.15, PQR = PRQ, then prove that PQS = PRT.

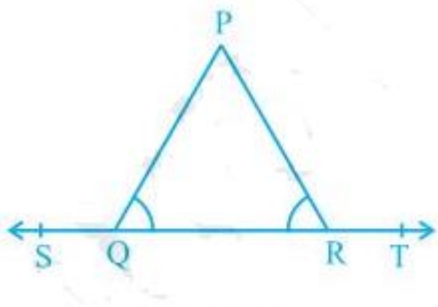


Fig. 6.15

Solution:

Since ST is a straight line so,

∠PQS+∠PQR = 180° (linear pair) and

∠PRT+∠PRQ = 180° (linear pair)

Now, ∠PQS + ∠PQR = ∠PRT+∠PRQ = 180°

Since ∠PQR =∠PRQ (as given in the question)

∠PQS = ∠PRT. (Hence proved).

4. In Fig. 6.16, if x+y = w+z, then prove that AOB is a line.

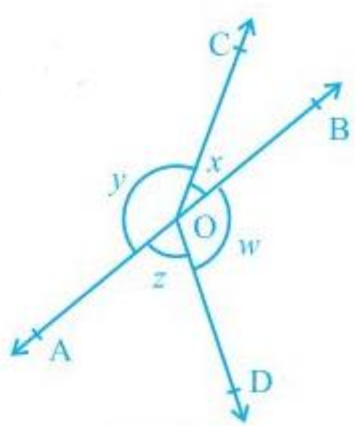


Fig. 6.16

Solution:

For proving AOB is a straight line, we will have to prove x+y is a linear pair

i.e. x+y = 180°

We know that the angles around a point are 360° so,

x+y+w+z = 360°

In the question, it is given that,

x+y = w+z

So, (x+y)+(x+y) = 360°

2(x+y) = 360°

∴ (x+y) = 180° (Hence proved).

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ROS = ½ (QOS – POS).

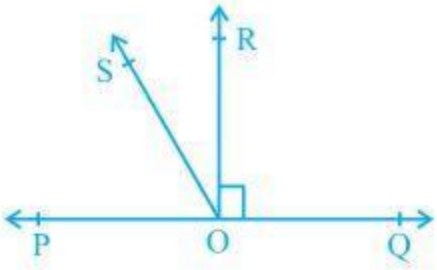


Fig. 6.17

Solution:

In the question, it is given that (OR ⊥ PQ) and POQ = 180°

So, POS+ROS+ROQ = 180°

Now, POS+ROS = 180°- 90° (Since POR = ROQ = 90°)

∴ POS + ROS = 90°

Now, QOS = ROQ+ROS

It is given that ROQ = 90°,

∴ QOS = 90° +ROS

Or, QOS – ROS = 90°

As POS + ROS = 90° and QOS – ROS = 90°, we get

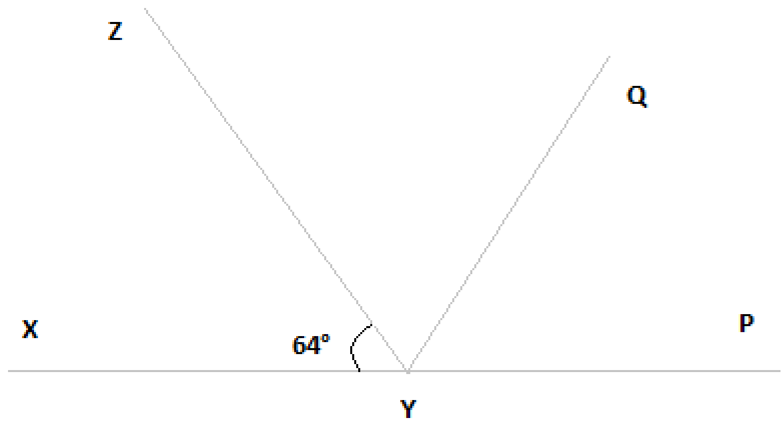
POS + ROS = QOS – ROS

2 ROS + POS = QOS

Or, ROS = ½ (QOS – POS) (Hence proved).

6. It is given that XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ZYP, find XYQ and reflex QYP.

Solution:



Here, XP is a straight line

So, XYZ +ZYP = 180°

Putting the value of XYZ = 64° we get,

64° +ZYP = 180°

∴ ZYP = 116°

From the diagram, we also know that ZYP = ZYQ + QYP

Now, as YQ bisects ZYP,

ZYQ = QYP

Or, ZYP = 2ZYQ

∴ ZYQ = QYP = 58°

Again, XYQ = XYZ + ZYQ

By putting the value of XYZ = 64° and ZYQ = 58° we get.

XYQ = 64°+58°

Or, XYQ = 122°

Now, reflex QYP = 180°+XYQ

We computed that the value of XYQ = 122°.

So,

QYP = 180°+122°

∴ QYP = 302°

Exercise: 6.2 (Page No: 103)

1. In Fig. 6.28, find the values of x and y and then show that AB CD.

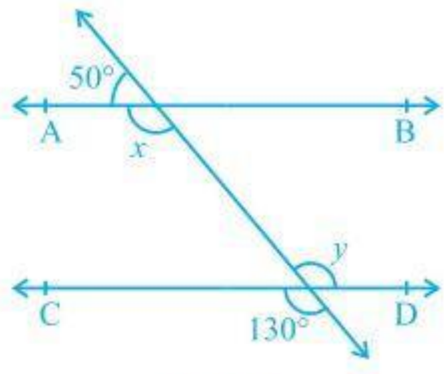


Fig. 6.28

Solution:

We know that a linear pair is equal to 180°.

So, x+50° = 180°

∴ x = 130°

We also know that vertically opposite angles are equal.

So, y = 130°

In two parallel lines, the alternate interior angles are equal. In this,

x = y = 130°

This proves that alternate interior angles are equal and so, AB CD.

2. In Fig. 6.29, if AB CD, CD EF and y : z = 3 : 7, find x.

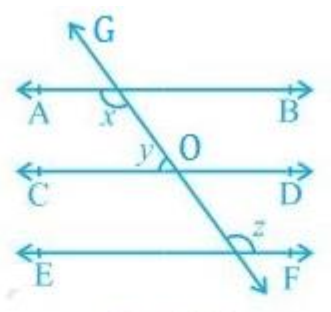


Fig. 6.29

Solution:

It is known that AB CD and CDEF

As the angles on the same side of a transversal line sums up to 180°,

x + y = 180° ——(i)

Also,

O = z (Since they are corresponding angles)

and, y +O = 180° (Since they are a linear pair)

So, y+z = 180°

Now, let y = 3w and hence, z = 7w (As y : z = 3 : 7)

∴ 3w+7w = 180°

Or, 10 w = 180°

So, w = 18°

Now, y = 3×18° = 54°

and, z = 7×18° = 126°

Now, angle x can be calculated from equation (i)

x+y = 180°

Or, x+54° = 180°

∴ x = 126°

3. In Fig. 6.30, if AB CD, EF ⊥ CD and GED = 126°, find AGE, GEF and FGE.

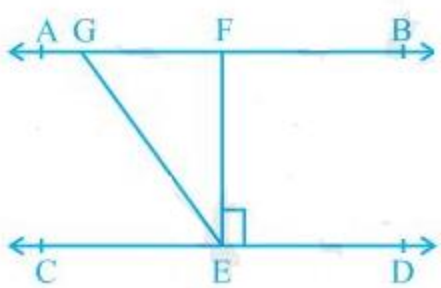


Fig. 6.30

Solution:

Since AB CD, GE is a transversal.

It is given that GED = 126°

So, GED = AGE = 126° (As they are alternate interior angles)

Also,

GED = GEF +FED

As EF⊥ CD, FED = 90°

∴ GED = GEF+90°

Or, GEF = 126° – 90° = 36°

Again, FGE +GED = 180° (Transversal)

Putting the value of GED = 126° we get,

FGE = 54°

So,

AGE = 126°

GEF = 36° and

FGE = 54°

4. In Fig. 6.31, if PQ ST, PQR = 110° and RST = 130°, find QRS.

[Hint : Draw a line parallel to ST through point R.]

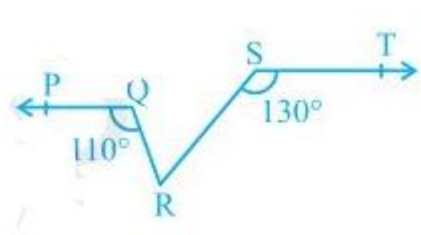
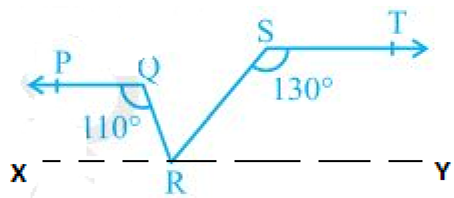


Fig. 6.31

Solution:

First, construct a line XY parallel to PQ.



We know that the angles on the same side of transversal is equal to 180°.

So, PQR+QRX = 180°

Or,QRX = 180°-110°

∴ QRX = 70°

Similarly,

RST +SRY = 180°

Or, SRY = 180°- 130°

∴ SRY = 50°

Now, for the linear pairs on the line XY-

QRX+QRS+SRY = 180°

Putting their respective values, we get,

QRS = 180° – 70° – 50°

Hence, QRS = 60°

5. In Fig. 6.32, if AB CD, APQ = 50° and PRD = 127°, find x and y.

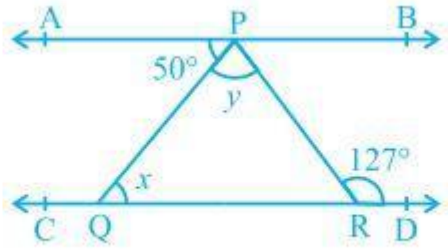


Fig. 6.32

Solution:

From the diagram,

APQ = PQR (Alternate interior angles)

Now, putting the value of APQ = 50° and PQR = x we get,

x = 50°

Also,

APR = PRD (Alternate interior angles)

Or, APR = 127° (As it is given that PRD = 127°)

We know that

APR = APQ+QPR

Now, putting values of QPR = y and APR = 127° we get,

127° = 50°+ y

Or, y = 77°

Thus, the values of x and y are calculated as:

x = 50° and y = 77°

6. In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB CD.

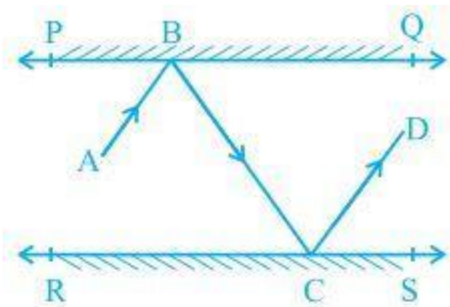


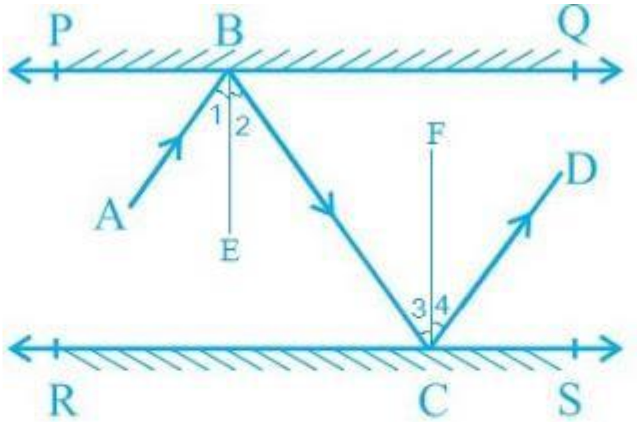
Fig. 6.33

Solution:

First, draw two lines BE and CF such that BE ⊥ PQ and CF ⊥ RS.

Now, since PQ RS,

So, BE CF



We know that,

Angle of incidence = Angle of reflection (By the law of reflection)

So,

1 = 2 and

3 = 4

We also know that alternate interior angles are equal. Here, BE ⊥ CF and the transversal line BC cuts them at B and C

So, 2 = 3 (As they are alternate interior angles)

Now, 1 +2 = 3 +4

Or, ABC = DCB

So, AB CD alternate interior angles are equal)

Exercise: 6.3 (Page No: 107)

1. In Fig. 6.39, sides QP and RQ of ΔPQR are produced to points S and T respectively. If SPR = 135° and PQT = 110°, find PRQ.

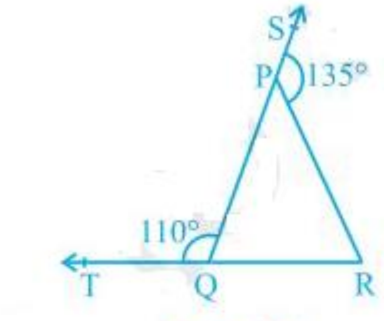


Fig. 6.39

Solution:

It is given the TQR is a straight line and so, the linear pairs (i.e. TQP and PQR) will add up to 180°

So, TQP +PQR = 180°

Now, putting the value of TQP = 110° we get,

PQR = 70°

Consider the ΔPQR,

Here, the side QP is extended to S and so, SPR forms the exterior angle.

Thus, SPR (SPR = 135°) is equal to the sum of interior opposite angles. (Triangle property)

Or, PQR +PRQ = 135°

Now, putting the value of PQR = 70° we get,

PRQ = 135°-70°

Hence, PRQ = 65°

2. In Fig. 6.40, X = 62°, XYZ = 54°. If YO and ZO are the bisectors of XYZ and XZY respectively of Δ XYZ, find OZY and YOZ.

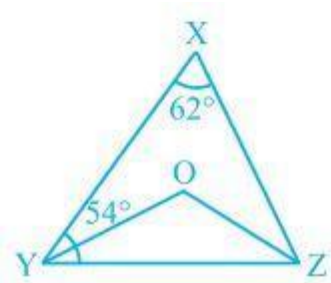


Fig. 6.40

Solution:

We know that the sum of the interior angles of the triangle.

So, X +XYZ +XZY = 180°

Putting the values as given in the question we get, 62°+54° +XZY = 180°

Or, XZY = 64°

Now, we know that ZO is the bisector so,

OZY = ½ XZY

∴ OZY = 32°

Similarly, YO is a bisector and so,

OYZ = ½ XYZ

Or, OYZ = 27° (As XYZ = 54°)

Now, as the sum of the interior angles of the triangle,

OZY +OYZ +O = 180°

Putting their respective values, we get,

O = 180°-32°-27°

Hence, O = 121°

3. In Fig. 6.41, if AB DE, BAC = 35° and CDE = 53°, find DCE.

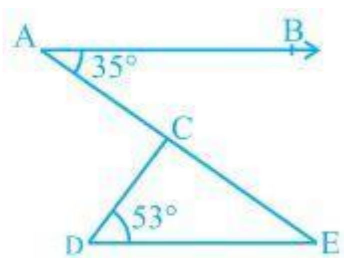


Fig. 6.41

Solution:

We know that AE is a transversal since AB DE

Here BAC and AED are alternate interior angles.

Hence, BAC = AED

It is given that BAC = 35°

AED = 35°

Now consider the triangle CDE. We know that the sum of the interior angles of a triangle is 180°.

∴ DCE+CED+CDE = 180°

Putting the values, we get

DCE+35°+53° = 180°

Hence, DCE = 92°

4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that PRT = 40°, RPT = 95° and TSQ = 75°, find SQT.

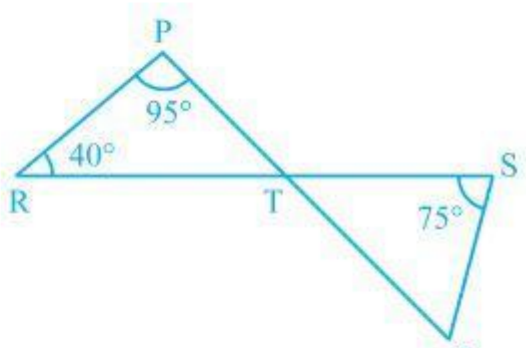


Fig. 6.42

Solution:

Consider triangle PRT.

PRT +RPT + PTR = 180°

So, PTR = 45°

Now PTR will be equal to STQ as they are vertically opposite angles.

So, PTR = STQ = 45°

Again, in triangle STQ,

TSQ +PTR + SQT = 180°

Solving this we get,

SQT = 60°

5. In Fig. 6.43, if PQ ⊥ PS, PQ SR, SQR = 28° and QRT = 65°, then find the values of x and y.

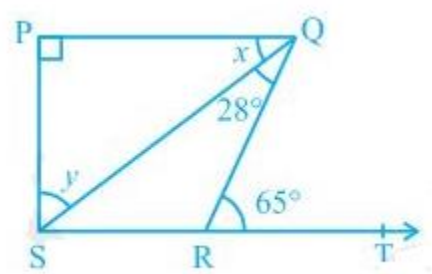


Fig. 6.43

Solution:

x +SQR = QRT (As they are alternate angles since QR is transversal)

So, x+28° = 65°

∴ x = 37°

It is also known that alternate interior angles are same and so,

QSR = x = 37°

Also, Now,

QRS +QRT = 180° (As they are a Linear pair)

Or, QRS+65° = 180°

So, QRS = 115°

Now, we know that the sum of the angles in a quadrilateral is 360°. So,

P +Q+R+S = 360°

Putting their respective values, we get,

S = 360°-90°-65°-115°

In Δ SPQ

∠SPQ + x + y = 180^0

$90^0 + 37^0 + y = 180^0$

$y = 180^0 – 127^0 = 53^0$

Hence, y = 53°

6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of PQR and PRS meet at point T, then prove that QTR = ½ QPR.

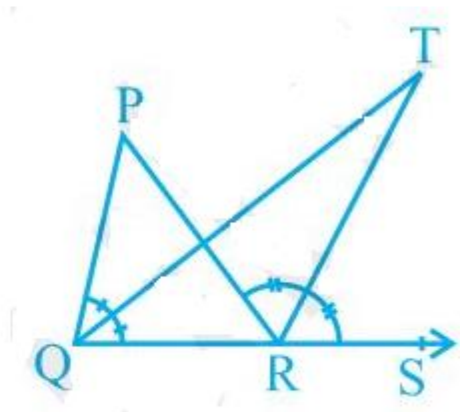


Fig. 6.44

Solution:

Consider the ΔPQR. PRS is the exterior angle and QPR and PQR are interior angles.

So, PRS = QPR+PQR (According to triangle property)

Or, PRS -PQR = QPR ———(i)

Now, consider the ΔQRT,

TRS = TQR+QTR

Or, QTR = TRS-TQR

We know that QT and RT bisect PQR and PRS respectively.

So, PRS = 2 TRS and PQR = 2TQR

Now, QTR = ½ PRS – ½PQR

Or, QTR = ½ (PRS -PQR)

From (i) we know that PRS -PQR = QPR

So, QTR = ½ QPR (hence proved).

What is Congruence?

- In geometry, two figures or objects are congruent if they have the same shape and size, or if one has the same shape and size as the mirror image of the other.
- A combination of rigid motions, namely a translation, a rotation, and a reflection is also permitted in Congruence. Translation means sliding, Rotation means turning and Reflection means Flipping. If two figures can be overlapped using translation, rotation or reflection then they are congruence
- It has come from Latin word congruere "agree, correspond with"

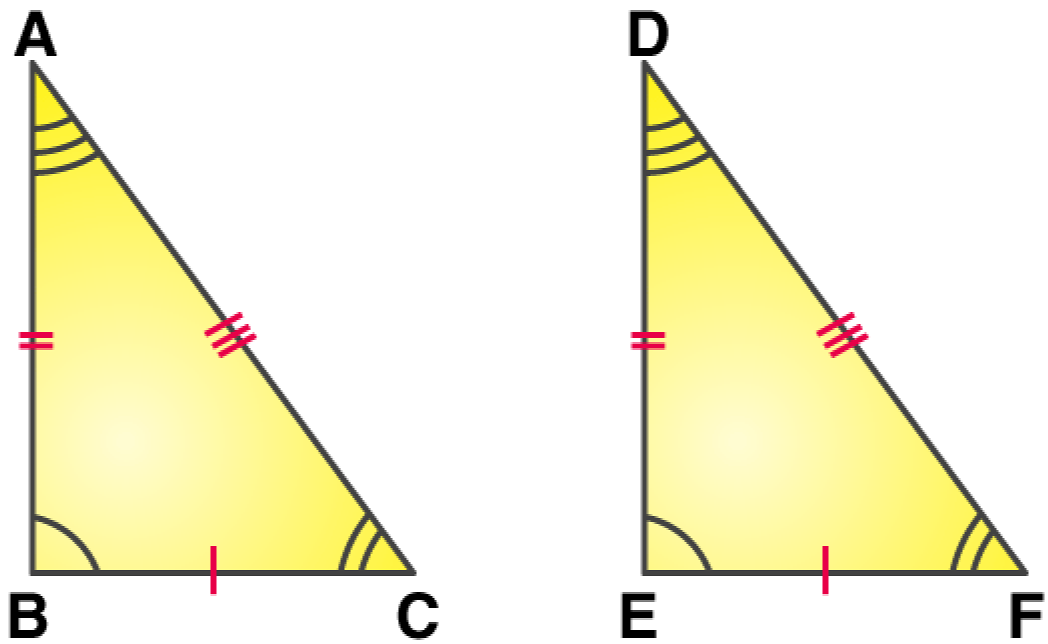
Congruence

Two Geometric figure are said to be congruence if they are exactly same size and shape

- Symbol used is ≅
- Two angles are congruent if they are equal
- Two circle are congruent if they have equal radii
- Two squares are congruent if the sides are equal
- Two line segments are congruent if they have equal length

Congruent Triangles

In a pair of triangles if all three corresponding sides and three corresponding angles are exactly equal, and then the triangles are said to be congruent.



In congruent triangles, the corresponding parts are equal and are written as CPCT (Corresponding part of the congruent triangle).

To know more about Congruency of Triangles,

Criteria for Congruency

The following are the criteria for the congruency of the triangles.

SSS Criteria for Congruency

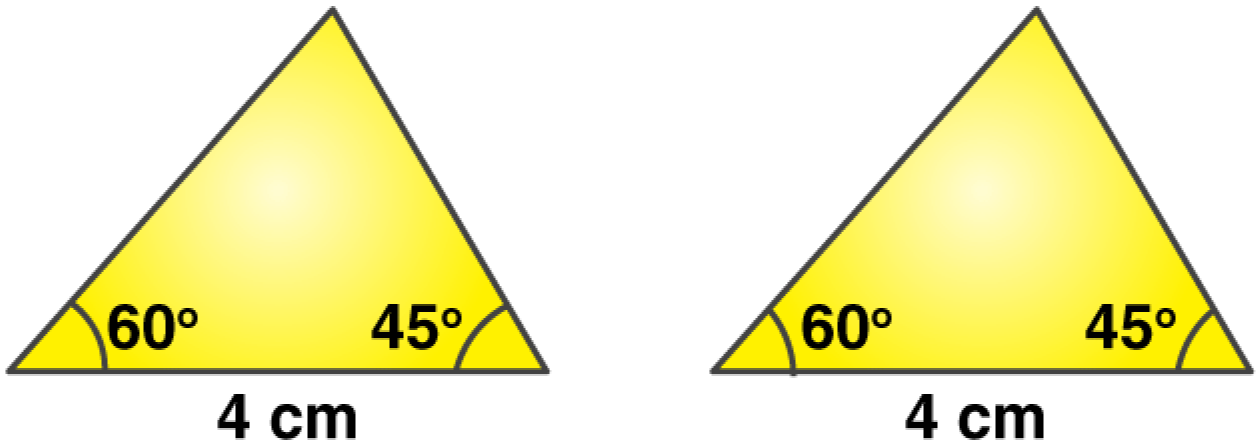
- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- If all sides are exactly the same, then their corresponding angles must also be exactly the same.

SAS Criteria for Congruency

– Axiom: Two triangles are congruent if two sides and the **included** angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.

ASA Criteria for Congruency

– Two triangles are congruent if two angles and the **included** side of one triangle are equal to the corresponding two angles and the included side of the other triangle



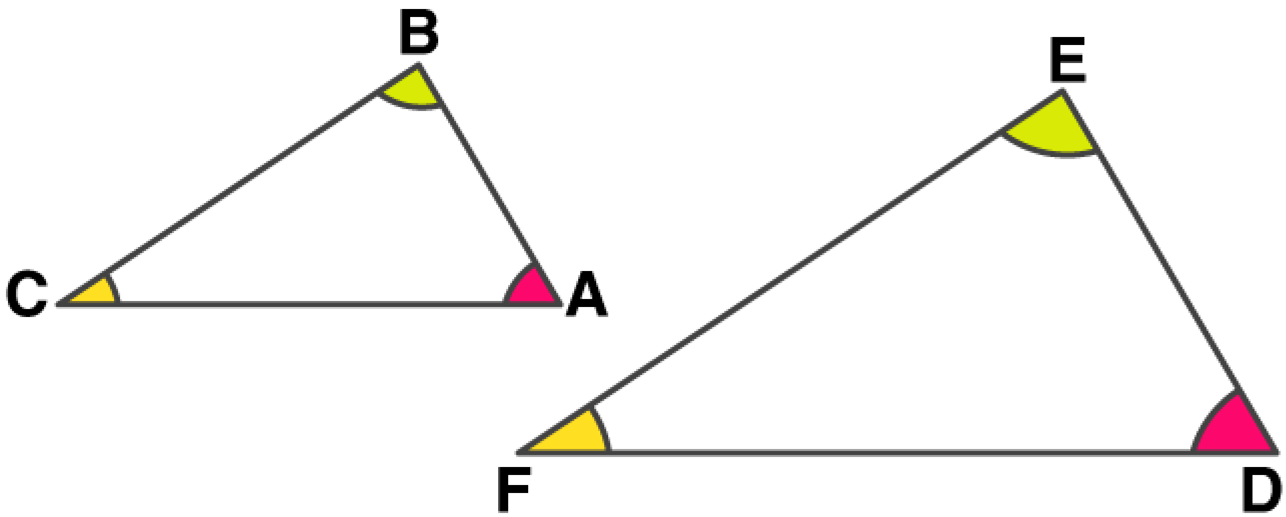
An included side is 4cm

AAS Criteria for Congruency

– Two triangles are said to be congruent to each other if two angles and one side of one triangle are equal to two angles and one side of the other triangle.

Why SSA and AAA congruency rules are not valid?

- SSA or ASS test is not a valid test for congruency as the angle is not included between the pairs of equal sides.-
- The AAA test also is not a valid test as even though 2 triangles can have all three same angles, the sides can be of differing lengths. This becomes a test for similarity (AA).



Angles of a triangle

RHS Criteria for Congruency

- If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.
- RHS stands for Right angle – Hypotenuse – Side.

Properties of Isosceles triangle

– If 2 sides of the triangle are equal, the angles opposite those sides are also equal and vice versa.

To know more about Properties of Isosceles Triangle,

Criteria for Congruency of triangles

– The criteria for congruency of triangles are :

- SAS
- ASA
- AAS
- SSS
- RHS

Symbolically, it is expressed as $\Delta ABC \cong \Delta XYZ$

Exercise: 7.1 (Page No: 118)

1. In quadrilateral ACBD, AC = AD and AB bisect A (see Fig. 7.16). Show that ΔABC≅ ΔABD. What can you say about BC and BD?

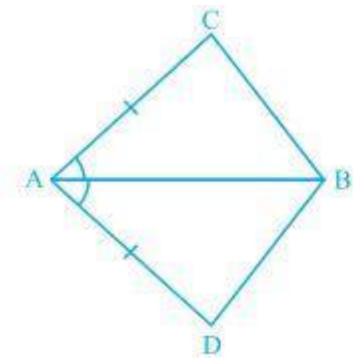


Fig. 7.16

Solution:

It is given that AC and AD are equal i.e. AC = AD and the line segment AB bisects A.

We will have to now prove that the two triangles ABC and ABD are similar i.e. **ΔABC≅ ΔABD**

Proof:

Consider the triangles ΔABC and ΔABD,

(i) AC = AD (It is given in the question)

(ii) AB = AB (Common)

(iii) ∠CAB = ∠DAB (Since AB is the bisector of angle A)

So, by **SAS congruency criterion**, ΔABC ≅ ΔABD.

For the 2nd part of the question, BC and BD are of equal lengths by the rule of C.P.C.T.

2. ABCD is a quadrilateral in which AD = BC and DAB = CBA (see Fig. 7.17). Prove that

(i) ΔABD ≅ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.

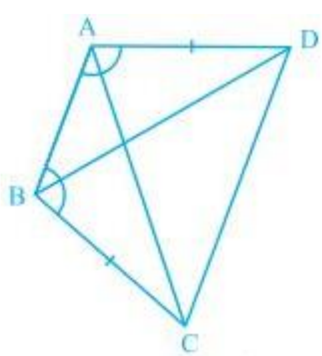


Fig. 7.17

Solution:

The given parameters from the questions are ∠DAB = ∠CBA and AD = BC.

(i) ΔABD and ΔBAC are similar by SAS congruency as

AB = BA (It is the common arm)

∠DAB = ∠CBA and AD = BC (These are given in the question)

So, triangles ABD and BAC are similar i.e. ΔABD≅ ΔBAC. (Hence proved).

(ii) It is now known that ΔABD ≅ΔBAC so,

BD = AC (by the rule of CPCT).

(iii) Since ΔABD ≅ΔBAC so,

Angles ∠ABD = ∠BAC (by the rule of CPCT).

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

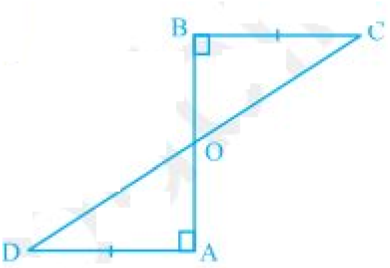


Fig. 7.18

Solution:

It is given that AD and BC are two equal perpendiculars to AB.

We will have to prove that **CD is the bisector of AB**

Now,

Triangles ΔAOD and ΔBOC are similar by AAS congruency since:

(i) A = B (They are perpendiculars)

(ii) AD = BC (As given in the question)

(iii) ∠AOD = ∠BOC (They are vertically opposite angles)

∴ ΔAOD ≅ΔBOC.

So, AO = OB (by the rule of CPCT).

Thus, CD bisects AB (Hence proved).

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that ΔABC ΔCDA.

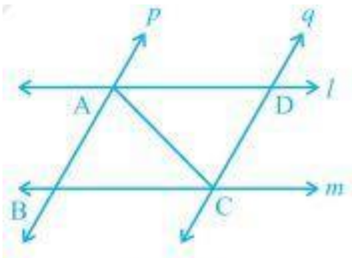


Fig. 7.19

Solution:

It is given that p q and l m

To prove:

Triangles ABC and CDA are similar i.e. ΔABC≅ ΔCDA

Proof:

Consider the ΔABC and ΔCDA,

(i) ∠ BCA = ∠DAC and ∠BAC = ∠DCA Since they are alternate interior angles

(ii) AC = CA as it is the common arm

So, by **ASA congruency criterion,** ΔABC≅ ΔCDA.

5. Line l is the bisector of an angle A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of A (see Fig. 7.20). Show that:

(i) ΔAPB ≅ΔAQB

(ii) BP = BQ or B is equidistant from the arms of A.

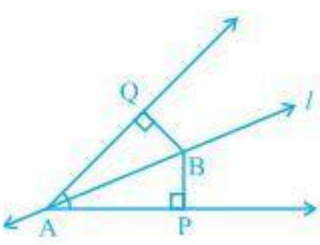


Fig. 7.20

Solution:

It is given that the line "*l*" is the bisector of angle A and the line segments BP and BQ are perpendiculars drawn from *l*.

(i) ΔAPB and ΔAQB are similar by AAS congruency because:

P = Q (They are the two right angles)

AB = AB (It is the common arm)

∠BAP = ∠BAQ (As line *l* is the bisector of angle A)

So, ΔAPB ≅ΔAQB.

(ii) By the rule of CPCT, BP = BQ. So, it can be said the point B is equidistant from the arms of A.

6. In Fig. 7.21, AC = AE, AB = AD and BAD = EAC. Show that BC = DE.

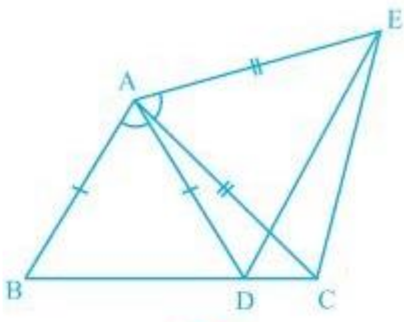


Fig. 7.21

Solution:

It is given in the question that AB = AD, AC = AE, and ∠BAD = ∠EAC

To prove:

The line segment BC and DE are similar i.e. BC = DE

Proof:

We know that ∠BAD = ∠EAC

Now, by adding DAC on both sides we get,

∠BAD + ∠DAC = ∠EAC + ∠DAC

This implies, ∠BAC = ∠ EAD

Now, ΔABC and ΔADE are similar by SAS congruency since:

(i) AC = AE (As given in the question)

(ii) ∠BAC = ∠EAD

(iii) AB = AD (It is also given in the question)

∴ Triangles ABC and ADE are similar i.e. ΔABC≅ ΔADE.

So, by the rule of CPCT, it can be said that BC = DE.

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠ EPA = ∠DPB (see Fig. 7.22). Show that

(i) ΔDAP ≅ΔEBP

(ii) AD = BE

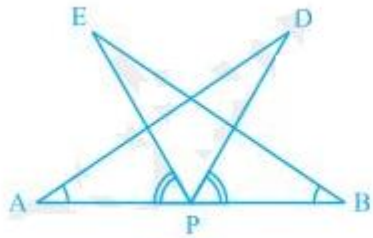


Fig. 7.22

Solutions:

In the question, it is given that P is the mid-point of line segment AB. Also, ∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) It is given that ∠EPA = ∠DPB

Now, add DPE on both sides,

∠EPA + ∠DPE = ∠DPB+ ∠DPE

This implies that angles DPA and EPB are equal i.e. DPA = EPB

Now, consider the triangles DAP and EBP.

∠DPA = ∠ EPB

AP = BP (Since P is the mid-point of the line segment AB)

∠BAD = ∠ABE (As given in the question)

So, by **ASA congruency**, ΔDAP≅ ΔEBP.

(ii) By the rule of CPCT, AD = BE.

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:

(i) ΔAMC ≅ΔBMD

(ii) DBC is a right angle.

(iii) ΔDBC ΔACB

(iv) CM = ½ AB

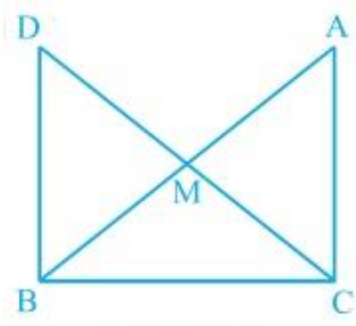


Fig. 7.23

Solution:

It is given that M is the mid-point of the line segment AB, C = 90°, and DM = CM

(i) Consider the triangles ΔAMC and ΔBMD:

AM = BM (Since M is the mid-point)

CM = DM (Given in the question)

∠CMA = ∠DMB (They are vertically opposite angles)

So, by **SAS congruency criterion**, ΔAMC≅ ΔBMD.

(ii) ∠ACM = ∠BDM (by CPCT)

Now, ∠ACB + ∠DBC = 180° (Since they are co-interiors angles)

⇒ 90° +B = 180°

∴ ∠DBC = 90°

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

∠ACB = ∠DBC (They are right angles)

DB = AC (by CPCT)

So, ΔDBC ≅ΔACB by **SAS congruency**.

(iv) DC = AB (Since ΔDBC ΔACB)

⇒ DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM+AM

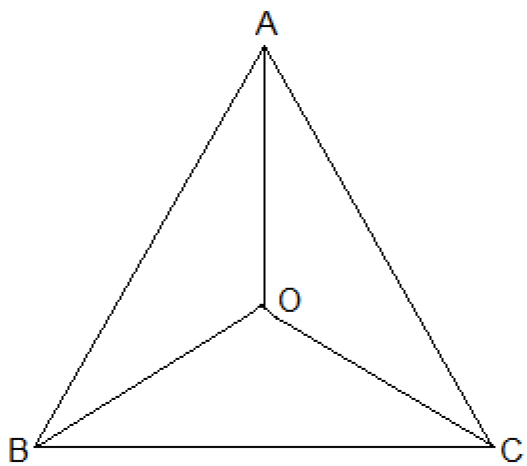
Hence, CM + CM = AB

⇒ CM = (½) AB

Exercise: 7.2 (Page No: 123)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of B and C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects A



Solution:

Given:

AB = AC and

the bisectors of B and C intersect each other at O

(i) Since ABC is an isosceles with AB = AC,

B = C

½ B = ½ C

⇒ ∠OBC = ∠OCB (Angle bisectors)

∴ OB = OC (Side opposite to the equal angles are equal.)

(ii) In ΔAOB and ΔAOC,

AB = AC (Given in the question)

AO = AO (Common arm)

OB = OC (As Proved Already)

So, ΔAOB ≅ΔAOC by SSS congruence condition.

∠BAO = ∠CAO (by CPCT)

Thus, AO bisects A.

2. In ΔABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that ΔABC is an isosceles triangle in which AB = AC.

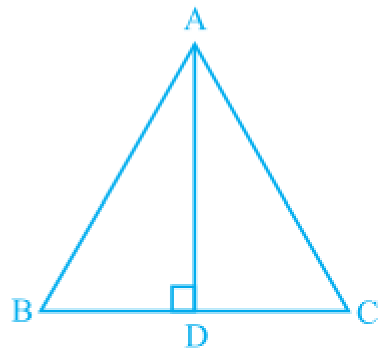


Fig. 7.30

Solution:

It is given that AD is the perpendicular bisector of BC

To prove:

AB = AC

Proof:

In ΔADB and ΔADC,

AD = AD (It is the Common arm)

∠ADB = ∠ADC

BD = CD (Since AD is the perpendicular bisector)

So, ΔADB ≅ΔADC by **SAS congruency criterion**.

Thus,

AB = AC (by CPCT)

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

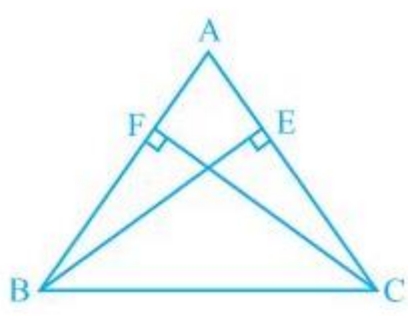


Fig. 7.31

Solution:

Given:

(i) BE and CF are altitudes.

(ii) AC = AB

To prove:

BE = CF

Proof:

Triangles ΔAEB and ΔAFC are similar by AAS congruency since

A = A (It is the common arm)

∠AEB = ∠ AFC (They are right angles)

AB = AC (Given in the question)

∴ ΔAEB ≅ΔAFC and so, BE = CF (by CPCT).

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) ΔABE ≅ΔACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.

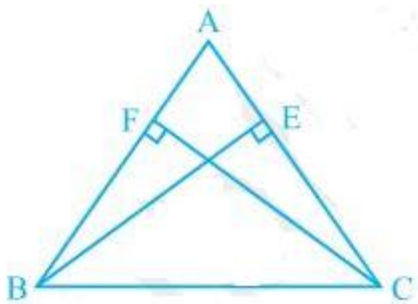


Fig. 7.32

Solution:

It is given that BE = CF

(i) In ΔABE and ΔACF,

A = A (It is the common angle)

∠AEB = ∠AFC (They are right angles)

BE = CF (Given in the question)

∴ ΔABE ≅ΔACF by **AAS congruency condition**.

(ii) AB = AC by CPCT and so, ABC is an isosceles triangle.

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ABD = ∠ACD.

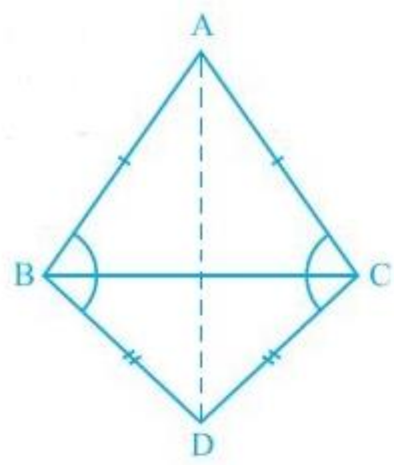


Fig. 7.33

Solution:

In the question, it is given that ABC and DBC are two isosceles triangles.

We will have to show that ∠ ABD = ∠ACD

Proof:

Triangles ΔABD and ΔACD are similar by SSS congruency since

AD = AD (It is the common arm)

AB = AC (Since ABC is an isosceles triangle)

BD = CD (Since BCD is an isosceles triangle)

So, ΔABD≅ ΔACD.

∴ ∠ABD = ∠ACD by CPCT.

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that BCD is a right angle.

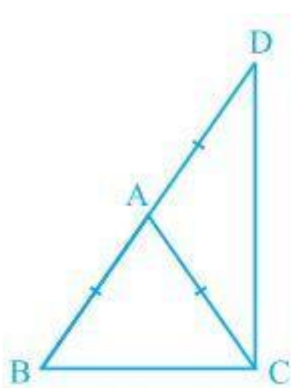


Fig. 7.34

Solution:

It is given that AB = AC and AD = AB

We will have to now prove BCD is a right angle.

Proof:

Consider ΔABC,

AB = AC (It is given in the question)

Also, ∠ACB = ∠ABC (They are angles opposite to the equal sides and so, they are equal)

Now, consider ΔACD,

AD = AB

Also, ∠ADC = ∠ ACD (They are angles opposite to the equal sides and so, they are equal)

Now,

In ΔABC,

∠CAB + ∠ ACB + ∠ABC = 180°

So, ∠CAB + 2 ∠ACB = 180°

⇒ ∠CAB = 180° – 2ACB — (i)

Similarly, in ΔADC,

∠CAD = 180° – 2 ∠ACD — (ii)

also,

∠CAB + ∠CAD = 180° (BD is a straight line.)

Adding (i) and (ii) we get,

∠CAB + ∠CAD = 180° – 2 ∠ACB+180° – 2 ∠ACD

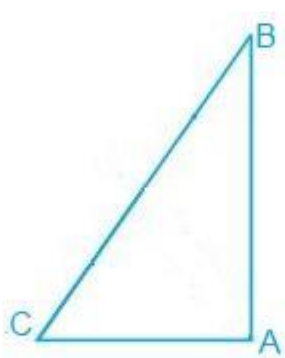
⇒ 180° = 360° – 2 ∠ACB-2 ∠ACD

⇒ 2(∠ACB+ ∠ACD) = 180°

⇒ ∠BCD = 90°

7. ABC is a right-angled triangle in which A = 90° and AB = AC. Find B and C.

Solution:



In the question, it is given that

A = 90° and AB = AC

AB = AC

⇒ B = C (They are angles opposite to the equal sides and so, they are equal)

Now,

∠A+ ∠B+ ∠C = 180° (Since the sum of the interior angles of the triangle)

∴ 90° + 2 ∠B = 180°

⇒ 2 ∠B = 90°

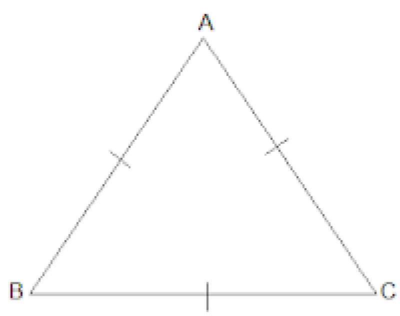
⇒ ∠ B = 45°

So, ∠B = ∠C = 45°

8. Show that the angles of an equilateral triangle are 60° each.

Solution:

Let ABC be an equilateral triangle as shown below:



Here, BC = AC = AB (Since the length of all sides is same)

⇒ ∠ A = ∠B = ∠C (Sides opposite to the equal angles are equal.)

Also, we know that

∠A+ ∠B+ ∠C = 180°

⇒ 3 ∠A = 180°

⇒ ∠A = 60°

∴ ∠A = ∠B = ∠C = 60°

So, the angles of an equilateral triangle are always 60° each.

Exercise: 7.3 (Page No: 128)

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ΔACD

(ii) ΔABP ≅ΔACP

(iii) AP bisects A as well as D.

(iv) AP is the perpendicular bisector of BC.

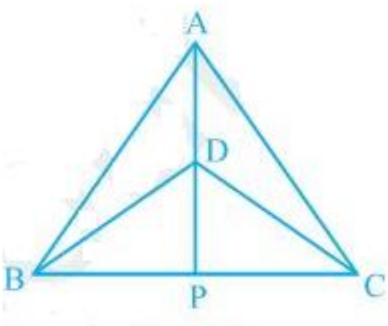


Fig. 7.39

Solution:

In the above question, it is given that ΔABC and ΔDBC are two isosceles triangles.

(i) ΔABD and ΔACD are similar by SSS congruency because:

AD = AD (It is the common arm)

AB = AC (Since ΔABC is isosceles)

BD = CD (Since ΔDBC is isosceles)

∴ ΔABD ≅ΔACD.

(ii) ΔABP and ΔACP are similar as:

AP = AP (It is the common side)

∠PAB = ∠PAC (by CPCT since ΔABD ≅ ΔACD)

AB = AC (Since ΔABC is isosceles)

So, ΔABP ≅ΔACP by SAS congruency condition.

(iii) ∠PAB = ∠PAC by CPCT as ΔABD≅ ΔACD.

AP bisects A. — (i)

Also, ΔBPD and ΔCPD are similar by SSS congruency as

PD = PD (It is the common side)

BD = CD (Since ΔDBC is isosceles.)

BP = CP (by CPCT as ΔABP≅ ΔACP)

So, ΔBPD ≅ΔCPD.

Thus, ∠BDP = ∠CDP by CPCT. — (ii)

Now by comparing (i) and (ii) it can be said that AP bisects A as well as D.

(iv) ∠BPD = ∠CPD (by CPCT as ΔBPD≅ ΔCPD)

and BP = CP — (i)

also,

∠BPD + ∠CPD = 180° (Since BC is a straight line.)

⇒ 2 ∠BPD = 180°

⇒ ∠BPD = 90° —(ii)

Now, from equations (i) and (ii), it can be said that

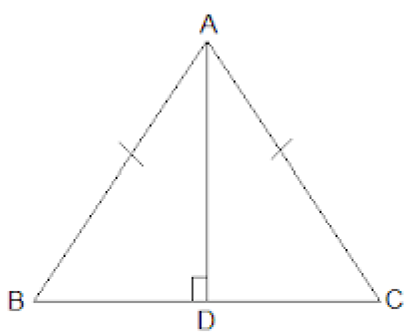
AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects A.

Solution:

It is given that AD is an altitude and AB = AC. The diagram is as follows:



(i) In ΔABD and ΔACD,

∠ADB = ∠ADC = 90°

AB = AC (It is given in the question)

AD = AD (Common arm)

∴ ΔABD ≅ΔACD by RHS congruence condition.

Now, by the rule of CPCT,

BD = CD.

So, AD bisects BC

(ii) Again, by the rule of CPCT, ∠ BAD = ∠CAD

Hence, AD bisects A.

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Fig. 7.40). Show that:

(i) ΔABM ≅ΔPQN

(ii) ΔABC ≅ΔPQR

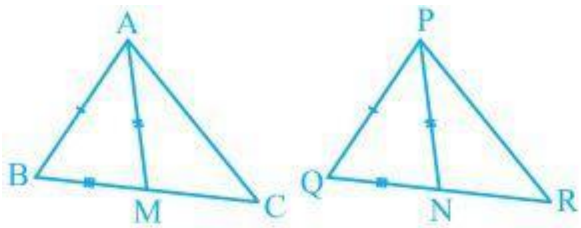


Fig. 7.40

Solution:

Given parameters are:

AB = PQ,

BC = QR and

AM = PN

(i) ½ BC = BM and ½ QR = QN (Since AM and PN are medians)

Also, BC = QR

So, ½ BC = ½ QR

⇒ BM = QN

In ΔABM and ΔPQN,

AM = PN and AB = PQ (As given in the question)

BM = QN (Already proved)

∴ ΔABM ≅ ΔPQN by SSS congruency.

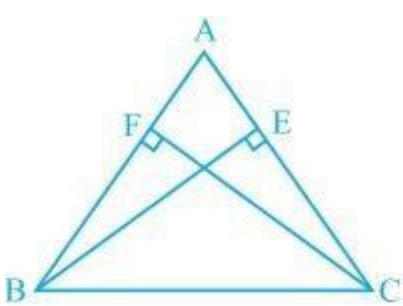
(ii) In ΔABC and ΔPQR,

AB = PQ and BC = QR (As given in the question)

∠ABC = ∠PQR (by CPCT)

So, ΔABC ≅ ΔPQR by SAS congruency.

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



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Solution:

It is known that BE and CF are two equal altitudes.

Now, in ΔBEC and ΔCFB,

∠BEC = ∠CFB = 90° (Same Altitudes)

BC = CB (Common side)

BE = CF (Common side)

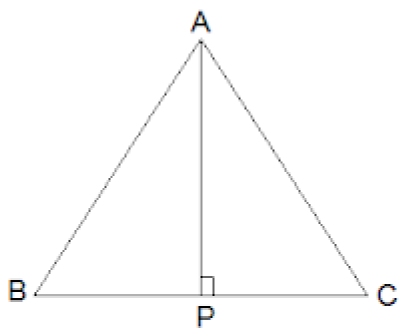
So, ΔBEC ≅ ΔCFB by RHS congruence criterion.

Also, ∠C = ∠B (by CPCT)

Therefore, AB = AC as sides opposite to the equal angles is always equal.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

Solution:



In the question, it is given that AB = AC

Now, ΔABP and ΔACP are similar by RHS congruency as

∠APB = ∠APC = 90° (AP is altitude)

AB = AC (Given in the question)

AP = AP (Common side)

So, ΔABP ≅ ΔACP.

∴ ∠B = ∠C (by CPCT)

Chapter-12

Heron's Formula

Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:
Based on **sides** – a) Equilateral b) Isosceles c) Scalene
Based on **angles** – a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

Area of a triangle

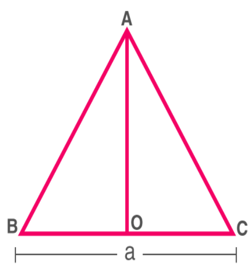
Area = (1/2) × base × height

In case of equilateral and isosceles triangles, if the lengths of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

To know more about Area of a Triangle,

Area of an equilateral triangle

Consider an equilateral ΔABC, with each side as a unit. Let AO be the perpendicular bisector of BC. In order to derive the formula for the area of an equilateral triangle, we need to find height AO.



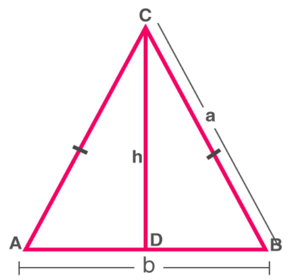
Using Pythagoras theorem,
$AC^2 = OA^2 + OC^2$
$OA^2 = AC^2 - OC^2$
Substitute AC = a,OC = a/2 in the above equation.
$OA^2 = a^2 - a^2/4$
OA = √3a/2

We know that the area of the triangle is:
A = (1/2) × base × height
A = (1/2) × a × (√3a/2)

∴ Area of Equilateral triangle = $\sqrt{3}a^2/4$

Area of an isosceles triangle

Consider an isosceles ΔABC with equal sides as a units and base as b units.



Isosceles triangle ABC

The height of the triangle can be found by Pythagoras' Theorem :

$CD^2 = AC^2 - AD^2$

$\Rightarrow h^2 = a^2 - (b^2/4) = (4a^2 - b^2)/4$

$\Rightarrow h = (1/2)\sqrt{(4a^2 - b^2)}$

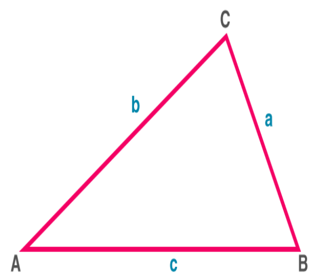
Area of triangle is $A = (1/2)bh$

$\therefore A = (1/2) \times b \times (1/2)\sqrt{(4a^2 - b^2)}$

$$\therefore A = (1/4) \times b \times \sqrt{(4a^2 - b^2)}$$

Area of a triangle – By Heron's formula

Area of a ΔABC, given sides a, b, c by **Heron's formula** (also known as Hero's Formula) is:



Triangle ABC

Find semi perimeter $(s) = (a + b + c)/2$

$$\text{Area} = \sqrt{[s(s - a)(s - b)(s - c)]}$$

This formula is helpful to find the area of a scalene triangle, given the lengths of all its sides.

Exercise: 12.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Solution:

Given,

Side of the signal board = a

Perimeter of the signal board = 3a = 180 cm

∴ a = 60 cm

Semi perimeter of the signal board (s) = 3a/2

By using Heron's formula,

Area of the triangular signal board will be =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$= \sqrt{(3a/2)(3a/2 - a)(3a/2 - a)(3a/2 - a)}$
$= \sqrt{3a/2 \times a/2 \times a/2 \times a/2}$
$= \sqrt{3a^4/16}$
$= \sqrt{3}a^2/4$
$= \sqrt{3}/4 \times 60 \times 60 = 900\sqrt{3}\ cm^2$

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 12.9). The advertisements yield an earning of ₹5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

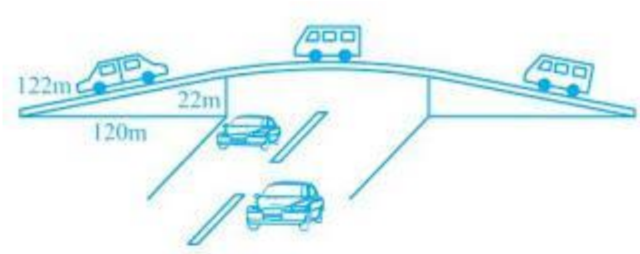


Fig. 12.9

Solution:

The sides of the triangle ABC are 122 m, 22 m and 120 m respectively.

Now, the perimeter will be (122+22+120) = 264 m

Also, the semi perimeter (s) = 264/2 = 132 m

Using Heron's formula,

Area of the triangle =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$= \sqrt{132(132 - 122)(132 - 22)(132 - 120)}\ m^2$
$= \sqrt{132 \times 10 \times 110 \times 12}\ m^2$

$=1320\ m^2$

We know that the rent of advertising per year = ₹ 5000 per m^2

∴ The rent of one wall for 3 months = Rs. (1320×5000×3)/12 = Rs. 1650000

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 12.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

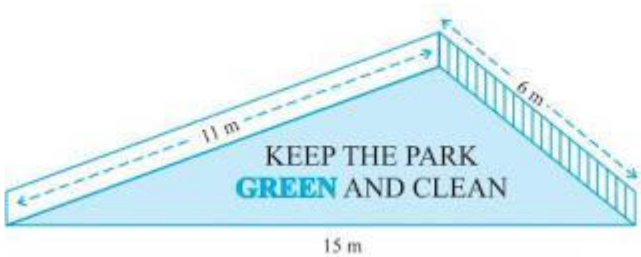


Fig. 12.10

Solution:

It is given that the sides of the wall as 15 m, 11 m and 6 m.

So, the semi perimeter of triangular wall (s) = (15+11+6)/2 m = 16 m

Using Heron's formula,

Area of the message =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

= √[16(16-15)(16-11) (16-6)] m^2

= √[16×1×5×10] m^2 = √800 m^2

= 20√2 m^2

4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42cm.

Solution:

Assume the third side of the triangle to be "x".

Now, the three sides of the triangle are 18 cm, 10 cm, and "x" cm

It is given that the perimeter of the triangle = 42cm

So, x = 42-(18+10) cm = 14 cm

∴ The semi perimeter of triangle = 42/2 = 21 cm

Using Heron's formula,

Area of the triangle,

=

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$= \sqrt{[21(21-18)(21-10)(21-14)]}\ cm^2$

$= \sqrt{[21×3×11×7]}\ m^2$

$= 21\sqrt{11}\ cm^2$

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

Solution:

The ratio of the sides of the triangle are given as 12 : 17 : 25

Now, let the common ratio between the sides of the triangle be "x"

∴ The sides are 12x, 17x and 25x

It is also given that the perimeter of the triangle = 540 cm

12x+17x+25x = 540 cm

54x = 540cm

So, x = 10

Now, the sides of triangle are 120 cm, 170 cm, 250 cm.

So, the semi perimeter of the triangle (s) = 540/2 = 270 cm

Using Heron's formula,

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \left[\sqrt{270(270-120)(270-170)(270-250)}\right] cm^2$$

$$= \left[\sqrt{270\times150\times100\times20}\right] cm^2$$

$= 9000\ cm^2$

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution:

First, let the third side be x.

It is given that the length of the equal sides is 12 cm and its perimeter is 30 cm.

So, 30 = 12+12+x

∴ The length of the third side = 6 cm

Thus, the semi perimeter of the isosceles triangle (s) = 30/2 cm = 15 cm

Using Heron's formula,

Area of the triangle

=

$$\sqrt{s(s-a)(s-b)(s-c)}$$

= $\sqrt{[15(15-12)(15-12)(15-6)]}$ cm^2

= $\sqrt{[15\times3\times3\times9]}$ cm^2

= $9\sqrt{15}$ cm^2

Chapter 14

Statistics

Introduction to Statistics

– A study dealing with the collection, presentation and interpretation and analysis of data is called as statistics.

Data

- Facts /figures numerical or otherwise collected for a definite purpose is called as data.
- data collected first-hand data:- Primary
- Secondary data: Data collected from a source that already had data stored

Frequency

– The number of times a particular instance occurs is called frequency in statistics.

Ungrouped data

Ungrouped data is data in its original or raw form. The observations are not classified in groups.

Grouped data

In grouped data, observations are organized in groups.

Class Interval

- The size of the class into which a particular data is divided.
- E.g divisions on a histogram or bar graph.
- **Class width** = upper class limit – lower class limit

Regular and Irregular class interval

- Regular class interval: When the class intervals are equal or of the same sizes.
- E.g 0-10, 10-20, 20-30….. 90-100
- Irregular class interval: When the class intervals are of varying sizes.
- E.g 0-35, 35-45, 45-55, 55- 80, 80-90, 90-95, 95-100

Frequency table

– A frequency table or distribution shows the occurrence of a particular variable in a tabular form.

Sorting

- Raw data needs to be sorted in order to carry out operations.-
- Sorting ⇒ ascending order or descending order

Ungrouped frequency table

– When the frequency of each class interval is not arranged or organised in any manner.

Grouped frequency table

– The frequencies of the corresponding class intervals are organised or arranged in a particular manner, either ascending or descending.

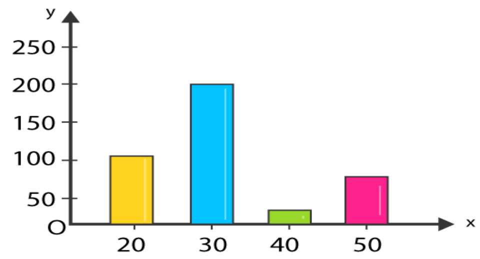
Graphical Representation of Data

Bar graphs

Graphical representation of data using bars of equal width and equal spacing between them (on one axis). The height

Savings (in percentage)	**Number of Employees (Frequency)**
20	105
30	199
40	29
50	73
Total	400

The data can be represented as:



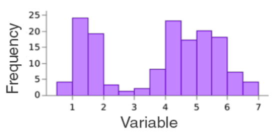
Variable being a number

- A variable can be a number such as ‘no. of students’ or ‘no. of months’.
- Can be represented by bar graphs or histograms depending on the type of data.

Discrete → bar graphs
Continuous → Histograms

Histograms

- Like bar graphs, but for continuous class intervals.
- Area of each rectangle is ∝ Frequency of a variable and the width is equal to the class interval.



Ex 14.1

Question 1.
Give five examples of data that you collect from your day-to-day life.
Solution:
Following are the five examples which are related to day-to-day life :

- Number of girl students in our class.
- Number of computer sets in our computer lab.
- Telephone bills of our house for last two years.
- Number of students appeared in an examination obtained from newspapers.
- Number of female teachers in all the schools in a state obtained from the education department.

Question 2.
Classify the data in Q.1 above as primary or secondary data.
Solution:

We have,
Primary data: (i), (ii) and (iii)
Secondary data: (iv) and (v)

Ex 14.2

Question 1.
The blood groups of 30 students of class VIII are recorded as follows
A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O
Represent this data in the form of a frequency distribution table. Which is the most common and which is the rarest blood group among these students?
Solution:
The required frequency distribution table is

Blood groups	Tally marks	Number of students
A	~~IIII~~ IIII	9
B	~~IIII~~ I	6
O	~~IIII~~ ~~IIII~~ II	12
AB	III	3
Total		30

From the above table, we have The most common blood group is O. The rarest blood group is AB.

Question 2.
The distance (in km) of 40 engineers from their residence to their place of work were found as follows

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?
Solution:
Here, the observation with minimum and maximum values are 2 and 32 respectively.
∴ The class intervals are as follows:
0 – 5, 5 – 10, 10 – 15, 15 – 20, 20 – 25, 25 – 30, 30 – 35
The required frequency distribution table is

Distance (in km)	Tally marks	Number of engineers
0 - 5	𝍸	5
5 - 10	𝍸 𝍸 I	11
10 - 15	𝍸 𝍸 I	11
15 - 20	𝍸 IIII	9
20 - 25	I	1
25 - 30	I	1
30 - 35	II	2
Total		40

From the above table we observe that:
(i) Frequencies of class intervals 5-10 and 10 – 15 are equal, i.e., 11 each. It shows that maximum number of engineers have their residences at 5 to 15 km away from their work place.
(ii) Frequencies of class intervals 20 – 25 and 25 – 30 are also equal, i.e., 1 each. It shows that minimum number of engineers have their residences at 20 to 30 km away from their work place.

Question 3.
The relative humidity (in %) of a certain city for a month of 30 days was as follows

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2
95.1	89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3
95.2	97.3	96.2	92.1	84.9	90.2	95.7	98.3	97.3
96.1	92.1	89						

(i) Construct a grouped frequency distribution table with classes 84-86, 86-88 etc.
(ii) Which month or season do you think this data is about?
(iii) What is the range of this data?
Solution:
Here, the lowest value of observation = 84.9
The highest value of observation = 99.2
So, class intervals are 84 – 86, 86 – 88, 88 – 90, ……. , 98 – 100

(i) Thus, the required frequency distribution table is

Relative humidity (in %)	Tally marks	Frequency
84 - 86	I	1
86 - 88	I	1
88 - 90	II	2
90 - 92	II	2
92 - 94	~~IIII~~ II	7
94 - 96	~~IIII~~ I	6
96 - 98	~~IIII~~ II	7
98 - 100	IIII	4
Total		30

(ii) Since, the relative humidity is high during the rainy season, so, the data appears to be taken in the rainy season.
(iii) Range = (Highest observation) – (Lowest observation) = 99.2 – 84.9 = 14.3

Question 4.
The heights of 50 students, measured to the nearest centimeters have been found to be as follows

161	**150**	**154**	**165**	**168**	**161**	**154**	**162**	**150**	**151**
162	**164**	**171**	**165**	**158**	**154**	**156**	**172**	**160**	**170**
153	**159**	**161**	**170**	**162**	**165**	**166**	**168**	**165**	**164**
154	**152**	**153**	**156**	**158**	**162**	**160**	**161**	**173**	**166**
161	**159**	**162**	**167**	**168**	**159**	**158**	**153**	**154**	**159**

(i) Represent the data given above by a grouped frequency distribution table, taking class intervals as 160 – 165, 165 – 170 etc.
(ii) What can you conclude about their heights form the table?
Solution:
(i) Here, the lowest value of the observation = 150
The highest value of the observation = 173
∴ Class intervals are 150 – 155, 155 -160, …, 170 – 175.
The required frequency distribution table is

Heights	Tally marks	Number of students
150 - 155	~~IIII~~ ~~IIII~~ II	12
155 - 160	~~IIII~~ IIII	9
160 - 165	~~IIII~~ ~~IIII~~ IIII	14
165 - 170	~~IIII~~ ~~IIII~~	10
170 - 175	~~IIII~~	5
Total		50

(ii) From the above table, we can conclude that more than 50% of the students are shorter than 165 cm.

Question 5.

A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is as follows

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	0.04

(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 – 0.04, 0.04 – 0.08 and so on.
(ii) For how many day's was the the concentration of sulphur dioxide more than 0.11 parts per million ?
Solution:
(i) Here, the lowest value of the observation = 0.01
The highest value of the observation = 0.22
∴ Class intervals are 0.00 – 0.04, 0.04 – 0.08,……., 0.20 – 0.24
The required frequency distribution table is

Concentration of sulphur dioxide (in ppm)	Tally marks	Number of days
0.00 - 0.04	IIII	4
0.04 - 0.08	~~IIII~~ IIII	9
0.08 - 0.12	~~IIII~~ IIII	9
0.12 - 0.16	II	2
0.16 - 0.20	IIII	4
0.20 - 0.24	II	2
Total		30

(ii) The concentration of sulphur dioxide was more than 0.11 ppm for 8 days.

Question 6.

Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.
Solution:

The required frequency distribution table is

Number of heads occurring	Tally marks	Frequency
0	~~||||~~ |	6
1	~~||||~~ ~~||||~~	10
2	~~||||~~ ||||	9
3	~~||||~~	5
Total		30

Question 7.
The value of π upto 50 decimal places is given below
3.14159265358979323846264338327950288419716939937510
(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.
(ii) What are the most and the least frequently occurring digits?
Solution:
(i) The required frequency distribution table

Digits	Tally marks	Frequency
0	||	2
1	~~||||~~	5
2	~~||||~~	5
3	~~||||~~ |||	8
4	||||	4
5	~~||||~~	5
6	||||	4
7	||||	4
8	~~||||~~	5
9	~~||||~~ |||	8
Total		50

(ii) The most frequently occurring digits are 3 and 9 and the least frequently occurring digit is 0.

Question 8.
Thirty children were asked about the number of hours they watched TV programmes in the previous week.
The results were found as follows

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 – 10.
(ii) How many children watched television for 15 or more hours a week?
Solution:
(i) Here, the lowest value of the observation = 1 and the highest value of the observation = 17
∴ Class intervats are 0 – 5, 5 – 10 ., 15 – 20
The required frequency distribution table is

Number of hours	Tally marks	Number of children
0 - 5	~~||||~~ ~~||||~~	10
5 - 10	~~||||~~ ~~||||~~ |||	13
10 - 15	~~||||~~	5
15 - 20	||	2
Total		30

(ii) Number of children who watched television for 15 or more hours in a week = 2.

Question 9.
A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows

2.6	3.0	3.7	3.2	2.2	4.1	3.5	4.5
3.5	2.3	3.2	3.4	3.8	3.2	4.6	3.7
2.5	4.4	3.4	3.3	2.9	3.0	4.3	2.8
3.5	3.2	3.9	3.2	3.2	3.1	3.7	3.4
4.6	3.8	3.2	2.6	3.5	4.2	2.9	3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 – 2.5.
Solution:
Here, the lowest value of the observation = 2.2
and the highest value of the observation = 4.6
∴ Class intervals are 2.0 – 2.5, 2.5 – 3.0, …., 4.5 – 5.0

The required frequency distribution table is

Life of batteries (in years)	Tally marks	Number of batteries
2.0 - 2.5	||	2
2.5 - 3.0	卌 |	6
3.0 - 3.5	卌 卌 ||||	14
3.5 - 4.0	卌 卌 |	11
4.0 - 4.5	||||	4
4.5 - 5.0	|||	3
Total		40

Ex 14.3

Question 1.

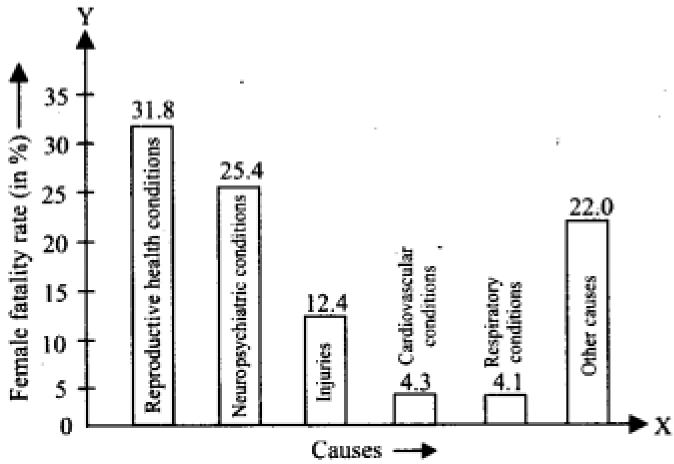
A survey conducted by an organization for the cause of illness and death among the women between the ages 15-44 (in years) worldwide, found the following figures (in %)

S.No.	Causes	Female fatality rate (in %)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiorascular conditions	4.3
5.	Repiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

Solution:

(i) The required graphical representation is shown as follows:

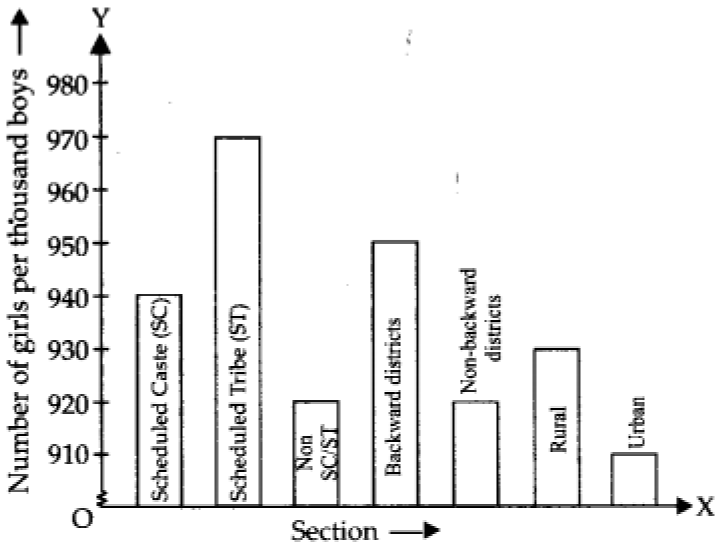


(ii) The major cause of women's ill health and death worldwide is 'reproductive health conditions'.
(iii) Two factors may be uneducation and poor background.

Question 2.
The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below

Section	Number of girls per thousands boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.
(ii) In the classroom discuss, what conclusions can be arrived at from the graph.
Solution:
(i) The required bar graph is shown below:

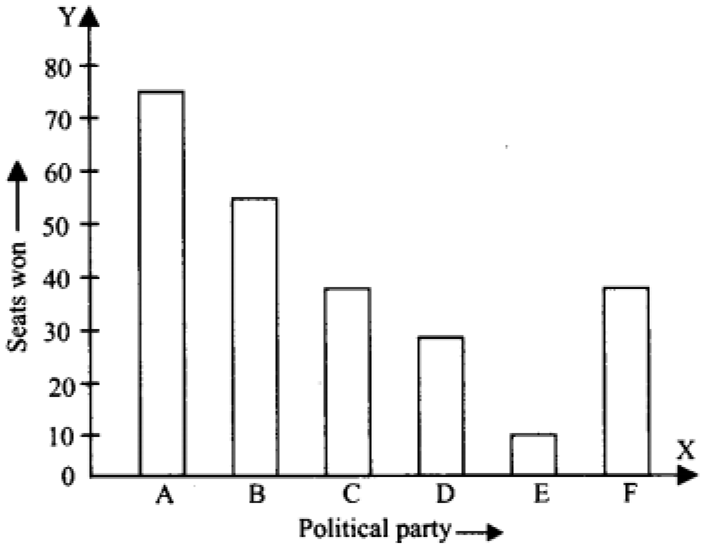


(ii) We conclude that number of girls per thousand boys are maximum in scheduled tribe section whereas minimum in urban section.

Question 3.
Given below are the seats won by different political parties in the polling outcome of a state assembly elections

Political party	Seats won
A	75
B	55
C	37
D	29
E	10
F	37

(i) Draw a bar graph to represent the polling results.
(ii) Which political party won the maximum number of seats?
Solution:
(i) The required bar graph is shown below:



(ii) The political party A won the maximum number of seats.

Question 4.
The length of 40 leaves of a plant measured correct to one millimetre and the obtained data is represented in the following table

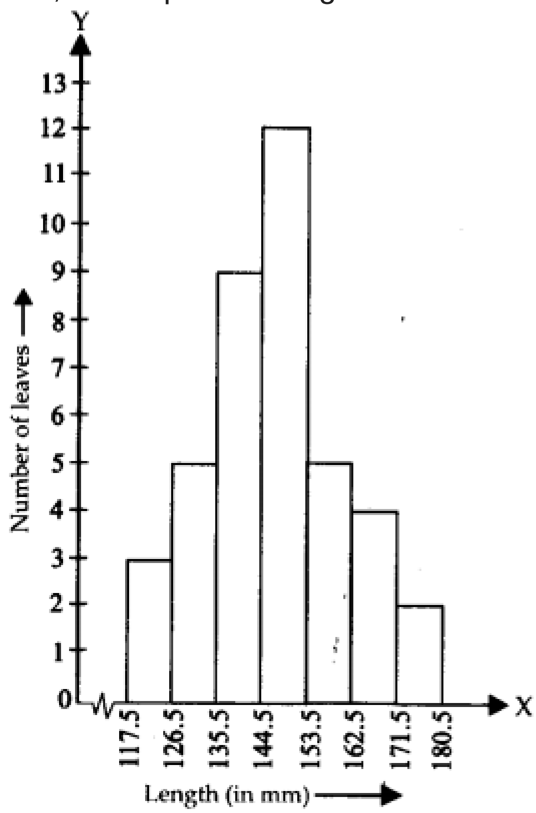
Length (in mm)	Number of leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

(i) Draw a histogram to represent the given data.
(ii) Is there any other suitable graphical representation for the same data?
(iii) Is it correct to conclude that the maximum number of leaves 153 mm long and Why?
Solution:
(i) The given frequency distribution table is not continuous. Therefore, first we have to modify it to be continuous distribution.

Thus, the modified frequency distribution table is:

Length (in mm)	Number of leaves
117.5 - 126.5	3
126.5 - 135.5	5
135.5 - 144.5	9
144.5 - 153.5	12
153.5 - 162.5	5
162.5 - 171.5	4
171.5 - 180.5	2

Now, the required histogram of the frequency distribution is shown below :



(ii) Yes, other suitable graphical representation is a ‘frequency polygon’.
(iii) No, it is not a correct statement. The maximum number of leaves lie in the class interval 145 – 153.

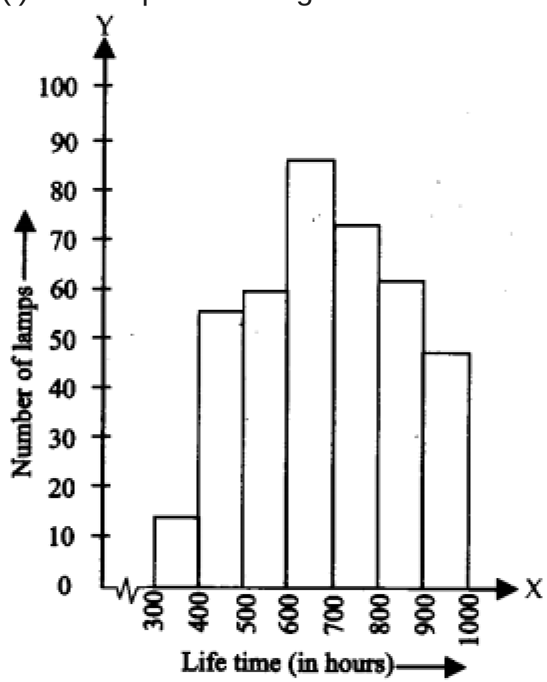
Question 5.
The following table gives the lifetimes of 400 neon lamps

Life time (in hours)	Number of lamps
300 - 400	14
400 - 500	56
500 - 600	60
600 - 700	86
700 - 800	74
800 - 900	62
900 -1000	48

(i) Represent the given information with the help of a histogram.
(ii) How many lamps have a lifetime of more 700 h?

Solution:

(i) The required histogram is shown below:



(ii) Number of lamps having life time of more than 700 hours = 74 + 62 + 48 = 184.

Chapter 4

Linear Equations in Two Variables

What is Linear equations

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. Linear equations can have one or more variables.

Example

$ax + b = 0$

a and b are constant

$ax + by + c = 0$

a, b, c are constants

In Linear equation, No variable is raised to a power greater than 1 or used as the denominator of a fraction.

So, $ax^2+b=0$ is not a linear equation.

similarly $a/x + b=0$ is not a linear equation.

Linear equations are straight lines when plotted on Cartesian plane.

Linear equations Solutions

S.no	Type of equation	Mathematical representation	Solutions
1	Linear equation in one Variable	$ax+b=0, a\neq 0$ ax+b=0,a≠0 a and b are real number	One solution
2	Linear equation in two Variables	$ax+by+c=0, a\neq 0$ and $b\neq 0$ ax+by+c=0,a≠0 and b≠0 i.e. a and b both should not be zero.	Infinite solution possible
3	Linear equation in three Variables	$ax+by+cz+d=0, a\neq 0, b\neq 0$ and $c\neq 0$ax+by+cz+d=0,a≠0,b≠0andc≠0 i,e a , b and c all should not be zero.	Infinite solution possible

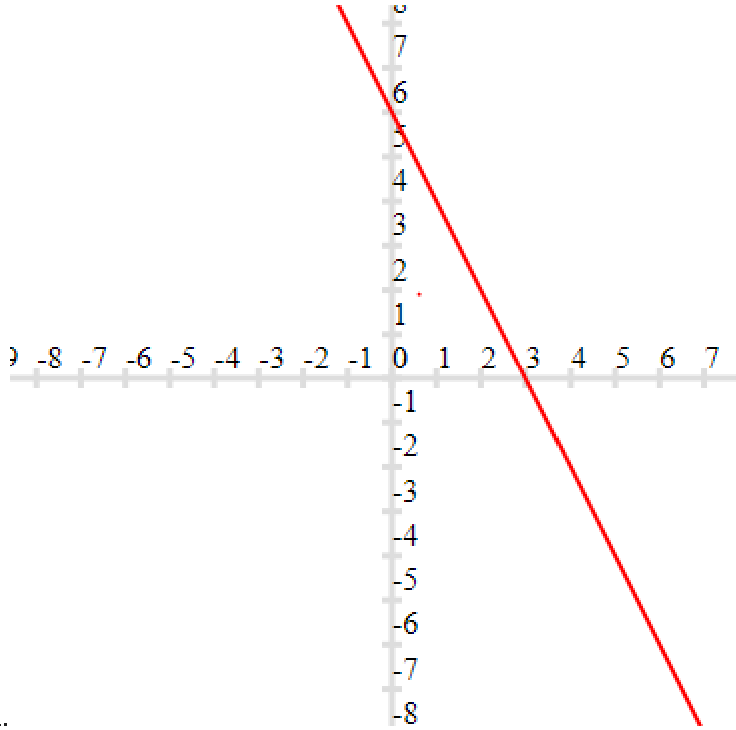
Graphical Representation of Linear equation in one and two variables

- Linear equation in two variables is represented by straight line the Cartesian plane.
- Every point on the line is the solution of the equation.
- In fact Linear equation in one variable can also be represented on Cartesian plane, it will be a straight line either parallel to x –axis or y –axis
  $x-2=0$, (straight line parallel to y axis). It means (2,<any value on y axis) will satisfy this line
  $y-2=0$, (straight line parallel to x axis). It means (<any value on x-axis),2) will satisfy this line

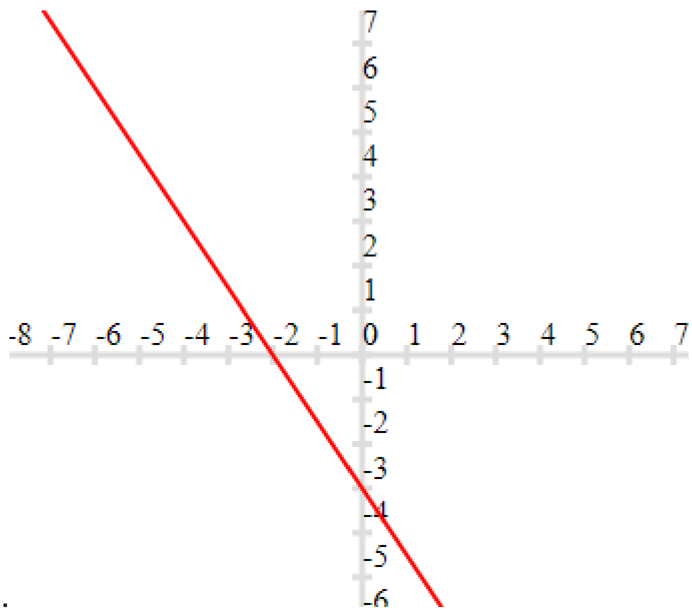
Steps to Draw the Given line on Cartesian plane| how to graph the linear equations

1. Suppose the equation given is $ax+by+c=0, a\neq 0$ and $b\neq 0$
2. Find the value of y for x=0; $y=-c/b$
   this point will lie on Y –axis. And the coordinates will be $(0,-c/b)$
3. Find the value of x for y=0; $x=-c/a$
   This point will lie on X –axis. And the coordinates will be $(-c/a,0)$
4. Now we can draw the line joining these two points.
5. It is easy to draw the points if the values are integers, So if you don't get integer in previous step you may choose point where the values are integers

1
Draw the graph of each of the equations given below. Also, find the co-ordinates of the point where the graph cuts the co-ordinates axis:
(a) 2x + y = 6
(b) 3x + 2y + 6 = 0
Solution
To draw the graph, we need at least two solutions of the equation We can get two points by putting x=0 and y=0. And then draw the graph. Coordinates will be given by (x,0) and (0,y)



a.
Coordinates are (3,0) and (0,6)



b.
Coordinates are (-2,0) and (0,-3)

2 Any point on the y-axis is of the form

(a) (x, 0)
(b) (x, y)
(c) (0, y)
(d) (y, y)

Solution C

3 At what point does the graph of the linear equation x + y = 5 meet a line which is parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of x-axis.

Solution

The coordinates of the points lying on the line parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of the x-axis are of the form (2, a).

Putting x = 2, y = a in the equation x + y = 5, we get a = 3. Thus, the required point is (2, 3).

Exercise 4.1

Question 1

The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be Rs x and that of a pen to be Rs y.)

Solution

Let cost of notebook and a pen be x and y respectively.

Cost of note book = 2 cost of pen

$x=2y$

$x-2y=0$

Question 2

Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b, c in each case:

(i) $2x+3y=9.3\overline{5}$

(ii) $x-y5-10=0$

(iii) $-2x+3y=6$

(iv) $x=3y$

(v) $2x=-5y$

(vi) $3x+2=0$

(vii) $y-2=0$

(viii) $5=2x$

Solution

(i) $2x+3y=9.3\overline{5}$

=> $2x+3y-9.3\overline{5}=0$

Comparing this equation with $ax+by+c=0$,

a=2 ,b=3 $c=-9.3\overline{5}$.

(ii) $x-y5-10=0$

Comparing this equation with $ax+by+c=0$, a = 1, b =-1/5,c = -10

(iii) $-2x+3y=6$
=> $-2x+3y-6=0$
Comparing this equation with $ax+by+c=0$,
a = -2, b = 3, c = -6
(iv) $x=3y$
=> $x-3y+0=0$
Comparing this equation with $ax+by+c=0$,
a = 1, b = -3, c = 0
(v) $2x=-5y$
=> $2x+5y+0=0$
Comparing this equation with $ax+by+c=0$,
a = 2, b = 5, c = 0
(vi) $3x+2=0$
=> $3x+0.y+2=0$
Comparing this equation with $ax+by+c=0$,
a = 3, b = 0, c = 2
(vii) $y-2=0$
=> y - 2 = 0
Comparing this equation with $ax+by+c=0$,
a = 0, b = 1, c = -2
(vii) $5=2x$
=> $-2x+5=0$
Comparing this equation with $ax+by+c=0$,
a = -2, b = 0, c = 5

Exercise 4.2

Question 1
Which one of the following options is true, and why?
y = 3x + 5 has
(i) a unique solution,
(ii) only two solutions,
(iii) infinitely many solutions
Solution:
Option (iii) is true because for every value of x, we get a corresponding value of y and vice-versa in the given equation.
Hence, given linear equation has an infinitely many solutions.

Question 2
Write four solutions for each of the following equations:
(i) $2x + y = 7$
(ii) $\pi x + y = 9$
(iii) $x = 4y$
Solution:
(i) $2x + y = 7$
When $x = 0$, $2(0) + y = 7 \Rightarrow y = 7$
∴ Solution is (0, 7)
When $x =1$, $2(1) + y = 7 \Rightarrow y = 7 – 2 \Rightarrow y = 5$
∴ Solution is (1, 5)
When $x = 2$, $2(2) + y =7y = 7 – 4 \Rightarrow y = 3$
∴ Solution is (2, 3)
When $x = 3$, $2(3) + y = 7y = 7 – 6 \Rightarrow y = 1$
∴ Solution is (3, 1).

(ii) $\pi x + y = 9$
When $x = 0$, $\pi(0) + y = 9 \Rightarrow y = 9 – 0 \Rightarrow y = 9$
∴ Solution is (0, 9)
When $x = 1$, $\pi(1) + y = 9 \Rightarrow y = 9 – \pi$
∴ Solution is $(1, (9 – \pi))$
When $x = 2$, $\pi(2) + y = 9 \Rightarrow y = 9 – 2\pi$
∴ Solution is $(2, (9 – 2\pi))$
When $x = -1$, $\pi(-1) + y = 9 \Rightarrow y = 9 + \pi$
∴ Solution is $(-1, (9 + \pi))$

(iii) $x = 4y$
When $x = 0$, $4y = 1 \Rightarrow y = 0$
∴ Solution is (0, 0)
When $x = 1$, $4y = 1 \Rightarrow y = 14$
∴ Solution is (1,14)
When $x = 4$, $4y = 4 \Rightarrow y = 1$
∴ Solution is (4, 1)
When $x = 4$, $4y = 4 \Rightarrow y = -1$
∴ Solution is (-4, -1)

Question 3
Check which of the following are solutions of the equation $x – 2y = 4$ and which are not:
(i) (0,2)
(ii) (2,0)
(iii) (4, 0)
(iv) $(\sqrt{2}, 4\sqrt{2})$
(v) (1, 1)
Solution:
(i) (0,2) means $x = 0$ and $y = 2$

Puffing x = 0 and y = 2 in x – 2y = 4, we get
L.H.S. = 0 – 2(2) = -4.
But R.H.S. = 4
∴ L.H.S. ≠ R.H.S.
∴ x =0, y =2 is not a solution.

(ii) (2, 0) means x = 2 and y = 0
Putting x = 2 and y = 0 in x – 2y = 4, we get
L.H:S. 2 – 2(0) = 2 – 0 = 2.
But R.H.S. = 4
∴ L.H.S. ≠ R.H.S.
∴ (2,0) is not a solution.

(iii) (4, 0) means x = 4 and y = 0
Putting x = 4 and y = o in x – 2y = 4, we get
L.H.S. = 4 – 2(0) = 4 – 0 = 4 =R.H.S.
∴ L.H.S. = R.H.S.
∴ (4, 0) is a solution.

(iv) (√2, 4√2) means x = √2 and y = 4√2
Putting x = √2 and y = 4√2 in x – 2y = 4, we get
L.H.S. = √2 – 2(4√2) = √2 – 8√2 = -7√2
But R.H.S. = 4
∴ L.H.S. ≠ R.H.S.
∴ (√2 , 4√2) is not a solution.

(v) (1, 1) means x =1 and y = 1

Putting x = 1 and y = 1 in x – 2y = 4, we get
LH.S. = 1 – 2(1) = 1 – 2 = -1. But R.H.S = 4
∴ LH.S. ≠ R.H.S.
∴ (1, 1) is not a solution.

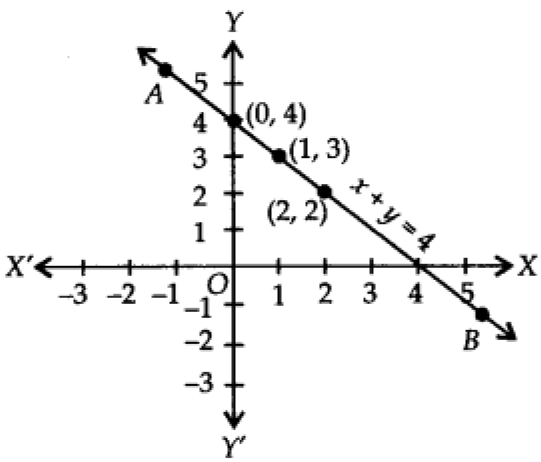
Question 4
Find the value of k, if x = 2, y = 1 ¡s a solution of the equation 2x + 3y = k.
Solution:
We have 2x + 3y = k
putting x = 2 and y = 1 in 2x+3y = k,we get
2(2) + 3(1) ⇒ k = 4 + 3 – k ⇒ 7 = k
Thus, the required value of k is 7.

Exercise 4.3

Question 1
Draw the graph of each of the following linear equations in two variables:
(i) x + y = 4
(ii) x – y = 2
(iii) y = 3x
(iv) 3 = 2x + y
Solution:
(i) x + y = 4 ⇒ y = 4 – x
If we have x = 0, then y = 4 – 0 = 4
x = 1, then y =4 – 1 = 3
x = 2, then y = 4 – 2 = 2
∴ We get the following table:

x	0	1	2
y	4	3	2

Plot the ordered pairs (0, 4), (1,3) and (2,2) on the graph paper. Joining these points, we get a straight line AB as shown.

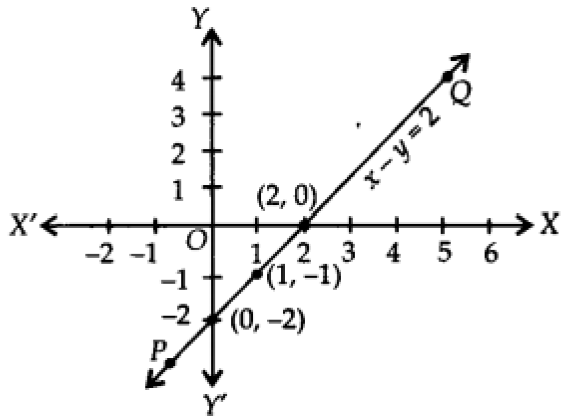


Thus, the line AB is the required graph of x + y = 4

(ii) x – y = 2 ⇒ y = x – 2
If we have x = 0, then y = 0 – 2 = -2
x = 1, then y = 1 – 2 = -1
x = 2, then y = 2 – 2 = 0
∴ We get the following table:

x	0	1	2
y	–2	–1	0

Plot the ordered pairs (0, -2), (1, -1) and (2, 0) on the graph paper. Joining these points, we get a straight line PQ as shown.

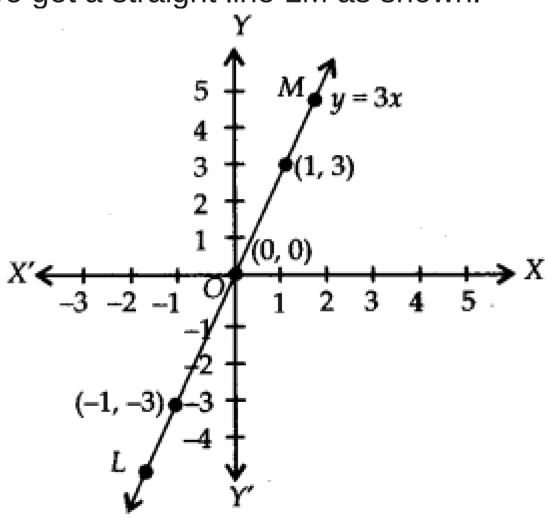


Thus, the ime is the required graph of x – y = 2

(iii) y = 3x
If we have x = 0,
then y = 3(0) ⇒ y = 0
x = 1, then y = 3(1) = 3
x= -1, then y = 3(-1) = -3
∴ We get the following table:

x	0	1	–1
y	0	3	–3

Plot the ordered pairs (0, 0), (1, 3) and (-1, -3) on the graph paper. Joining these points, we get a straight line LM as shown.

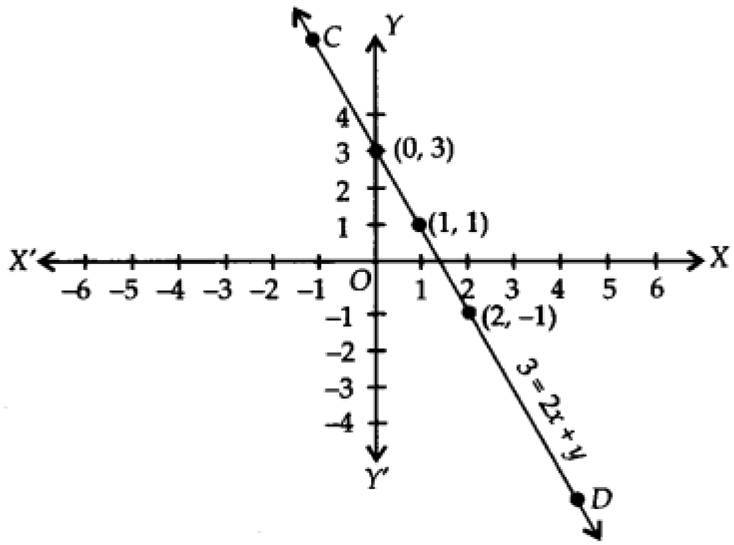


Thus, the line LM is the required graph of y = 3x.

(iv) 3 = 2x + y ⇒ y = 3 – 2x
If we have x = 0, then y = 3 – 2(0) = 3
x = 1,then y = 3 – 2(1) = 3 – 2 = 1
x = 2,then y = 3 – 2(2) = 3 – 4 = -1
∴ We get the following table:

x	0	1	2
y	3	1	–1

Plot the ordered pairs (0, 3), (1, 1) and (2, – 1) on the graph paper. Joining these points, we get a straight line CD as shown.



Thus, the line CD is the required graph of 3 = 2x + y.

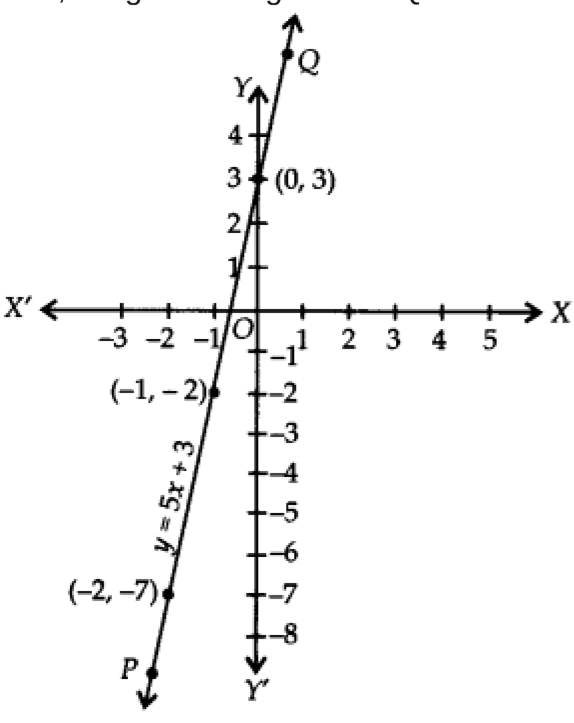
Question 2
Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?
Solution:
(2, 14) means x = 2 and y = 14
Equations which have (2,14) as the solution are (i) x + y = 16, (ii) 7x – y = 0
There are infinite number of lines which passes through the point (2, 14), because infinite number of lines can be drawn through a point.

Question 3
If the point (3, 4) lies on the graph of the equation 3y = ax + 7, find the value of a.
Solution:
The equation of the given line is 3y = ax + 7
∵ (3, 4) lies on the given line.
∴ It must satisfy the equation 3y = ax + 7
We have, (3, 4) ⇒ x = 3 and y = 4.
Putting these values in given equation, we get
3 x 4 = a x 3 + 7
⇒ 12 = 3a + 7
⇒ 3a = 12 – 7 = 5 ⇒ a = 53
Thus, the required value of a is 53
Question 4
The taxi fare In a city Is as follows: For the first kilometre, the fare Is Rs. 8 and for the

subsequent distance it is Rs. 5 per km. Taking the distance covered as x km and total fare as Rs.y, write a linear equation for this Information, and draw Its graph.
Solution:
Here, total distance covered = x km and total taxi fare = Rs. y
Fare for 1km = Rs. 8
Remaining distance = (x – 1) km
∴ Fare for (x – 1)km = Rs.5 x(x – 1)
Total taxi fare = Rs. 8 + Rs. 5(x – 1)
According to question,
$y = 8 + 5(x – 1) = y = 8 + 5x – 5$
$\Rightarrow y = 5x + 3$,
which is the required linear equation representing the given information.
Graph: We have $y = 5x + 3$
Wben x = 0, then $y = 5(0) + 3 \Rightarrow y = 3$
x = -1, then $y = 5(-1) + 3 \Rightarrow y = -2$
x = -2, then $y = 5(-2) + 3 \Rightarrow y = -7$
∴ We get the following table:

x	0	–1	–2
y	3	–2	–7

Now, plotting the ordered pairs (0, 3), (-1, -2) and (-2, -7) on a graph paper and joining them, we get a straight line PQ as shown.



Thus, the line PQ is the required graph of the linear equation y = 5x + 3.

Question 5
From the choices given below, choose the equation whose graphs are given ¡n Fig. (1)

and Fig. (2).
For Fig. (1)
(i) y = x
(ii) x + y = 0
(iii) y = 2x
(iv) 2 + 3y = 7x

For Fig. (2)
(i) y = x + 2
(ii) y = x – 2
(iii) y = -x + 2
(iv) x + 2y = 6

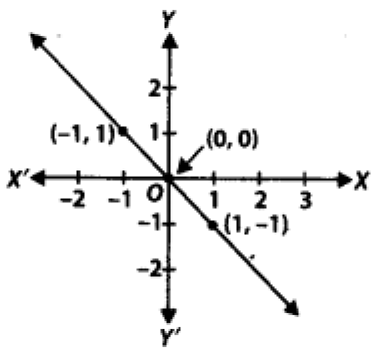


Fig. (1)

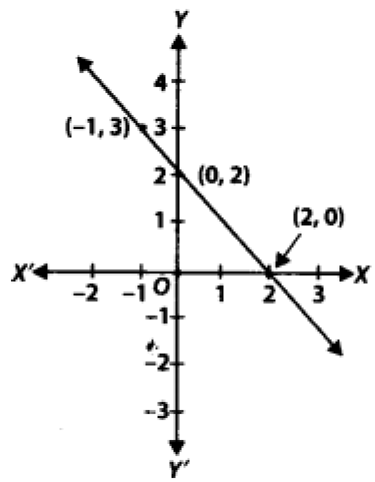


Fig. (2)

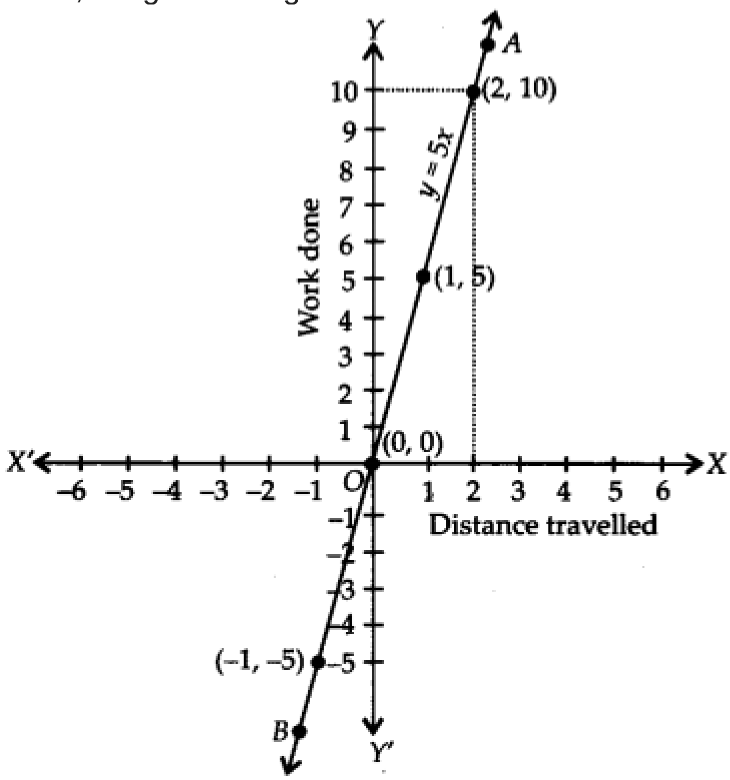
Solution:
For Fig. (1), the correct linear equation is x + y = 0
[As (-1, 1) = -1 + 1 = 0 and (1,-1) = 1 + (-1) = 0]
For Fig.(2), the correct linear equation is y = -x + 2
[As(-1,3) 3 = -1(-1) + 2 = 3 = 3 and (0,2)
⇒ 2 = -(0) + 2 ⇒ 2 = 2]

Question 6
If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is
(i) 2 units
(ii) 0 unit
Solution:
Constant force is 5 units.
Let the distance travelled = x units and work done = y units.
Work done = Force x Distance
⇒ y = 5 x x ⇒ y = 5x

For drawing the graph, we have y = 5x
When x = 0, then y = 5(0) = 0
x = 1, then y = 5(1) = 5
x = -1, then y = 5(-1) = -5
∴ We get the following table:

x	0	1	–1
y	0	5	–5

Ploffing the ordered pairs (0, 0), (1, 5) and (-1, -5) on the graph paper and joining the points, we get a straight line AB as shown.



From the graph, we get
(i) Distance travelled =2 units i.e., x = 2
∴ If x = 2, then y = 5(2) = 10
⇒ Work done = 10 units.

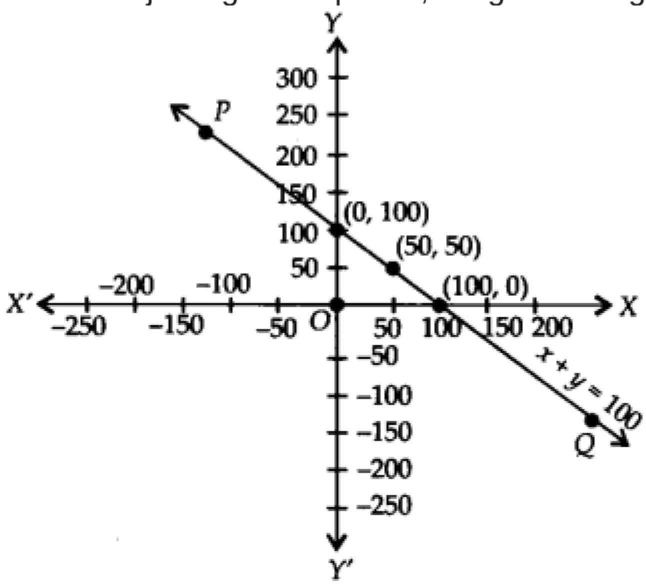
(ii) Distance travelled = 0 unit i.e., x = 0
∴ If x = 0 ⇒ y = 5(0) – 0
⇒ Work done = 0 unit.

Question 7
Yamini and Fatima, two students of Class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as Rs.xand Rs.y.) Draw the graph of the same.

Solution:
Let the contribution of Yamini = Rs. x
and the contribution of Fatima Rs. y
∴ We have x + y = 100 ⇒ y = 100 – x
Now, when x = 0, y = 100 – 0 = 100
x = 50, y = 100 – 50 = 50
x = 100, y = 100 – 100 = 0
∴ We get the following table:

x	0	50	100
y	100	50	0

Plotting the ordered pairs (0,100), (50,50) and (100, 0) on a graph paper using proper scale and joining these points, we get a straight line PQ as shown.



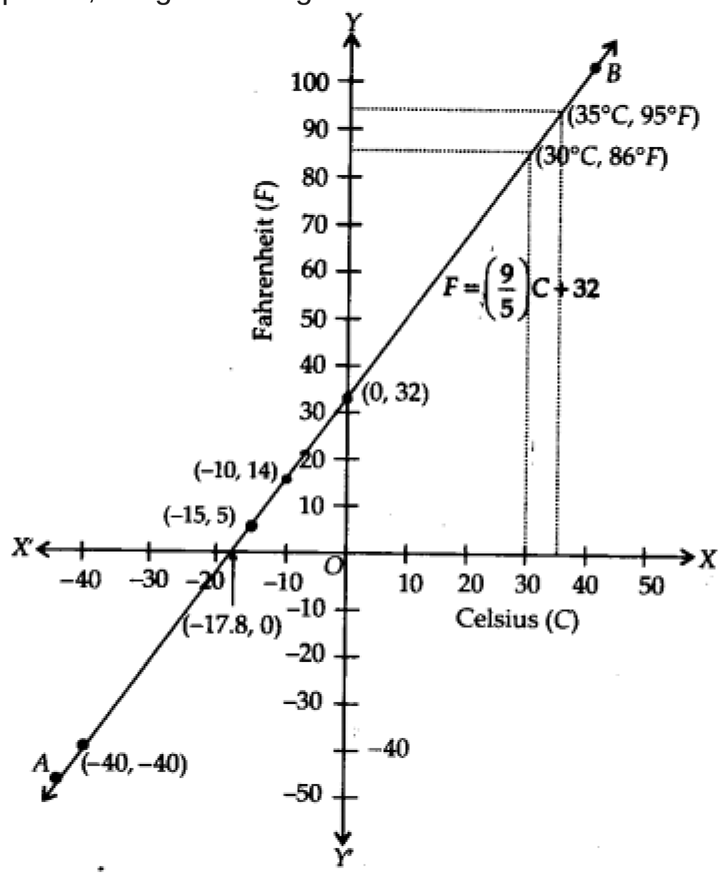
Thus, the line PQ is the required graph of the linear equation x + y = 100.

Question 8
In countries like USA and Canada, temperature is measured In Fahrenheit, whereas in countries like India, it is measured in Celsius. Here Is a
linear equation that converts Fahrenheit to Celsius:
$F = (\frac{9}{5})C + 32$
(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
(ii) If the temperature Is 30°C, what is the temperature in Fahrenheit?
(iii) If the temperature is 95°F, what is the temperature in Celsius?
(iv) If the temperature is 0°C, what Is the temperature In Fahrenheit and If the temperature is 0°F, what Is the temperature In Celsius?
(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find It.
Solution:
(i) We have

F = (95)C + 32
When C = 0 , F = (95) x 0 + 32 = 32
When C = 15, F = (95)(-15) + 32= -27 + 32 = 5
When C = -10, F = 95 (-10)+32 = -18 + 32 = 14
We have the following table:

C	0	–15	–10
F	32	5	14

Plotting the ordered pairs (0, 32), (-15, 5) and (-10,14) on a graph paper. Joining these points, we get a straight line AB.



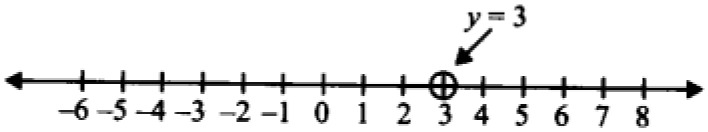
(ii) From the graph, we have 86°F corresponds to 30°C.
(iii) From the graph, we have 95°F corresponds 35°C.
(iv) We have, C = 0
From (1), we get
F = (95)0 + 32 = 32
Also, F = 0
From (1), we get
0 = (95)C + 32 ⇒ −32×59 = C ⇒ C = -17.8
(V) When F = C (numerically)
From (1), we get
F = 95F + 32 ⇒ F – 95F = 32

⇒ −45F = 32 ⇒ F = -40
∴ Temperature is – 40° both in F and C.

Exercise 4.4

Question 1
Give the geometric representations of y = 3 as an equation
(i) in one variable
(ii) in two variables

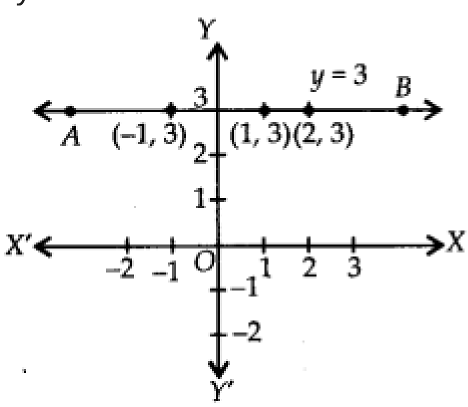
Solution:
(i) y = 3
∵ y = 3 is an equation in one variable, i.e., y only.
∴ y = 3 is a unique solution on the number line as shown below:



(ii) y = 3
We can write y = 3 in two variables as 0.x + y = 3
Now, when x = 1, y = 3
x = 2, y = 3
x = -1, y = 3
∴ We get the following table:

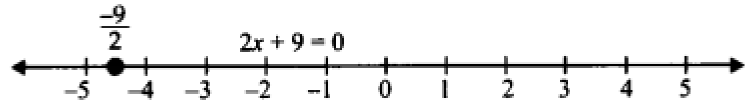
x	1	2	–1
y	3	3	3

Plotting the ordered pairs (1, 3), (2, 3) and (-1, 3) on a graph paper and joining them, we get aline AB as solution of 0. x + y = 3,
i.e. y = 3.



Question 2
Give the geometric representations of 2x + 9 = 0 as an equation
(i) in one variable

(ii) in two variables
Solution:
(i) 2x + 9 = 0
We have, 2x + 9 = 0 ⇒ 2x = – 9 ⇒ x = $\frac{-9}{2}$
which is a linear equation in one variable i.e., x only.
Theref ore, x = $-\frac{9}{2}$ is a unique solution on the number line as shown below:



(ii) 2x +9=0
We can write 2x + 9 = 0 in two variables as 2x + 0, y + 9 = 0
or $x = \frac{-9-0.y}{2}$
∴ When y = 1, x = $x = \frac{-9-0(1)}{2} = -\frac{9}{2}$

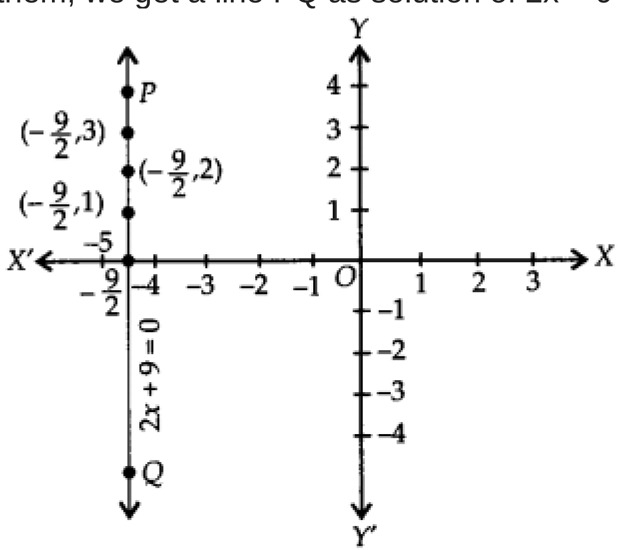
$$y = 2,\ x = \frac{-9-0(2)}{2} = \frac{-9}{2}$$

$$y = 3,\ x = \frac{-9-0(3)}{2} = \frac{-9}{2}$$

Thus, we get the following table:

x	$\frac{-9}{2}$	$\frac{-9}{2}$	$\frac{-9}{2}$
y	1	2	3

Now, plotting the ordered pairs $\left(\frac{-9}{2}, 3\right)$,$\left(\frac{-9}{2}, 3\right)$ and $\left(\frac{-9}{2}, 3\right)$ on a graph paper and joining them, we get a line PQ as solution of 2x + 9 = 0.



DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER4 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

MIND MAP

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 FORMATION OF LINEAR EQUATIONS IN TWO VARIABLES

Q.1 Write the equation $12x + 3y = 20$ in the form of $ax + by + c = 0$ and find out the values of a, b and c.

Q.2 Write $2x = 3y + 5$ in standard form of equation in two variables.

Q.3 Write the equation of the line parallel to the y-axis.

Q.4 Write the equation of the line parallel to x-axis.

TYPE: 2 GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

Q.1 The taxi fare in a city is as follows: For the first kilometer, the fare is ₹20 and for the subsequent distance it is ₹6 per km. Taking x km as the distance covered and ₹y as the total fare, write a linear equation for this information and draw its graph.

Q.2 Draw the graph of equation $3x + 4y = 24$.

Q.3 Draw the graph of equation $2y + 5 = 0$.

Chapter 3

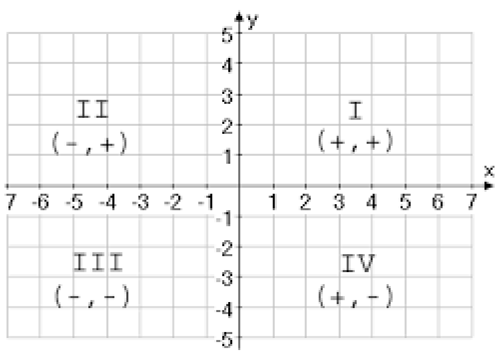
Coordinate Geometry

What is Coordinate Geometry?

It a branch of geometry which sets up a definite correspondence between the position of a point in a plane and a pair of algebraic numbers called co-ordinates.

Important points

(1) We require two perpendicular axes to locate a point in the plane. One of them is horizontal and other is vertical.
(2) The plane is called Cartesian plane and axis are called the coordinates axis.
(3) The horizontal axis is called x-axis and Vertical axis is called Y-axis
(4) The point of intersection of axis is called origin.



(5) The distance of a point from y axis is called x - coordinate or abscissa and the distance of the point from x axis is called y - coordinate or Ordinate.

(6) The x-coordinate and y - coordinate of the point in the plane is written as (x, y) for point and is called the coordinates of the point.

(7) The Origin has zero distance from both x-axis and y-axis so that its abscissa and ordinate both are zero. So the coordinate of the origin is (0, 0).

(8) A point on the x - axis has zero distance from x-axis so coordinate of any point on the x - axis will be (x, 0).

(9) A point on the y - axis has zero distance from y-axis so coordinate of any point on the y - axis will be (0, y).

(10) The axes divide the Cartesian plane in to four parts. These Four parts are called the quadrants.

(11) The coordinates of the points in the four quadrants will have sign according to the below table

Quadrant	x-coordinate	y-coordinate
Ist Quadrant	+	+
IInd quadrant	-	+
IIIrd quadrant	-	-
IVth quadrant	+	-

PRACTICE SUMS

Given below are the important Questions for Coordinate Geometry

1. State the quadrant for each of these points in Cartesian plane

a. (2,3)
b. (3,-9)
c. (-9,-8)
d. (1,1)
e. (-2,7)
f. (2,0)
g. (0,3)

2. Plot the following points in the Cartesian plane

a (9, 8)

b (-1,-1)

c (6, 6)
d (4, 4)
Also find which of these three lie are collinear

3) **True or False statement**
a) x –coordinate is positive in Ist and IIIrd quadrants
b) The (0,0) is the coordinate of origin
c) The point (0,2) lies on y axis
d) The ordinate of the point Q (2,3) is 2
e) Abscissa of all points on y axis is zero
f) The points P (2,3) and Q (-3,2) lie in the same quadrant

Exercise 3.1

Question 1

How will you describe the position of a table lamp on your study table to another person?

Answer

We can use the concept of Coordinate Geometry to describe the position of a table lamp on the study table. we have to take two lines, a perpendicular and horizontal. Considering the table as a plane and taking perpendicular line as Y axis and horizontal as X axis. Take one corner of table as origin where both X and Y axes intersect each other. Now, the length of table is Y axis and breadth is X axis. From The origin, join the line to the lamp and mark a point. Calculate the distance of this point from both X and Y axes and then write it in terms of coordinates.
Let the distance of point from X axis is x and from Y axis is y then the position of the table lamp in terms of coordinates is (x, y).

Question 2

(Street Plan): A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

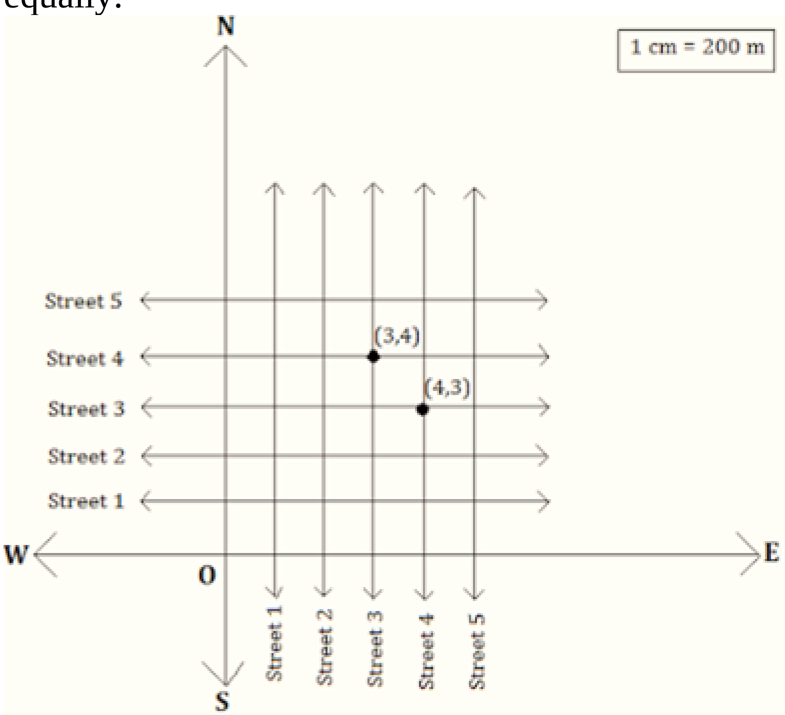
All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1cm = 200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines. There are many cross- streets in your model. A particular cross-street is made by two streets, one running in the North - South direction and another in the East - West direction. Each cross street is referred to in the following manner: If the 2nd street running in the North - South direction and 5th in the East - West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) How many cross - streets can be referred to as (4, 3).

(ii) How many cross - streets can be referred to as (3, 4)

Answer

Let us taken X axis along S-N direction and Y axis along W-E direction. Now drawing the 5 lines representing roads parallel to X and Y axis each separated equally.



(i) Only one street can be referred to as (4, 3) as we see from the figure.

(ii) Only one street can be referred to as (3, 4) as we see from the figure.

Exercise 3.2

Question 1

Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

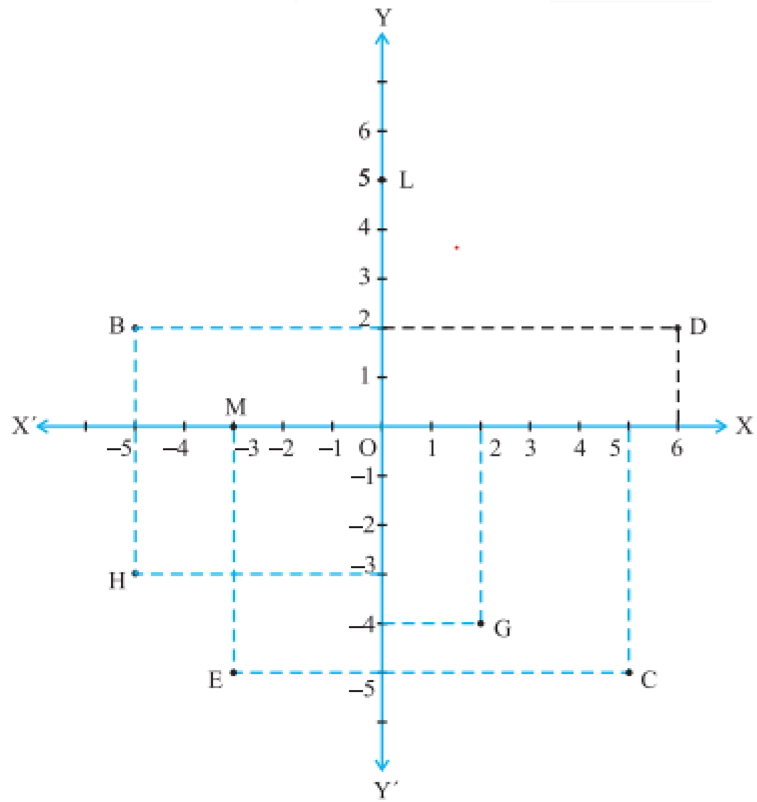
(ii) What is the name of each part of the plane formed by these two lines?
(iii) Write the name of the point where these two lines intersect.

Answer

What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?	The name of horizontal lines and vertical lines drawn to determine the position of any point in the Cartesian plane is x-axis and y-axis respectively.
What is the name of each part of the plane formed by these two lines?	The name of each part of the plane formed by these two lines x-axis and y-axis is quadrants.
Write the name of the point where these two lines intersect	The point where these two lines intersect is called origin.

Question 2

Refer the below figure and write the following:



(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (-3, -5).
(iv) The point identified by the coordinates (2, -4).
(v) The abscissa of the point D.
(vi) The ordinate of the point H.
(vii)The coordinates of the point L.
(viii) The coordinates of the point M.

Answer

1) The distance of a point from y axis is called x –coordinate or abscissa and the distance of the point from x –axis is called y – coordinate or Ordinate

2) The x-coordinate and y –coordinate of the point in the plane is written as (x, y) for point and is called the coordinates of the point

3) A point on the x –axis has zero distance from x-axis so coordinate of any point on the x-axis will be (x, 0)

4) A point on the y –axis has zero distance from y-axis so coordinate of any point on the y-axis will be (0, y)

i)	The coordinates of B is (-5, 2)
ii)	The coordinates of C is (5, -5)
iii)	The point identified by the coordinates (-3, -5) is E.
iv)	The point identified by the coordinates (2, -4) is G.
v)	Abscissa means x coordinate of point D. So, abscissa of the point D is 6.
vi)	Ordinate means y coordinate of point H. So, ordinate of point H is -3.
vii)	The coordinates of the point L is (0, 5).
viii)	The coordinates of the point M is (- 3, 0).

Exercise 3.3

Question 1

In which quadrant or on which axis do each of the points (-2, 4), (3, -1), (-1, 0), (1, 2) and (-3, -5) lie? Verify your answer by locating them on the Cartesian plane.

Answer

We know the quadrant signs are given below

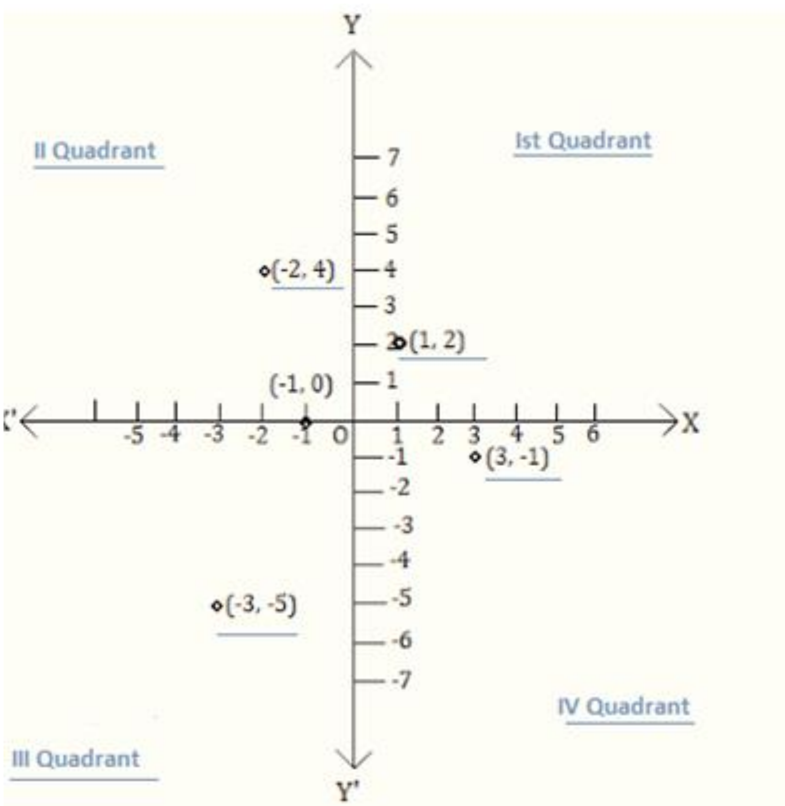
Quadrant	**x-coordinate**	**y-coordinate**
Ist Quadrant	+	+
IInd quadrant	-	+

IIIrd quadrant	-	-
IVth quadrant	+	-

So on that basis we can derive the answer as

(-2, 4)	Second quadrant
(3, -1)	Fourth quadrant
(-1, 0)	Second Quadrant and X axis
(1, 2)	First quadrant
(-3, -5)	Third quadrant

Now Let us draw the Cartesian plane as given below to verify the answer



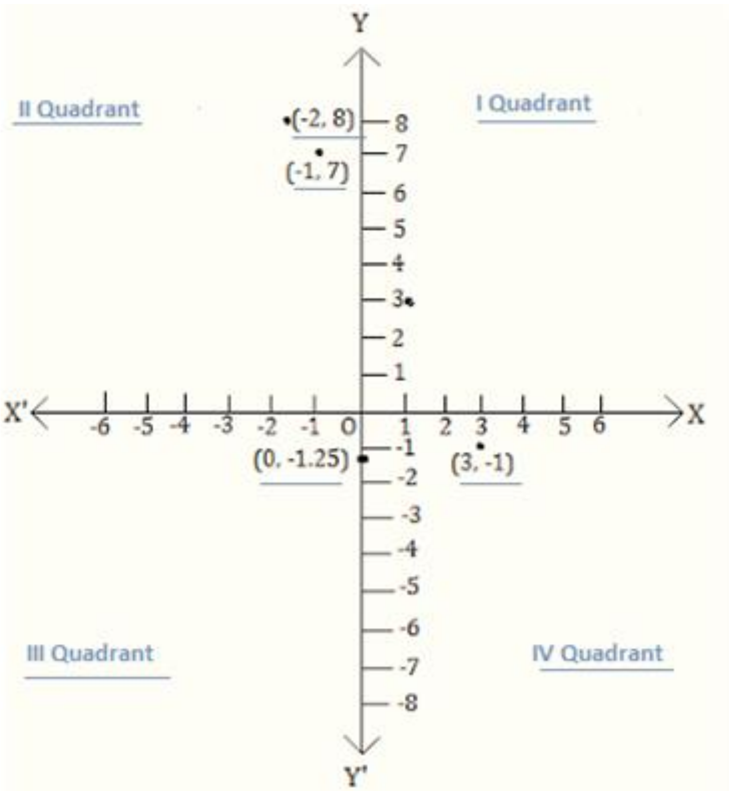
Question 2

Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	-2	-1	0	1	3
y	8	7	-1.25	3	-1

Answer

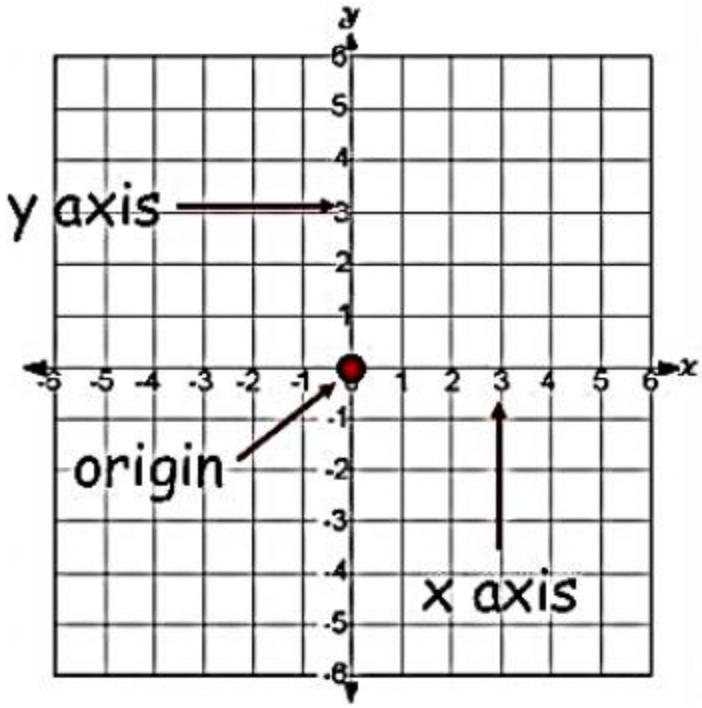
We can draw the Cartesian plane by taking 1 unit as 1cm



Revision Notes on Coordinate Geometry

Cartesian System

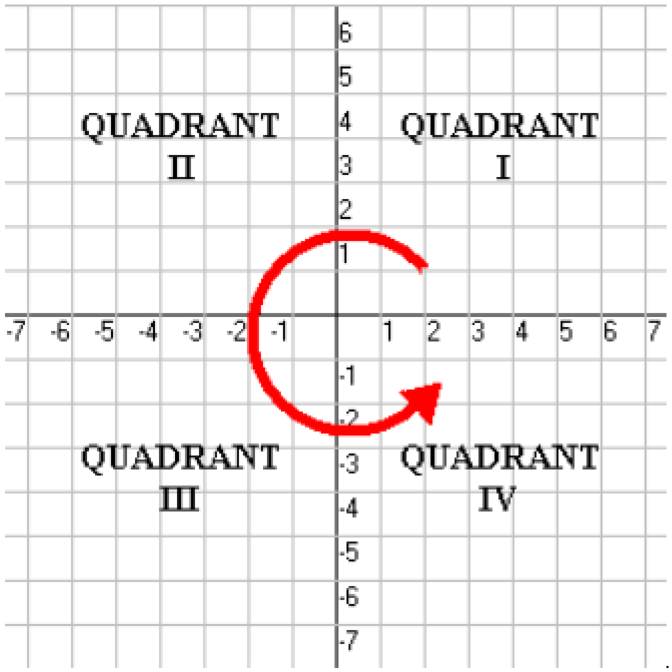
If we take two number lines, one horizontal and one vertical, and then combine them in such a way that they intersect each other at their zeroes, and then they form a **Cartesian Plane.**



- The horizontal line is known as **the x-axis** and the vertical line is known as the **y-axis**.
- The point where these two lines intersects each other is called the **origin**. It is represented as ‘O’.
- OX and OY are the positive directions as the positive numbers lie in the right and upward direction.
- Similarly, the left and the downward directions are the negative directions as all the negative numbers lie there.

Quadrants of the Cartesian Plane

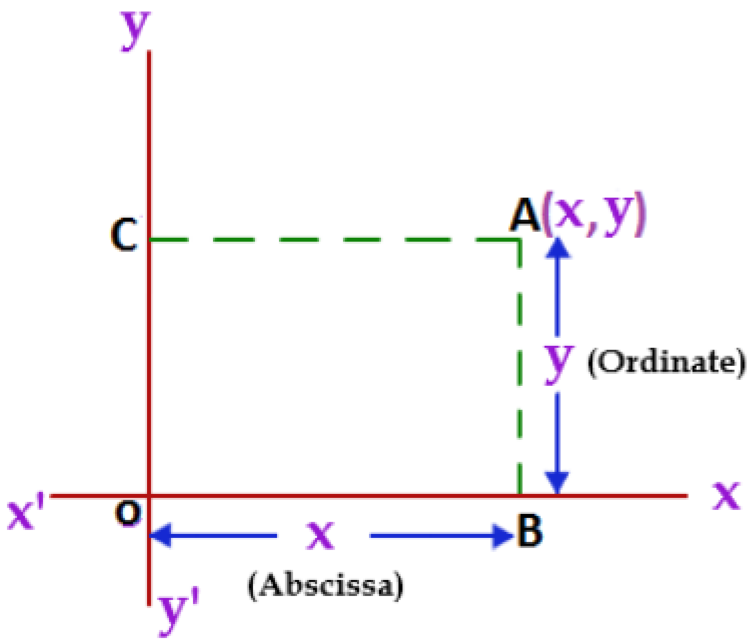
The Cartesian plane is dividing into four quadrants named as **Quadrant I, II, III, and IV** anticlockwise from OX.



Coordinates of a Point

To write the coordinates of a point we need to follow these rules-

- The **x - coordinate** of a point is marked by drawing perpendicular from the y-axis measured a length of the x-axis .It is also called the **Abscissa.**
- The **y - coordinate** of a point is marked by drawing a perpendicular from the x-axis measured a length of the y-axis .It is also called the **Ordinate.**
- While writing the coordinates of a point in the coordinate plane, the x - coordinate comes first, and then the y - coordinate. We write the coordinates in brackets.

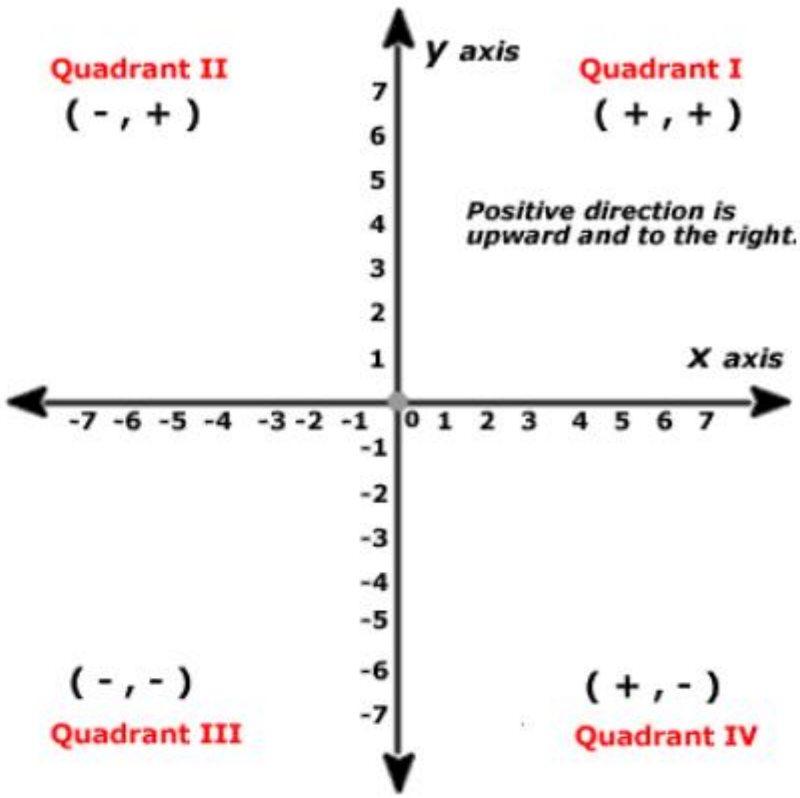


In the above figure, OB = CA = x coordinate (Abscissa), and CO = AB = y coordinate (Ordinate).

We write the coordinate as (x, y).

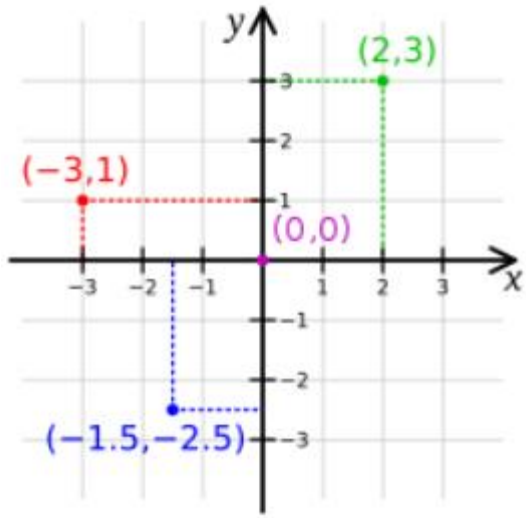
Remark: As the origin O has zero distance from the x-axis and the y-axis so its abscissa and ordinate are zero. Hence the **coordinate of the origin is (0, 0)**.

The relationship between the signs of the coordinates of a point and the quadrant of a point in which it lies.



Quadrant	Coordinate	Sign in the quadrant
I	(+, +)	1st quadrant is enclosed by the positive x-axis and the positive y-axis.
II	(-, +)	2nd quadrant is enclosed by the negative x-axis and the positive y-axis.
III	(-, -)	3rd quadrant is enclosed by the negative x-axis and the negative y-axis.
IV	(+, -)	4th quadrant is enclosed by the positive x-axis and the negative y-axis

Plotting a Point in the Plane if its Coordinates are Given



Steps to plot the point (2, 3) on the Cartesian plane -

- First of all, we need to draw the Cartesian plane by drawing the coordinate axes with 1 unit = 1 cm.
- To mark the x coordinates, starting from 0 moves towards the positive x-axis and count to 2.
- To mark the y coordinate, starting from 2 moves upwards in the positive direction and count to 3.
- Now this point is the coordinate (2, 3)

Likewise, we can plot all the other points, like (-3, 1) and (-1.5,-2.5) in the right site figure.

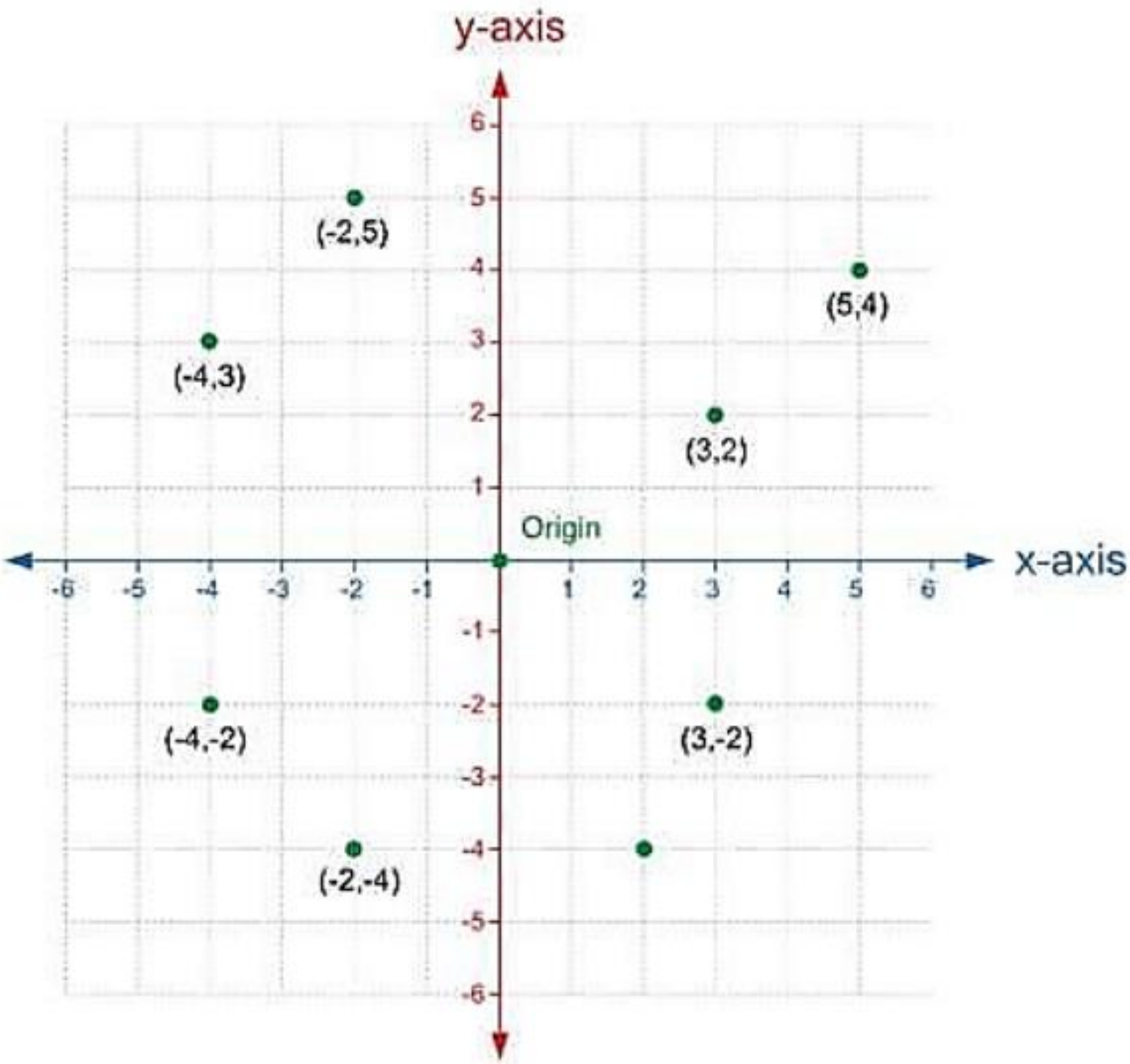
Is the coordinates (x, y) = (y, x)?

Let x = (-4) and y = (-2)

So (x, y) = (- 4, – 2)

(y, x) = (- 2, - 4)

Let’s mark these coordinates on the Cartesian plane.



You can see that the positions of both the points are different in the Cartesian plane. So,

If x ≠ y, then (x, y) ≠ (y, x), and (x, y) = (y, x), if x = y.
Example:

Plot the points (6, 4), (- 6,- 4), (- 6, 4) and (6,- 4) on the Cartesian plane.

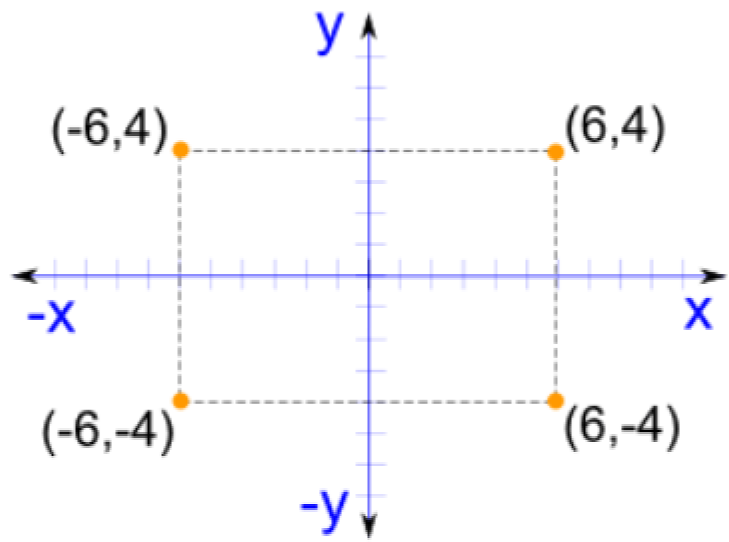
Solution:

As you can see in (6, 4) both the numbers are positive so it will come in the first quadrant.

For x coordinate, we will move towards the right and count to 6.

Then from that point go upward and count to 4.

Mark that point as the coordinate (6, 4).



Similarly, we can plot all the other three points.

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CH. 3 COORDINATE GEOMETRY

MIND MAP

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 COORDINATES OF A POINT

Q.1 Write the perpendicular distance of point P (3, 4) from the y –axis.

Q.2 Write the point which lies on y-axis at a distance of 5 units in the negative direction of y-axis

Q.3 Write the point whose ordinate is 4 and which lies on y-axis.

Q.4 The points whose abscissa and ordinate have different signs will lie in which quadrant?

Q.5 Write the abscissa of all the points on the x-axis.

Q.6 If the coordinates of the two points are P (–2, 3) and Q (–3, 5), then find

(abscissa of P) – (abscissa of Q).

Q.7 What is the name to the horizontal and vertical line in a coordinate system?

The origin is indicated by what coordinates?

Q.8 How many quadrants are there in the Cartesian Plane?

Q.9 In which quadrant will the coordinates (-2,3) lie?

Q.10 In which quadrant will the coordinates (-3, -4) lie?

Q.11 What is the abscissa and the ordinate in the coordinates (3, -5)

Q.12 Write the abscissa and the ordinate of the coordinates of the points (0,3) (3,0) (0, 0).

TYPE: 2 PLOTTING OF POINTS IN THE CARTESIAN PLANE

Q.1 Plot the following point on the number line using a graph and join the points.

(a) (3, -4) (b) (-3, 2)

Q.2 Plot the points P (1, 0), Q (4, 0) and S (1, 3). Find the coordinates of the point R such that PQRS is a square.

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MIND MAP

CH.1 NUMBER SYSTEMS

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE: 1 RATIONAL AND IRRATIONAL NUMBERS

Q.1. Find 5 rational numbers between $\frac{3}{4}$ and $\frac{5}{8}$.

Q.2 Find two irrational numbers between 1.5 and 1.6.

Q.3 Represent $\sqrt{11}$, $\sqrt{13}$ and $\sqrt{5.6}$ on the number line.

Q.4 Express $0.56\overline{28}$ in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$

TYPE: 2 POWERS AND EXPONENTS

Q.1 Find the value of $\frac{3^{49}+3^{50}-9^{24}}{3^{48}+\ 3^{47}+9^{23}}$

Q.2 Prove that $\frac{2}{1+x^{2a-2b}}+\frac{2}{1+x^{2b-2a}}=2$

Q.3 Prove that $\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}=1$

Q.4 Simplify: $\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

TYPE: 3 RATIONALIZING THE DENOMINATOR

Q.1. Find the value of a and b in $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}=\frac{a}{2}+\frac{b\sqrt{5}}{2}$

Q.2 If $x=2+\sqrt{3}$, find the value of $x^2+\frac{1}{x^2}$

Q.3 If $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x^2+y^2

Ch. 1 Number Systems

Exercise 1.1

Question 1 :

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q≠0

Answer:

Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2:

Find six rational numbers between 3 and 4.

Answer:

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$ respectively.

Therefore, rational numbers between 3 and 4 are

$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

There are infinite rational numbers between . $\frac{3}{5}$ and $\frac{4}{5}$

$$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$$

Therefore, the required rational numbers are

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer:

(i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: $\frac{1}{5}$ is a rational number but not a whole number.

Exercise 1.2 Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer:

(i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer:

If numbers such as $\sqrt{4} = 2,\ \sqrt{9} = 3$ are considered,
Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

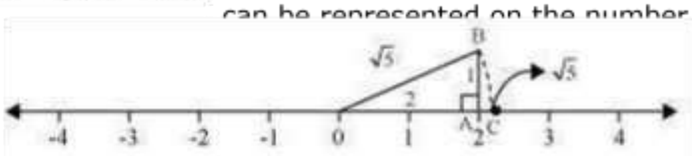
Question 3:

Show how $\sqrt{5}$ can be represented on the number line.

Answer:

We know that, $\sqrt{4} = 2$

And, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$



Mark a point 'A' representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw an arc intersecting number line at C.

C is representing $\sqrt{5}$.

has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909...... = 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769.... = 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818........ = 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

Question 2:

You know that $\frac{1}{7} = 0.\overline{142857}$

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

Exercise 1.3 Question 1:

Write the following in decimal form and say what kind of decimal expansion each . Can you predict what the decimal expansion of are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.] Answer:

Yes. It can be done as follows.

$$\frac{2}{7} = 2\times\frac{1}{7} = 2\times 0.\overline{142857} = 0.\overline{285714}$$
$$\frac{3}{7} = 3\times\frac{1}{7} = 3\times 0.\overline{142857} = 0.\overline{428571}$$
$$\frac{4}{7} = 4\times\frac{1}{7} = 4\times 0.\overline{142857} = 0.\overline{571428}$$
$$\frac{5}{7} = 5\times\frac{1}{7} = 5\times 0.\overline{142857} = 0.\overline{714285}$$
$$\frac{6}{7} = 6\times\frac{1}{7} = 6\times 0.\overline{142857} = 0.\overline{857142}$$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6} = 0.666...$

Let x = 0.666...

10x = 6.666...

10x = 6 + x

9x = 6

$$x = \frac{2}{3}$$

(ii) $0.4\overline{7} = 0.4777.....$

$$= \frac{4}{10} + \frac{0.777}{10}$$

Let x = 0.777...
10x = 7.777...

10x = 7 + x

$$x = \frac{7}{9}$$

$$\frac{4}{10} + \frac{0.777...}{10} = \frac{4}{10} + \frac{7}{90} = \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001...$

Let x = 0.001001...

1000x = 1.001001...

1000x = 1 + x

999x = 1

$$x = \frac{1}{999}$$

Question 4:

Express 0.99999...in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer:

Let x = 0.9999...
10x = 9.9999...

$10x = 9 + x$

$9x = 9$ $x =$

1

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$

$$\frac{11}{8} = 1.375$$

$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.
Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005...

0.7207200720007200007200000... 0.080080008000080000080000008...

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
Answer:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are as follows.

0.73073007300073000073...

0.75075007500075000075... 0.79079007900079000079...

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001...

(i) $\sqrt{23} = 4.79583152331\ldots$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 … $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 …
As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

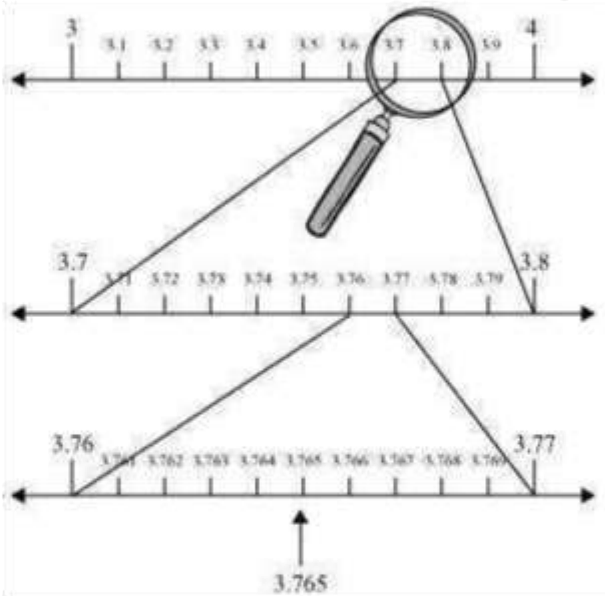
Exercise 1.4 Question

1:

Visualise 3.765 on the number line using successive magnification.

Answer:

3.765 can be visualised as in the following steps.



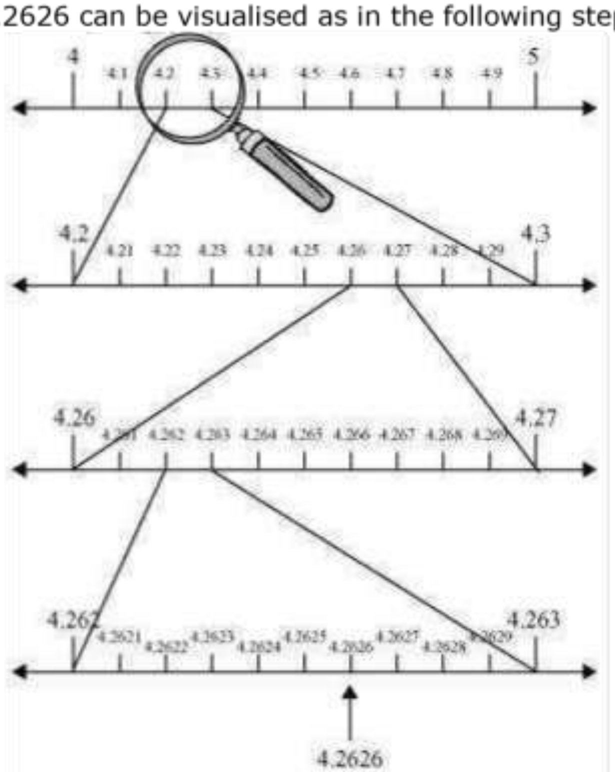
Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer:

$4.\overline{26}$ = 4.2626…

4.2626 can be visualised as in the following steps.



Exercise 1.5 Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ = 2 − 2.2360679…

= − 0.2360679…

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

form, therefore, it is a rational

number. form, therefore, it is a

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$ rational number.

As it can be represented in $\frac{p}{q}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$

(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811...$ As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number. (v) 2π = 2(3.1415 …)

= 6.2830 …

As the decimal expansion of this expression is non-terminating non-recurring, therefore,

it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Answer:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=(3)^2-\left(\sqrt{3}\right)^2$

= 9 − 3 = 6

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{2}\right)$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$

= 5 − 2 = 3

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore, the $\frac{c}{d}$ fraction is irrational. Hence, π is irrational.

Question 4:

Represent on the number $\sqrt{9.3}$ line. Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the midpoint

D of OC and draw a semi-circle on OC while taking D as its centre. Draw a

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

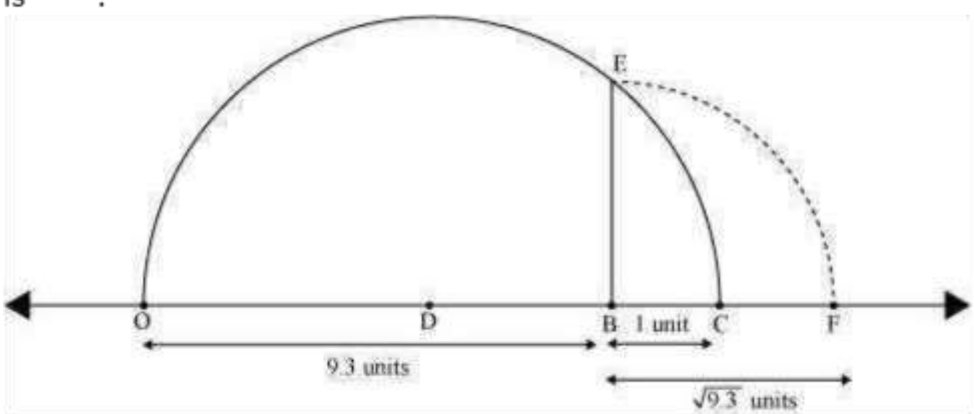
(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}}=\frac{1\times\sqrt{7}}{1\times\sqrt{7}}=\frac{\sqrt{7}}{7}$

perpendicular to line OC passing through point B. Let it intersect the semi-circle at E.

Taking B as centre and BE as radius, draw an arc in tersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(ii)

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}\left(\sqrt{7}+\sqrt{6}\right)$$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

(iii)

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}\left(\sqrt{5}-\sqrt{2}\right)$$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv)

$$\frac{1}{\sqrt{7}-2}=\frac{1}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}\left(\sqrt{7}+2\right)$$

$$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$$

Exercise 1.6 Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:

Answer:

(i)

$$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}}$$
$$= 2^{6\times\frac{1}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$
$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^1 = 2$$

(iii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}$$
$$= 5^{3\times\frac{1}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 5^1 = 5$$

Question 2:

(i)

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}}$$
$$= 3^{2\times\frac{3}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 3^3 = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}}$$
$$= 2^{5\times\frac{2}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^2 = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}}$$
$$= 2^{4\times\frac{3}{4}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$
$$= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}$$
$$= \frac{1}{5^{3\times\frac{1}{3}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \frac{1}{5}$$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer:

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$

$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times 7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = \left(7\times 8\right)^{\frac{1}{2}} \qquad \left[a^m.b^m = \left(ab\right)^m\right]$$

$$= \left(56\right)^{\frac{1}{2}}$$

CHAPTER 2
POLYNOMIALS

Introduction

In this chapter, we shall study a particular type of algebraic expression, called polynomial, the Remainder Theorem and Factor Theorem and their use in the factorisation of polynomials.

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH. 2 POLYNOMIALS

MIND MAP

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE: 1 RATIONAL AND IRRATIONAL NUMBERS

Q.1. Find 5 rational numbers between $\frac{3}{4}$ and $\frac{5}{8}$.

Q.2 Find two irrational numbers between 1.5 and 1.6.

Q.3 Represent $\sqrt{11}$, $\sqrt{13}$ and $\sqrt{5.6}$ on the number line.

Q.4 Express $0.56\overline{28}$ in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$

TYPE: 2 POWERS AND EXPONENTS

Q.1 Find the value of $\frac{3^{49}+3^{50}-9^{24}}{3^{48}+\ 3^{47}+9^{23}}$

Q.2 Prove that $\frac{2}{1+x^{2a-2b}}+\frac{2}{1+x^{2b-2a}}=2$

Q.3 Prove that $\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}=1$

Q.4 Simplify: $\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

TYPE: 3 RATIONALIZING THE DENOMINATOR

Q.1. Find the value of $a\ and\ b\ in\ \frac{7+3\sqrt{5}}{7-3\sqrt{5}}=\frac{a}{2}+\frac{b\sqrt{5}}{2}$

Q.2 If $x=2+\sqrt{3}$, find the value of $x^2+\frac{1}{x^2}$

Q.3 If $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x^2+y^2

Exercise 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since x is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 – 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation $y^2 + \sqrt{2}$ can be written as $y^2 + \sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{\frac{1}{2}} + \sqrt{2}t$

Though, t is the only variable in the given equation, the powers of t (i.e., $\frac{1}{2}$) is not a whole number.

Hence, we can say that the expression $3\sqrt{t} + t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) $y + \frac{2}{y}$

Solution:

The equation $y + \frac{2}{y}$ can be written as $y+2y^{-1}$

Though, y is the only variable in the given equation, the powers of y (i.e.,-1) is not a whole number.

Hence, we can say that the expression $y + \frac{2}{y}$ is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)\, x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

∴, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1)\, x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

∴, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\frac{\pi}{2}x^2 + x$

Solution:

The equation $\frac{\pi}{2}x^2 + x$ can be written as $(\frac{\pi}{2})\, x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\frac{\pi}{2}$

∴, the coefficients of x^2 in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

(iv) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2 + \sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

∴, the coefficients of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

∴, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 - y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 - y^2$,

The power of the variable y is: 2

∴, the degree of $4 - y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t - \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t - \sqrt{7}$,

The power of the variable y is: 1

∴, the degree of $5t - \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3 = 3 \times 1 = 3 \times x^0$

The power of the variable here is: 0

∴, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

∴, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) **$x - x^3$**

Solution:

The highest power of $x - x^3$ is 3

∴, the degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) **$y + y^2 + 4$**

Solution:

The highest power of $y + y^2 + 4$ is 2

∴, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) **$1 + x$**

Solution:

The highest power of $1 + x$ is 1

∴, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) **$3t$**

Solution:

The highest power of $3t$ is 1

∴, the degree is 1

Hence, $3t$ is a linear polynomial

(vi) **r^2**

Solution:

The highest power of r^2 is 2

∴, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) **$7x^3$**

Solution:

The highest power of $7x^3$ is 3

∴, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2

1. Find the value of the polynomial (x)=$5x-4x^2+3$

(i) x= 0

(ii) x = – 1

(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When x=0

$f(0)=5(0)+4(0)^2+3$
$=3$

(ii) When x= -1

$f(x)=5x-4x^2+3$
$f(-1)=5(-1) -4(-1)^2+3$
$=-5-4+3$
$=-6$

(iii) When x=2

$f(x)=5x-4x^2+3$
$f(2)=5(2) -4(2)^2+3$
$=10-16+3$
$=-3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$
$\therefore p(0)=(0)^2-(0)+1=1$
$p(1)=(1)^2-(1)+1=1$
$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$
$\therefore p(0)=2+0+2(0)^2-(0)^3=2$
$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$
$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii)$p(x)=x^3$

Solution:

$p(x)=x^3$
$\therefore p(0)=(0)^3=0$
$p(1)=(1)^3=1$
$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1,\ x=-\frac{1}{3}$

Solution:

For, $x=-\frac{1}{3}$, $p(x)=3x+1$

$\therefore p(-\frac{1}{3})=3(-\frac{1}{3})+1=-1+1=0$

$\therefore -\frac{1}{3}$ is a zero of $p(x)$.

(ii) $p(x)=5x-\pi,\ x=\frac{4}{5}$

Solution:

For, $x=\frac{4}{5}$ $p(x)=5x-\pi$

$\therefore p(\frac{4}{5})=5(\frac{4}{5})-\pi=4-\pi$

$\therefore \frac{4}{5}$ is not a zero of $p(x)$.

(iii) $p(x)=x^2-1,\ x=1,\ -1$

Solution:

For, $x=1,\ -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1,\ -1$ are zeros of $p(x)$.

(iv) $p(x)=(x+1)(x-2),\ x=-1,\ 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of $p(x)$.

(v) $p(x)=x^2,\ x=0$

Solution:

For, x=0 p(x)= x^2
$p(0)=0^2=0$
∴0 is a zero of p(x).

(vi)p(x)=lx+m, x=$-\frac{m}{l}$

Solution:

For, x=$-\frac{m}{l}$; p(x)=lx+m
∴$p(-\frac{m}{l})=l(-\frac{m}{l})+m=-m+m=0$
∴$-\frac{m}{l}$ is a zero of p(x).

(vii) p(x)=$3x^2-1$,x=$-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Solution:

For, x=$-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$; p(x)=$3x^2-1$
∴$p(-\frac{1}{\sqrt{3}})=3(-\frac{1}{\sqrt{3}})^2-1=3(\frac{1}{3})-1=1-1=0$
∴$p(\frac{2}{\sqrt{3}})=3(\frac{2}{\sqrt{3}})^2-1=3(\frac{4}{3})-1=4-1=3\neq0$
∴$-\frac{1}{\sqrt{3}}$ is a zero of p(x) but $\frac{2}{\sqrt{3}}$ is not a zero of p(x).

(viii) p(x)=2x+1,x=$\frac{1}{2}$

Solution:

For, x=$\frac{1}{2}$ p(x)=2x+1
∴$p(\frac{1}{2})=2(\frac{1}{2})+1=1+1=2\neq0$
∴$\frac{1}{2}$ is not a zero of p(x).

4. Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

Solution:

p(x)=x+5
⇒x+5=0
⇒x=−5
∴-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

p(x)=x−5
⇒x−5=0

⇒x=5
∴5 is a zero polynomial of the polynomial p(x).

(iii)p(x) = 2x + 5
Solution:
p(x)=2x+5
⇒2x+5=0
⇒2x=−5
⇒x=$-\frac{5}{2}$
∴x= $-\frac{5}{2}$ is a zero polynomial of the polynomial p(x).

(iv)p(x) = 3x – 2
Solution:
p(x)=3x–2
⇒3x−2=0
⇒3x=2
⇒x=$\frac{2}{3}$
∴x=$\frac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x
Solution:
p(x)=3x
⇒3x=0
⇒x=0
∴0 is a zero polynomial of the polynomial p(x).

(vi)p(x) = ax, a≠0
Solution:
p(x)=ax
⇒ax=0
⇒x=0
∴x=0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.
Solution:
p(x)= cx + d
⇒ cx + d =0
⇒x=$\frac{-d}{c}$
∴ x=$\frac{-d}{c}$ is a zero polynomial of the polynomial p(x).

Exercise 2.3

1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) x+1

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-\frac{1}{2}$

Solution:

$x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{2}$

$\therefore$Remainder:

$p(\frac{1}{2})= (\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1$

$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$=\frac{27}{8}$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

∴Remainder:

$(-\frac{5}{2})^3+3(-\frac{5}{2})^2+3(-\frac{5}{2})+1=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$

$=-\frac{27}{8}$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a=5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=\frac{-7}{3}$

∴Remainder:

$3(\frac{-7}{3})^3+7(\frac{-7}{3})=-\frac{-343}{9}+\frac{-49}{3}$

$=\frac{-343-(49)3}{9}$

$=\frac{-343-147}{9}$

$=\frac{-490}{9}\neq 0$

∴7+3x is not a factor of $3x^3+7x$

Exercise 2.4

1. **Determine which of the following polynomials has (x + 1) a factor:**

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii)$x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv)$x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+ \sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3x2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii)$p(x)=x^3–4x^2+x+6$, $g(x) = x – 3$

Solution:

$p(x)= x^3–4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow1+1+k=0$

$\Rightarrow2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow2k-3=0$

$\Rightarrow k=\frac{3}{2}$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=1×12=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4=12]

$12x^2–7x+1=12x^2-4x-3x+1$
$=4x (3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=7 and product=2× 3=6
We get 6 and 1 as the numbers [6+1=7 and 6× 1=6]

$2x^2+7x+3 =2x^2+6x+1x+3$
$=2x (x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=5 and product=6× −6= -36
We get -4 and 9 as the numbers [-4+9=5 and -4× 9=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$
$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-1 and product=3× −4= -12
We get -4 and 3 as the numbers [-4+3=-1 and -4× 3=-12]

$3x^2 – x – 4=3x^2–x–4$
$=3x^2–4x+3x–4$
$=x(3x–4)+1(3x–4)$
$=(3x–4)(x+1)$

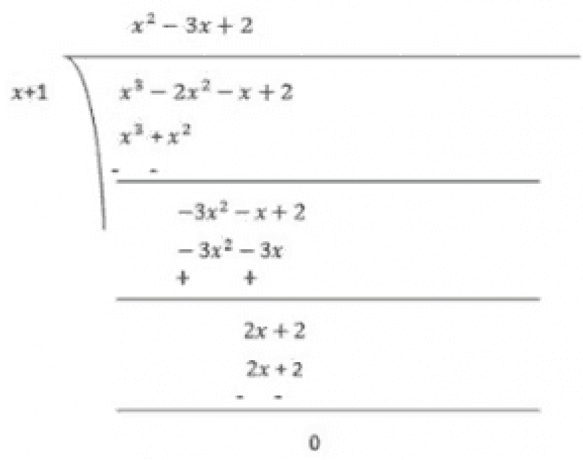
5. Factorize:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
p(1) = 0
So, (x+1) is factor of p(x)

Now,
p(x)= $x^3–2x^2–x+2$
p(−1)=$(−1)^3–2(−1)^2–(−1)+2$
=−1−1+1+2
=0
Therefore, (x+1) is the factor of p(x)



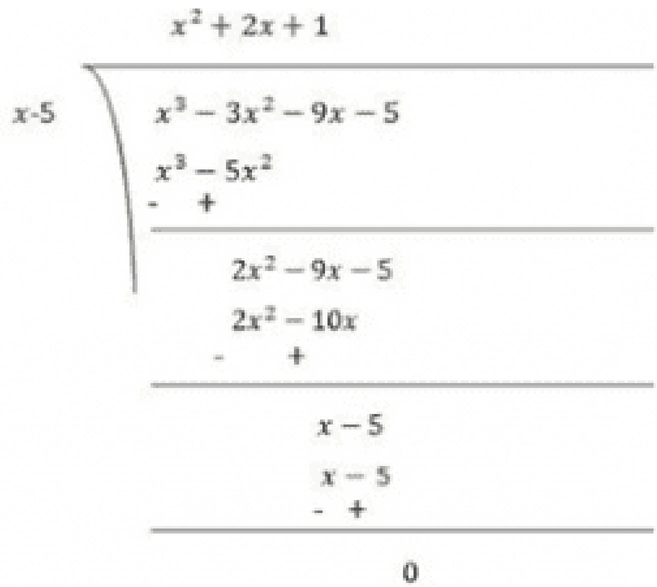
Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$
$=(x+1)(x(x−1)−2(x−1))$
$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let p(x) = $x^3–3x^2–9x–5$
Factors of 5 are ±1 and ±5
By trial method, we find that
p(5) = 0
So, (x-5) is factor of p(x)
Now,
p(x) = $x^3–3x^2–9x–5$
p(5) = $(5)^3–3(5)^2–9(5)–5$
=125−75−45−5
=0
Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$
$= (x-5)(x(x+1)+1(x+1))$
$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

$p(x) = x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$
$= -1+13-32+20$
$= 0$

Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x + 20 \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$
$= (x+1)x(x+2)+10(x+2)$
$= (x+1)(x+2)(x+10)$

(iv) $\mathbf{2y^3+y^2-2y-1}$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1) = -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, $(y-1)$ is the factor of $p(y)$

$$2y^2 + 3y + 1$$

$y-1$) $2y^3 + y^2 - 2y - 1$

$2y^3 - 2y^2$

$-\quad +$

$3y^2 - 2y - 1$

$3y^2 - 3y$

$-\quad +$

$y - 1$

$y - 1$

$-\quad +$

0

Now, Dividend = Divisor × Quotient + Remainder

$$(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1)$$
$$= (y-1)(2y(y+1)+1(y+1))$$
$$= (y-1)(2y+1)(y+1)$$

Exercise 2.5

1. **Use suitable identities to find the following products:**

(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$$
$$=x^2+14x+40$$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]

We get,

$$(x+8)(x-10)=x^2+(8+(-10))x+(8\times(-10))$$
$$=x^2+(8-10)x-80$$
$$=x^2-2x-80$$

(iii)(3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, x=3x, a=4 and b= −5]

We get,

$$(3x+4)(3x-5)=(3x)^2+4+(-5)3x+4\times(-5)$$
$$=9x^2+3x(4-5)-20$$
$$=9x^2-3x-20$$

(iv)$(y^2+\frac{3}{2})(y^2-\frac{3}{2})$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=\frac{3}{2}$]

We get,

$$(y^2+\tfrac{3}{2})(y^2-\tfrac{3}{2})=(y^2)^2-(\tfrac{3}{2})^2$$
$$=y^4-\frac{9}{4}$$

2. **Evaluate the following products without multiplying directly:**

(i) 103 × 107

Solution:

$103\times107=(100+3)\times(100+7)$

Using identity, [(x+a)(x+b)=x2+(a+b)x+ab

Here, x=100

a=3

b=7

We get, $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)100+(3\times7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95\times96=(100-5)\times(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, x=100

a=-5

b=-4

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii)104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, a=100

b=4

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, x=3x

y=y

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$
$=(3x+y)^2$
$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution: $4y^2-4y+1=(2y)^2-$

$(2\times2y\times1)+12$
Using identity, $x^2 - 2xy + y^2= (x - y)^2$
Here, $x=2y$
$y=1$
$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$
$=(2y-1)^2$
$=(2y-1)(2y-1)$

(iii) $x^2-\frac{y^2}{100}$

Solution:

$x^2-\frac{y^2}{100} = x^2-(\frac{y}{10})^2$
Using identity, $x^2 - y^2= (x - y)\ (x\ y)$
Here, $x=x$
$y=\frac{y}{10}$

$x^2 - \frac{y^2}{100} = x^2-(\frac{y}{10})^2$
$=(x-\frac{y}{10})(x+\frac{y}{10})$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii)$(-2x+3y+2z)^2$
(iv)$(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi)$(\frac{1}{4}a-\frac{1}{2}b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= - 7b$

$z= - c$

$(3a - 7b - c)^2 = (3a)^2+(- 7b)^2+(- c)^2+(2\times 3a \times - 7b)+(2\times - 7b \times - c)+(2\times - c \times 3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y= 5y$

$z= -3z$

$(-2x+5y-3z)^2 =(-2x)^2+(5y)^2+(-3z)^2+(2\times-2x \times 5y)+(2\times 5y \times-3z)+(2\times-3z \times-2x)$

$=4x^2 + 25y^2 + 9z^2- 20xy-30yz+12zx$

(vi) $(\frac{1}{4}a - \frac{1}{2}b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= \frac{1}{4}a$

$y= -\frac{1}{2}b$

$z= 1$

$(\frac{1}{4}a - \frac{1}{2}b +1)^2 =(\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+(1)^2+(2\times\frac{1}{4}a \times -\frac{1}{2}b)+(2\times -\frac{1}{2}b\times1)+(2\times1\times\frac{1}{4}a)$

$=\frac{1}{16}a^2+\frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b+\frac{2}{4}a$

$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz =(2x)^2+(3y)^2+(-4z)^2+(2\times2x\times3y)+(2\times3y\times-4z)+(2\times-4z\times2x)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$=(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times2\sqrt{2}z)+(2\times2\sqrt{2}z\times-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $(\frac{3}{2}x+1)^3$

(iv) $(x-\frac{2}{3}y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times2x\times1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(2a-3b)^3=(2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $(\frac{3}{2}x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(\frac{3}{2}x+1)^3 =(\frac{3}{2}x)^3+1^3+(3\times\frac{3}{2}x\times1)(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$

$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv) $\mathbf{(x-\frac{2}{3}y)^3}$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(x-\frac{2}{3}y)^3 = (x)^3-(\frac{2}{3}y)^3-(3\times x\times\frac{2}{3}y)(x-\frac{2}{3}y)$

$= (x)^3-\frac{8}{27}y^3-2xy(x-\frac{2}{3}y)$

$= (x)^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

7. Evaluate the following using suitable identities:

(i) $\mathbf{(99)^3}$

(ii) $\mathbf{(102)^3}$

(iii) $\mathbf{(998)^3}$

Solutions:

(i) $\mathbf{(99)^3}$

Solution:

We can write 99 as 100–1

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(99)^3 = (100-1)^3$

$=(100)^3-1^3-(3\times100\times1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $\mathbf{(102)^3}$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(100+2)^3 =(100)^3+2^3+(3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $\mathbf{(998)^3}$

Solution:

We can write 99 as 1000–2

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(998)^3 =(1000-2)^3$

$=(1000)^3-2^3-(3\times1000\times2)(1000-2)$

$= 1000000000 - 8 - 6000(1000 - 2)$

$= 1000000000 - 8- 6000000 + 12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 – \frac{1}{216} – \frac{9}{2}p^2 + \frac{1}{4}p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$
$=(2a+b)^3$
$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3–12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3–12a^2b+6ab^2 =(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$
$=(2a–b)^3$
$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$ $27–125a^3–135a+225a^2=3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$
$=(3–5a)^3$
$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv)64a3–27b3–144a^{2}b+108ab^2

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$ $64a^3–27b^3–144a^2b+108ab^2=(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$
$=(4a–3b)^3$
$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solution:

The expression, $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can be written as $(3p)^3 - (\frac{1}{6})^3 - 3(3p)^2(\frac{1}{6}) + 3(3p)(\frac{1}{6})^2$

$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - (\frac{1}{6})^3 - 3(3p)^2(\frac{1}{6}) + 3(3p)(\frac{1}{6})^2$

$= (3p - \frac{1}{6})^3$

$= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})$

9. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$
(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2-3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)-3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2-xy)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

We know that, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)^3+3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)[(x-y)^2+3xy]$

Taking(x+y) common $\Rightarrow x^3-y^3=(x-y)[(x^2+y^2-2xy)+3xy]$

$\Rightarrow x^3+y^3=(x-y)(x^2+y^2+xy)$

10. Factorize each of the following:

(i) $27y^3+125z^3$
(ii) $64m^3-343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 = (3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2-xy+y^2)$

$\therefore 27y^3+125z^3 = (3y)^3+(5z)^3$

$= (3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$= (3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$ can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 = (4m)^3-(7n)^3$

We know that, $x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$\therefore 64m^3 - 343n^3 = (4m)^3 - (7n)^3$
$= (4m+7n)[(4m)^2+(4m)(7n)+(7n)^2]$
$= (4m+7n)(16m^2+28mn+49n^2)$

11. Factorise : $27x^3+y^3+z^3-9xyz$

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz \quad =(3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\therefore 27x^3+y^3+z^3-9xyz \quad =(3x)^3+y^3+z^3-3(3x)(y)(z)$
$=(3x+y+z)(3x)^2+y^2+z^2-3xy-yz-3xz$
$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify that: $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

Solution:

We know that,
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz \quad =\frac{1}{2}\times(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$
$=\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$
$=\frac{1}{2}(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$
$=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,
$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$
Now, according to the question, let $(x + y + z) = 0$,
then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz =0$
$\Rightarrow \quad x^3+y^3+z^3 =3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let a= −12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, −12+7+5=0

$\therefore (-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3 \times -12 \times 7 \times 5$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$\therefore (28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

$=16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2-35a+12$

(ii) Area : $35y^2+13y-12$

Solution:

(i) Area : $25a^2-35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=25×12=300
We get -15 and -20 as the numbers [-15+-20=-35 and -3×-4=300]

$25a^2–35a+12 =25a^2–15a–20a+12$
$=5a(5a–3)–4(5a–3)$
$=(5a–4)(5a–3)$
Possible expression for length = 5a – 4
Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$
Using the splitting the middle term method,
We have to find a number whose sum= 13 and product=35× −12=420
We get -15 and 28 as the numbers [-15+28=-35 and -15× 28=420]

$35y^2+13y–12 =35y^2–15y+28y–12$
$=5y(7y–3)+4(7y–3)$
$=(5y+4)(7y–3)$
Possible expression for length = (5y + 4)
Possible expression for breadth = (7y – 3)

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:
(i) Volume : $3x^2–12x$
$3x^2–12x$ can be written as 3x(x – 4) by taking 3x out of both the terms.
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = (x – 4)

(ii) Volume : $12ky^2+8ky –20k$
$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.
$12ky^2+8ky–20k =4k(3y^2+2y–5)$
[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$
$=4k[y(3y+5)–1(3y+5)]$
$=4k(3y+5)(y–1)$

Possible expression for length = 4k
Possible expression for breadth = (3y +5)
Possible expression for height = (y -1)

CHAPTER 2
POLYNOMIALS

Introduction

In this chapter, we shall study a particular type of algebraic expression, called polynomial, the Remainder Theorem and Factor Theorem and their use in the factorisation of polynomials.

ALGEBRAIC EXPRESSION

- The combination of constants and variables are called algebraic expression.

For Example:-

2x+5

-3xy + 4

2a+5b +50

-6x+5y+3z + 10

Polynomials in One Variable

- $x^3 - x^2 + 4x + 7$ is a polynomial in one variable x.

- $3y^2 + 5y$ is a polynomial in one variable y.
- $t^2 + t + 4$ is a polynomial in one variable t.
- $6x^{3/4}+5$ is not a polynomial . WHY?

IMPORTANT TERMS

- **POLYNOMIAL:**

  AN ALGEBRAIC EXPRESSION IN WHICH THE VARIABLE INVOLVED HAVE ONLY NON-NEGATIVE INTEGRAL POWERS IS CALLED A POLYNOMIAL.

  $3y^2 + 5y$ IS A POLYNOMIAL.

  $6x^{3/4}+5$ IS NOT A POLYNOMIAL .

- **CONSTANT:**

  A SYMBOL HAVING A FIXED NUMERICAL VALUE IS CALLED A CONSTANT.

  FOR EXAMPLE IN POLYNOMIAL $-3xy + 4$, THE CONSTANTS ARE -3 AND 4 .

  IN POLYNOMIAL $2x+5$, THE CONSTANTS ARE 2 AND 5 .

- **VARIABLE:**

  A SYMBOL WHICH MAY BE ASSIGNED DIFFERENT NUMERICAL VALUES IS CALLED A VARIABLE.
  FOR EXAMPLE IN POLYNOMIAL -3XY + 4,
  X AND Y ARE VARIBLES.

- **COEFFICIENT:**

  THE NUMERICAVALUE (NUMBER/CONSTANT) THAT IS MULTIPLIED TO THE VARIABLE IN A TERM OF AN ALGEBRAIC EXPRESSION IS CALLED NUMERICAL COEFFICIENT.
  FOR EXAMPLE IN POLYNOMIAL 2X+5,
  THE NUMERICAL COEFFICIENT OF X IS 2.

DEGREE OF A POLYNOMIAL

- The highest power of the variable in a polynomial is called the degree of the polynomial.

  For example the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7. The degree of the polynomial $5y^6 - 4y^2 - 6$ is 6.

CONSTANT POLYNOMIALS

- A polynomial containing one term only, consisting of a constant is called constant polynomial.

Note: The degree of a non-zero constant polynomial is zero.
For example the degree of the polynomial 51 is 0.

ZERO POLYNOMIAL

- 0 IS A ZERO POLYNOMIAL.
- The degree of the zero polynomial is not defined.

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TYPES OF ALGEBRAIC EXPRESSION (ON THE BASIS OF TERMS)

- MONOMIALS
- BINOMIALS
- TRINOMIALS
- POLYNOMIALS

MONOMIALS

- Polynomials having only one term are called monomials (‘mono’ means ‘one’).
- For example the polynomials

  $2x$

  $5x^3$

  y

  u^4

BINOMIALS

- Polynomials having only two terms are called binomials ('bi' means 'two').

Observe each of the following polynomials:

$p(z) = z + 1$

$q(x) = x^2 - x$

$r(y) = y + 1$

$t(u) = u^{43} - u$

- How many terms are there in each of these?

TRINOMIALS

- Polynomials having only three terms are called trinomials (‘tri’ means ‘three’).
- Some examples of trinomials are

  $p(x) = x + x^2 + \pi$,

  $q(x) = 2 + x - x^2$,

  $r(u) = u + u^2 - 2$,

  $t(y) = y^4 + y + 5$.

POLYNOMIAL

- Polynomials having many terms are called polynomials.

  For example $p(x) = 3x^7 - 4x^6 + x + 9$ has more than three terms is called a polynomial.

- A polynomial in one variable x of degree n is an expression of the form:

  $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

  where $a_0, a_1, a_2, \ldots, a_n$ are constants and $a_n \neq 0$.

TYPES OF POLYNOMIALS (ON THE BASIS OF DEGREE)

- A polynomial of degree one is called a linear polynomial.

General form: $ax + b, where\ a \neq 0$

Eg. x + 2

- A polynomial of degree two is called a quadratic polynomial.

General form: $ax^2 + bx + c, where\ a \neq 0$

Eg. q^2 + 21

- A polynomial of degree three a cubic polynomial.

General form: $ax^3 + bx^2 + cx + d, where\ a \neq 0$

Eg. y^3

- A polynomial of degree four is called a biquadratic polynomial.

Eg. $p^4 - p^3 + p^2 - p + 33$

EXERCISE 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

(ii) $y^2 + 2^{1/2}$

(iii) $3X-7X^{8/9}$

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 + x^3$

(iii) $2x + \pi + 7$

(iv) 21

- Give one example of a binomial of degree 35.
- Give one example of a monomial of degree 100.

Write the degree of each of the following polynomials:

POLYNOMIALS	DEGREE
$5x^3 + 4x^2 + 7x$	
$4 - y^2$	
3	

Classify the following as linear, quadratic and cubic polynomials:

POLYNOMIALS	DEGREE	KIND OF POLYNOMIAL
$x^3 + x$	3	
$y + y^2 + 4$	2	
$1 + x$	1	

SOLVE EX. 2.1 IN YOUR CW NB

ZERO OF A POLYNOMIAL

- A zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

- Consider the polynomial $p(x) = x - 1$.

  What is $p(1)$?

  $p(1) = 1 - 1 = 0$.

  As $p(1) = 0$, we say that 1 is a zero of the polynomial $p(x)$.

Example 3 :

- Check whether –2 and 2 are zeroes of the polynomial $x + 2$.

- Solution : Let $p(x) = x + 2$.

  Then $p(2) = 2 + 2 = 4$,

  $p(-2) = -2 + 2 = 0$

  Therefore, –2 is a zero of the polynomial $x + 2$, but 2 is not.

Example 4 :

- Find a zero of the polynomial $p(x) = 2x + 1$

- Solution : Finding a zero of $p(x)$, is the same as solving the equation $p(x) = 0$

Now, $2x + 1 = 0$ gives us $x = -\frac{1}{2}$
So, $-\frac{1}{2}$ is a zero of the polynomial $2x + 1$.

Solve Ex. 2.2 in your CW NB

Zero of a Polynomial

In a polynomial with one variable, the value of the variable where the value of the polynomial becomes zero, is the **zero of a polynomial**.

HOME WORK

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$ (ii) $y^2 + 23$

(iii) $4x - 31y^3 + z^{30}$

Q2. Write the degree of the polynomial: $7x^3 + 4x^2 - 3x + 12$

Q3. Classify the following polynomials on the basis of their degree:

(i) $x + x^2 + 4$ (ii) $3y$ (iii) 7

Q4. Write a trinomial of degree 25.

Q5. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ (ii) $p(x) = x – 5$ (iii) $p(x) = 2x + 5$ (iv) $p(x) = 3x – 2$

Remainder Theorem

$$
\begin{array}{r|l}
 & 3x - 2 \\
\hline
x + 1 & 3x^2 + x - 1 \\
 & 3x^2 + 3x \\
 & - \quad\; - \\
\hline
 & -2x - 1 \\
 & -2x - 2 \\
 & + \quad\; + \\
\hline
 & 1
\end{array}
$$

Find the remainder when $p(x) = 3x^2 + x - 1$ is divided by $g(x) = x + 1$.

Soln.

Let $g(x) = x + 1 = 0$

$\Rightarrow \quad x = -1$

$p(-1) = 3(-1)^2 + (-1) - 1$

$= 3 - 1 - 1$

$= 1$

Remainder Theorem

- Remainder Theorem :

  Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

Example 7 :

- Find the remainder when the polynomial $3x^4 - 4x^3 - 3x - 1$ is divided by $x - 1$.
- Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$.

- Solution :

  The root of $x + 1 = 0$ is $x = -1$.

  We see that $p(-1) = (-1)^3 + 1$

  $= -1 + 1 = 0,$

  which is equal to the remainder.

Example 10 :

- Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$

Solution :

As you know, q(t) will be a multiple of 2t + 1 only, if 2t + 1 divides q(t) leaving remainder zero.

Let, $2t + 1 = 0$, $\Rightarrow t = -\frac{1}{2}$

Now, $q(t) = 4t^3 + 4t^2 - t - 1$

$q(-\frac{1}{2}) = 4(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 - (-\frac{1}{2}) - 1 = 0$

As the remainder obtained on dividing q(t) by 2t + 1 is 0. So, 2t + 1 is a factor of the given polynomial q(t), That is q(t) is a multiple of 2t + 1.

HW: Ex. 2.3 Q1, Q2 and Q3

Factorisation of Polynomials

- Factor Theorem :

  If p(x) is a polynomial of degree $n \geq 1$ and a is any real number, then

  (i) $x - a$ is a factor of p(x)

  $\Rightarrow$ p(a) = 0, and

  (ii) p(a) = 0

  $\Rightarrow$ x – a is a factor of p(x).

Example 11 : Examine whether x + 2 is a factor of $x^3 + 3x^2 + 5x + 6$ and of 2x + 4.

- Solution : The zero of x + 2 is –2.

  Let $p(x) = x^3 + 3x^2 + 5x + 6$ and $s(x) = 2x + 4$

  Then, $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$

  $= -8 + 12 - 10 + 6 = 0$

  So, by the Factor Theorem, x + 2 is a factor of $x^3 + 3x^2 + 5x + 6$.

  Again, $s(-2) = 2(-2) + 4 = 0$

  So, x + 2 is a factor of 2x + 4.

Example 12 :

- Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.

- Solution : As $x - 1$ is a factor of

  $p(x) = 4x^3 + 3x^2 - 4x + k$

  $\Rightarrow p(1) = 0$

  $\Rightarrow p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k = 0$

  $\Rightarrow 4 + 3 - 4 + k = 0$

  $\Rightarrow k = -3$

Factorisation by splitting the middle term

- Example 13 : Factorise $6x^2 + 17x + 5$

- Solution:

  Let us look for the pairs of factors of 30 (6×5)

  Some are 1 and 30, 2 and 15, 3 and 10, 5 and 6.

  Of these pairs, 2 and 15 will give us

$$
\begin{aligned}
6x^2 + 17x + 5 &= 6x^2 + 2x + 15x + 5 \\
&= 2x(3x + 1) + 5(3x + 1) \\
&= (3x + 1)(2x + 5)
\end{aligned}
$$

To determine the factors in case of cubic polynomials

(i) If sum of coefficients is zero then (x - 1) is the factor.

(ii) If sum of coefficients of even powers of x = sum of coefficients of odd powers of x, then (x + 1) is the factor.

(iii) If both are not the factors, then to check for other factors we have to apply trial and error method

Example 15 : Factorise $x^3 – 23x^2 + 142x – 120$.

- Solution :

  Let $p(x) = x^3 – 23x^2 + 142x – 120$

  Sum of coefficients = 1-23+142-120 = 0

  ∴ $(x – 1)$ is the factor

  $x^3 – 23x^2 + 142x – 120 = x^3 – x^2 – 22x^2 + 22x + 120x – 120$

  $= x^2(x –1) – 22x(x – 1) + 120(x – 1)$

  $= (x – 1)(x^2 – 22x + 120)$

  Now by splitting the middle term,

  $x^2 – 22x + 120 = (x – 12)(x – 10)$

  So, $x^3 – 23x^2 + 142x – 120 = (x – 1)(x – 10)(x – 12)$

Note: We can also divide the given polynomial by $(x - 1)$ to get the quotient $(x^2 – 22x + 120)$ and proceed further.

FACTORISE $x^3+13x^2+32x+20$

Given polynomial is $x^3+13x^2+32x+20$

Sum of coefficients of even powers of x
=sum of coefficients of odd power of x.
$\therefore (x+1)$ is the factor.
The remaining factors can be found by long division method

$$\begin{aligned} \text{Quotient } &= x^2+12x+20 \\ &= x^2+10x+2x+20 \\ &= x(x+10)+2(x+10) \\ &= (x+2)(x+10) \end{aligned}$$

Hence, $x^3+13x^2+32x+20 = (x+1)(x+2)(x+10)$

Algebraic Identities

1. $(x + y)^2 = x^2 + 2xy + y^2$
2. $(x - y)^2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

6. $$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$
$$= x^3 + y^3 + 3x^2y + 3xy^2$$

---

7. $$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$
$$= x^3 - y^3 - 3x^2y + 3xy^2$$

8. $$x^3 + y^3 + z^3 - 3xyz = (x+y+z)$$
$$(x^2 + y^2 + z^2 - xy - yz - zx)$$

9. $$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

10. $$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

1. Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

$= x^2 + (4 + 10)x + (4)(10)$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x – 10)$

$=x^2 + (8 - 10)x + (8)(-10)$

$= x^2 - 2x - 80$

2. Evaluate the following products without multiplying directly:

(i) 103×107

$(100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7).100 + (3)(7)$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

$(100 - 5)(100 - 4)$

$= (100)^2 + (-5-4).100 + (-5)(-4)$

$= 10000 - 900 - 20$

$= 9079$

3. Factorise the following using appropriate identity

(i) $4y^2 - 4y + 1$

$=(2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(ii) $x^2 - \frac{y2}{100}$

$=(x)^2 - \left(\frac{y}{10}\right)^2$

$=\left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $(2x - y + z)^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$=(2x)^2 + (3y)^2 + (-4z)^2 +$

$2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + y - z)^2$

6. Write the following cubes in expanded form:

(i) $(2a - 3b)^3$

$= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

7. Evaluate the following using suitable identities:

(i) $(998)^3$

$= (1000 – 2)^3$

$=(1000)^3–(2)^3–3(1000)(2)(1000–2)$

$= 1000000000-8-6000(998)$

$= 999999992- 5988000$

$= 994011992$

8. Factorise each of the following:

- (i) $27 - 125a^3 - 135a + 225a^2$

Identity : $x^3 - y^3 - 3xy(x - y) = (x - y)^3$

$=3^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

9. Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3$

$= x^3 + y^3$

10. Factorise each of the following:

(i) $64m^3 - 343n^3$

Identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Factorise :

- $8x^3 + y^3 + 27z^3 - 18xyz$

- Solution : $8x^3 + y^3 + 27z^3 - 18xyz$

$$= (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$$

$$= (2x + y + 3z)\left[(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)\right]$$

$$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$$

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

- We know that:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Putting $x+y+z=0$,

$$\therefore x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = 0$$

$$\therefore x^3 + y^3 + z^3 = 3xyz.$$

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

- Solution:

  Here, $x + y + z = (-12) + (7) + (5) = 0$

  So, $x^3 + y^3 + z^3 = 3xyz$

  $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

  $= -1260$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

- Area : $25a^2 - 35a + 12$

$= 25a^2 - 15a - 20a + 12$

$= 5a\,(5a - 3) - 4\,(5a - 3)$

$= (5a - 4)\,(5a - 3)$

LENGTH	BREADTH
5a - 4	5a – 3
5a – 3	5a - 4

- Solve Ex. 2.5 in your note book.

Summary

. A polynomial of one term is called a monomial.
. A polynomial of two terms is called a binomial.
. A polynomial of three terms is called a trinomial.
. A polynomial of degree one is called a linear polynomial.
. A polynomial of degree two is called a quadratic polynomial.
. A polynomial of degree three is called a cubic polynomial.
. A real number 'a' is a zero of a polynomial p(x) if p(a) = 0. In this case, a is also called a root of the equation p(x) = 0.
. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial.
. Remainder Theorem : If p(x) is any polynomial of degree greater than or equal to 1 and p(x) is divided by the linear polynomial x – a, then the remainder is p(a).
. Factor Theorem : x – a is a factor of the polynomial p(x), if p(a) = 0. Also, if x – a is a factor of p(x), then p(a) = 0.

ALGEBRAIC IDENTITIES

1. $(x + y)2 = x^2 + 2xy + y^2$
2. $(x - y)2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
   $= x^3 + y^3 + 3x^2y + 3xy^2$
7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
   $= x^3 - y^3 - 3x^2y + 3xy^2$
8. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)$
   $(x^2 + y^2 + z^2 - xy - yz - zx)$
9. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
10. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

ALGEBRAIC EXPRESSION

- The combination of constants and variables are called algebraic expression.

For Example:-

2x+5

-3xy + 4

2a+5b +50

-6x+5y+3z + 10

Polynomials in One Variable

- $x^3 - x^2 + 4x + 7$ is a polynomial in one variable x.

- $3y^2 + 5y$ is a polynomial in one variable y.
- $t^2 + t + 4$ is a polynomial in one variable t.
- $6x^{3/4}+5$ is not a polynomial . WHY?

IMPORTANT TERMS

- **POLYNOMIAL:**

  AN ALGEBRAIC EXPRESSION IN WHICH THE VARIABLE INVOLVED HAVE ONLY NON-NEGATIVE INTEGRAL POWERS IS CALLED A POLYNOMIAL.

  $3y^2 + 5y$ IS A POLYNOMIAL.

  $6x^{3/4}+5$ IS NOT A POLYNOMIAL .

- **CONSTANT:**

  A SYMBOL HAVING A FIXED NUMERICAL VALUE IS CALLED A CONSTANT.

  FOR EXAMPLE IN POLYNOMIAL $-3xy + 4$, THE CONSTANTS ARE -3 AND 4 .

  IN POLYNOMIAL $2x+5$, THE CONSTANTS ARE 2 AND 5 .

- **VARIABLE:**

  A SYMBOL WHICH MAY BE ASSIGNED DIFFERENT NUMERICAL VALUES IS CALLED A VARIABLE.
  FOR EXAMPLE IN POLYNOMIAL -3XY + 4,
  X AND Y ARE VARIBLES.

- **COEFFICIENT:**

  THE NUMERICAVALUE (NUMBER/CONSTANT) THAT IS MULTIPLIED TO THE VARIABLE IN A TERM OF AN ALGEBRAIC EXPRESSION IS CALLED NUMERICAL COEFFICIENT.
  FOR EXAMPLE IN POLYNOMIAL 2X+5,
  THE NUMERICAL COEFFICIENT OF X IS 2.

DEGREE OF A POLYNOMIAL

- The highest power of the variable in a polynomial is called the degree of the polynomial.

  For example the degree of the polynomial $3x^7 – 4x^6 + x + 9$ is 7. The degree of the polynomial $5y^6 – 4y^2 – 6$ is 6.

CONSTANT POLYNOMIALS

- A polynomial containing one term only, consisting of a constant is called constant polynomial.

  Note: The degree of a non-zero constant polynomial is zero.
  For example the degree of the polynomial 51 is 0.

ZERO POLYNOMIAL

- 0 IS A ZERO POLYNOMIAL.
- The degree of the zero polynomial is not defined.

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TYPES OF ALGEBRAIC EXPRESSION (ON THE BASIS OF TERMS)

- MONOMIALS
- BINOMIALS
- TRINOMIALS
- POLYNOMIALS

MONOMIALS

- Polynomials having only one term are called monomials (‘mono’ means ‘one’).
- For example the polynomials

  $2x$

  $5x^3$

  y

  u^4

BINOMIALS

- Polynomials having only two terms are called binomials (‘bi’ means ‘two’).

Observe each of the following polynomials:

$$p(z) = z + 1$$
$$q(x) = x^2 – x$$
$$r(y) = y + 1$$
$$t(u) = u^{43} – u$$

- How many terms are there in each of these?

TRINOMIALS

- Polynomials having only three terms are called trinomials (‘tri’ means ‘three’).

- Some examples of trinomials are

$$p(x) = x + x^2 + \pi,$$
$$q(x) = 2 + x - x^2,$$
$$r(u) = u + u^2 - 2,$$
$$t(y) = y^4 + y + 5.$$

POLYNOMIAL

- Polynomials having many terms are called polynomials.

  For example $p(x) = 3x^7 - 4x^6 + x + 9$ has more than three terms is called a polynomial.

- A polynomial in one variable x of degree n is an expression of the form:

  $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

  where $a_0, a_1, a_2, \ldots, a_n$ are constants and $a_n \neq 0$.

TYPES OF POLYNOMIALS (ON THE BASIS OF DEGREE)

- A polynomial of degree one is called a linear polynomial.

General form: $ax + b, where\ a \neq 0$

Eg. x + 2

- A polynomial of degree two is called a quadratic polynomial.

General form: $ax^2 + bx + c, where\ a \neq 0$

Eg. q^2 + 21

- A polynomial of degree three a cubic polynomial.

General form: $ax^3 + bx^2 + cx + d, where\ a \neq 0$

Eg. y^3

- A polynomial of degree four is called a biquadratic polynomial.

Eg. $p^4 - p^3 + p^2 - p + 33$

EXERCISE 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

(ii) $y^2 + 2^{1/2}$

(iii) $3X-7X^{8/9}$

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 + x^3$

(iii) $2x + \pi + 7$

(iv) 21

- Give one example of a binomial of degree 35.

- Give one example of a monomial of degree 100.

Write the degree of each of the following polynomials:

POLYNOMIALS	DEGREE
$5x^3 + 4x^2 + 7x$	
$4 - y^2$	
3	

Classify the following as linear, quadratic and cubic polynomials:

POLYNOMIALS	DEGREE	KIND OF POLYNOMIAL
$x^3 + x$	3	
$y + y^2 + 4$	2	
$1 + x$	1	

SOLVE EX. 2.1 IN YOUR CW NB

ZERO OF A POLYNOMIAL

- A zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

- Consider the polynomial $p(x) = x - 1$.

  What is $p(1)$?

  $p(1) = 1 - 1 = 0$.

  As $p(1) = 0$, we say that 1 is a zero of the polynomial $p(x)$.

Example 3 :

- Check whether –2 and 2 are zeroes of the polynomial x + 2.

- Solution : Let p(x) = x + 2.

  Then p(2) = 2 + 2 = 4,

  p(–2) = –2 + 2 = 0

  Therefore, –2 is a zero of the polynomial x + 2, but 2 is not.

Example 4 :

- Find a zero of the polynomial $p(x) = 2x + 1$

- Solution : Finding a zero of p(x), is the same as solving the equation $p(x) = 0$

  Now, $2x + 1 = 0$ gives us $x = -\frac{1}{2}$
  So, $-\frac{1}{2}$ is a zero of the polynomial $2x + 1$.

Solve Ex. 2.2 in your CW NB

POLYNOMIALS - POLYNOMIALS IN ONE VARIABLE

Zero of a Polynomial

In a polynomial with one variable, the value of the variable where the value of the polynomial becomes zero, is the **zero of a polynomial**.

HOME WORK

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$ **(ii)** $y^2 + 23$

(iii) $4x - 31y^3 + z^{30}$

Q2. Write the degree of the polynomial: $7x^3 + 4x^2 - 3x + 12$

Q3. Classify the following polynomials on the basis of their degree:

(i) $x + x^2 + 4$ **(ii)** $3y$ **(iii)** 7

Q4. Write a trinomial of degree 25.

Q5. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ **(ii)** $p(x) = x – 5$ **(iii)** $p(x) = 2x + 5$ **(iv)** $p(x) = 3x – 2$

Remainder Theorem

$$
\begin{array}{r|l}
 & 3x - 2 \\
\hline
x + 1 & 3x^2 + x - 1 \\
 & 3x^2 + 3x \\
 & - \quad\; - \\
\hline
 & \quad - 2x - 1 \\
 & \quad - 2x - 2 \\
 & \quad\; + \quad\; + \\
\hline
 & \qquad\qquad 1
\end{array}
$$

Find the remainder when $p(x) = 3x^2 + x - 1$ is divided by $g(x) = x + 1$.

Soln.

Let $g(x) = x + 1 = 0$

$\Rightarrow \qquad x = -1$

$p(-1) = 3(-1)^2 + (-1) - 1$

$= 3 - 1 - 1$

$= 1$

Remainder Theorem

- Remainder Theorem :

  Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

Example 7 :

- Find the remainder when the polynomial $3x^4 - 4x^3 - 3x - 1$ is divided by $x - 1$.
- Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$.

- Solution :

  The root of $x + 1 = 0$ is $x = -1$.

  We see that $p(-1) = (-1)^3 + 1$

  $= -1 + 1 = 0,$

  which is equal to the remainder.

Example 10 :

- Check whether the polynomial $q(t) = 4t^3 + 4t^2 – t – 1$ is a multiple of $2t + 1$

Solution :

As you know, q(t) will be a multiple of 2t + 1 only, if 2t + 1 divides q(t) leaving remainder zero.

Let, $2t + 1 = 0$, $\Rightarrow t = -\frac{1}{2}$

Now, $q(t) = 4t^3 + 4t^2 – t – 1$

$q(-\frac{1}{2}) = 4(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 – (-\frac{1}{2}) – 1 = 0$

As the remainder obtained on dividing q(t) by 2t + 1 is 0. So, 2t + 1 is a factor of the given polynomial q(t), That is q(t) is a multiple of 2t + 1.

HW: Ex. 2.3 Q1, Q2 and Q3

Factorisation of Polynomials

- Factor Theorem :

  If p(x) is a polynomial of degree $n \geq 1$ and a is any real number, then

  (i) $x - a$ is a factor of p(x)

  $\Rightarrow$ p(a) = 0, and

  (ii) p(a) = 0

  $\Rightarrow$ x – a is a factor of p(x).

Example 11 : Examine whether x + 2 is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.

- Solution : The zero of x + 2 is –2.

  Let $p(x) = x^3 + 3x^2 + 5x + 6$ and $s(x) = 2x + 4$

  Then, $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$

  $= -8 + 12 - 10 + 6 = 0$

  So, by the Factor Theorem, x + 2 is a factor of $x^3 + 3x^2 + 5x + 6$.

  Again, $s(-2) = 2(-2) + 4 = 0$

  So, x + 2 is a factor of 2x + 4.

Example 12 :

- Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.

- Solution : As $x - 1$ is a factor of

  $p(x) = 4x^3 + 3x^2 - 4x + k$

  $\Rightarrow p(1) = 0$

  $\Rightarrow p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k = 0$

  $\Rightarrow 4 + 3 - 4 + k = 0$

  $\Rightarrow k = -3$

Factorisation by splitting the middle term

- Example 13 : Factorise $6x^2 + 17x + 5$

- Solution:

  Let us look for the pairs of factors of 30 (6×5)

  Some are 1 and 30, 2 and 15, 3 and 10, 5 and 6.

  Of these pairs, 2 and 15 will give us

$$
\begin{aligned}
6x^2 + 17x + 5 &= 6x^2 + 2x + 15x + 5 \\
&= 2x(3x + 1) + 5(3x + 1) \\
&= (3x + 1)(2x + 5)
\end{aligned}
$$

To determine the factors in case of cubic polynomials

(i) If sum of coefficients is zero then (x - 1) is the factor.

(ii) If sum of coefficients of even powers of x = sum of coefficients of odd powers of x, then (x + 1) is the factor.

(iii) If both are not the factors, then to check for other factors we have to apply trial and error method

Example 15 : Factorise $x^3 - 23x^2 + 142x - 120$.

- Solution :

  Let $p(x) = x^3 - 23x^2 + 142x - 120$

  Sum of coefficients = 1-23+142-120 = 0

  $\therefore$ (x – 1) is the factor

  $$x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$$
  $$= x^2(x-1) - 22x(x-1) + 120(x-1)$$
  $$= (x-1)(x^2 - 22x + 120)$$

  Now by splitting the middle term,

  $x^2 - 22x + 120 = (x - 12)(x - 10)$

  So, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$

Note: We can also divide the given polynomial by (x - 1) to get the quotient $(x^2 - 22x + 120)$ and proceed further.

FACTORISE $x^3+13x^2+32x+20$

Given polynomial is $x^3+13x^2+32x+20$

Sum of coefficients of even powers of x
=sum of coefficients of odd power of x.
$\therefore (x+1)$ is the factor.
The remaining factors can be found by long division method

$$\begin{aligned}\text{Quotient} &= x^2+12x+20\\ &= x^2+10x+2x+20\\ &= x(x+10)+2(x+10)\\ &= (x+2)(x+10)\end{aligned}$$

Hence, $x^3+13x^2+32x+20 = (x+1)(x+2)(x+10)$

Algebraic Identities

1. $(x + y)^2 = x^2 + 2xy + y^2$

2. $(x - y)^2 = x^2 - 2xy + y^2$

3. $x^2 - y^2 = (x + y)(x - y)$

4. $(x + a)(x + b) = x^2 + (a + b)x + ab$

5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

6. $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
$= x^3 + y^3 + 3x^2y + 3xy^2$

---

7. $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
$= x^3 - y^3 - 3x^2y + 3xy^2$

8. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)$
$(x^2 + y^2 + z^2 - xy - yz - zx)$

9. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

10. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

1. Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

$= x^2 + (4 + 10)x + (4)(10)$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

$= x^2 + (8 - 10)x + (8)(-10)$

$= x^2 - 2x - 80$

2. Evaluate the following products without multiplying directly:

(i) 103×107

$(100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7).100 + (3)(7)$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

$(100 - 5)(100 - 4)$

$= (100)^2 + (-5-4).100 + (-5)(-4)$

$= 10000 - 900 - 20$

$= 9079$

3. Factorise the following using appropriate identity

(i) $4y^2 - 4y + 1$

$=(2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(ii) $x^2 - \frac{y2}{100}$

$=(x)^2 - \left(\frac{y}{10}\right)^2$

$=\left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $(2x - y + z)^2$

$$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$$

$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$=(2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + y - z)^2$

6. Write the following cubes in expanded form:

(i) $(2a - 3b)^3$

$= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

7. Evaluate the following using suitable identities:

(i) $(998)^3$

$= (1000 - 2)^3$

$=(1000)^3-(2)^3-3(1000)(2)(1000-2)$

$= 1000000000-8-6000(998)$

$= 999999992- 5988000$

$= 994011992$

8. Factorise each of the following:

- (i) $27 - 125a^3 - 135a + 225a^2$

Identity : $x^3 - y^3 - 3xy(x - y) = (x - y)^3$

$=3^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

9. Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3$

$= x^3 + y^3$

10. Factorise each of the following:

(i) $64m^3 – 343n^3$

Identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$64m^3 – 343n^3 = (4m)^3 – (7n)^3$$

$$= (4m – 7n)(16m^2 + 28mn + 49n^2)$$

Factorise :

- $8x^3 + y^3 + 27z^3 - 18xyz$

- Solution : $8x^3 + y^3 + 27z^3 - 18xyz$

$$= (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$$

$$= (2x + y + 3z)\left[(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)\right]$$

$$= (2x + y + 3z)\,(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$$

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

- We know that:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Putting $x+y+z=0$,

$$\therefore x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = 0$$

$$\therefore x^3 + y^3 + z^3 = 3xyz.$$

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

- Solution:

  Here, $x + y + z = (-12) + (7) + (5) = 0$

  So, $x^3 + y^3 + z^3 = 3xyz$

  $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

  $= -1260$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

- Area : $25a^2 - 35a + 12$

  $= 25a^2 - 15a - 20a + 12$

  $= 5a(5a - 3) - 4(5a - 3)$

  $= (5a - 4)(5a - 3)$

LENGTH	BREADTH
5a - 4	5a – 3
5a – 3	5a - 4

- Solve Ex. 2.5 in your note book.

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. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial.
. Remainder Theorem : If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the linear polynomial $x – a$, then the remainder is $p(a)$.
. Factor Theorem : $x – a$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also, if $x – a$ is a factor of $p(x)$, then $p(a) = 0$.

ALGEBRAIC IDENTITIES

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2. $(x - y)2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
   $= x^3 + y^3 + 3x^2y + 3xy^2$
7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
   $= x^3 - y^3 - 3x^2y + 3xy^2$
8. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)$
   $(x^2 + y^2 + z^2 - xy - yz - zx)$
9. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
10. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

CHAPTER 8

Quadrilaterals

A **quadrilateral** is a closed shape and a type of polygon that has four sides, four vertices and four angles. It is formed by joining four non-collinear points. The sum of interior angles of quadrilaterals is always equal to 360 degrees.

The word quadrilateral is derived from the Latin words '**Quadra**' which means four and '**Latus**' means 'sides'

What is a Quadrilateral?

A quadrilateral is a plane figure that has four sides or edges, and also has four corners or vertices. The angles are present at the four vertices or corners of the quadrilateral. If ABCD is a quadrilateral then angles at the vertices are ∠A, ∠B, ∠C and ∠D. The sides of a quadrilateral are AB, BC, CD and DA.

If we join the opposite vertices of the quadrilateral, we get the diagonals. In the below figure AC and BD are the diagonals of quadrilateral ABCD.

Types of Quadrilaterals

The types of quadrilaterals are defined based on the measure of the angles and lengths of their sides. As the word 'Quad' means four, all these types of a quadrilateral have four sides, and the sum of angles of these shapes is 360 degrees. The list of types of quadrilaterals are:

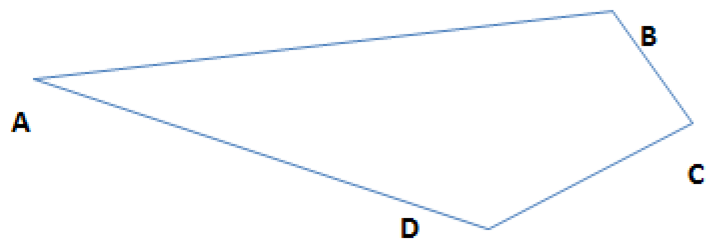
1.
    1. Trapezium
    2. Parallelogram
    3. Squares
    4. Rectangle
    5. Rhombus
    6. Kite

Convex, Concave and Intersecting Quadrilaterals

Another way to classify the types of quadrilaterals are:

1. Convex Quadrilaterals: Both the diagonals of a quadrilateral are completely contained within a figure.
2. Concave Quadrilaterals: At least one of the diagonals lies partly or entirely outside of the figure.

Properties of Quadrilaterals



Let us understand in a better way with the help of an example:

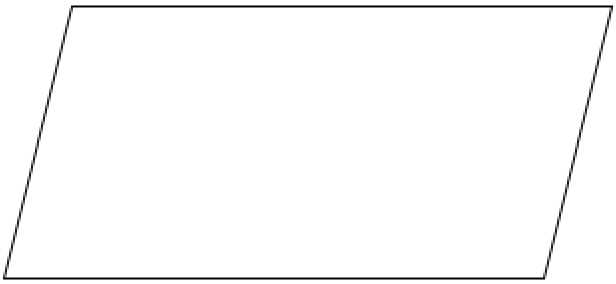
- It has four sides: AB, BC, CD, and DA
- It has four vertices: Points A, B, C, and D
- It has four angles: ∠ABC, ∠BCD, ∠CDA, and ∠DAB
- ∠A and ∠B are adjacent angles
- ∠A and ∠C are the opposite angles
- AB and CD are the opposite sides
- AB and BC are the adjacent sides

A quadrilateral is a 4-sided plane figure. Below are some important properties of quadrilaterals :

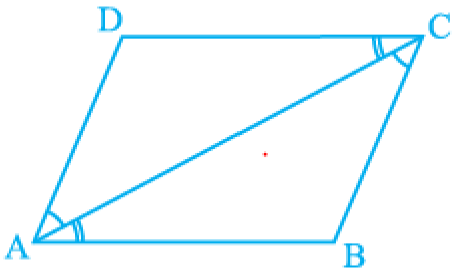
- Every quadrilateral has 4 vertices, 4 angles, and 4 sides
- The total of its interior angles = 360 degrees

Parallelogram

A quadrilateral which has both pairs of opposite sides parallel is called a parallelogram.



Its properties are:
(a)**The diagonal of a parallelogram divide into two congruent triangles**



In triangle ABC and CDA,

$\angle BCA=\angle DAC \angle BCA=\angle DAC$ (Pair of alternate angles between parallels BC and AD)

$\angle BAC=\angle DCA \angle BAC=\angle DCA$ (Pair of alternate angles between parallels AB and DC)

AC=CA

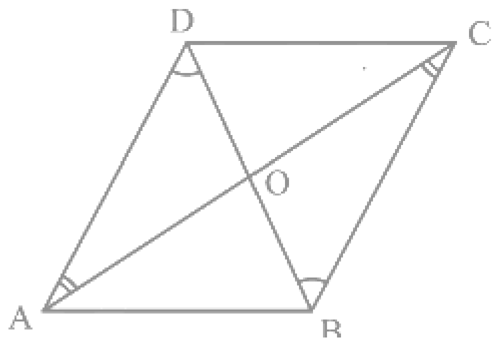
So, $\Delta ABC \cong \Delta CDA$

(b)**The opposite sides of a parallelogram are equal**
This can be seen from above proof only. Since it forms two congruent triangle
AB=DC and AD =BC

(c)**The opposite angles of a parallelogram are equal**
This can be seen from above proof only. Since it forms two congruent triangle

$\angle D=\angle B$ and $\angle A=\angle C$

(d)**The diagonals of a parallelogram bisect each other**



In triangle AOD and COB,

$\angle OAD=\angle OCB$ (Pair of alternate angles between parallels BC and AD)

$\angle ODA=\angle OBC$ (Pair of alternate angles between parallels AD and BC)

AD=BC So, $\Delta AOD\cong\Delta COB$

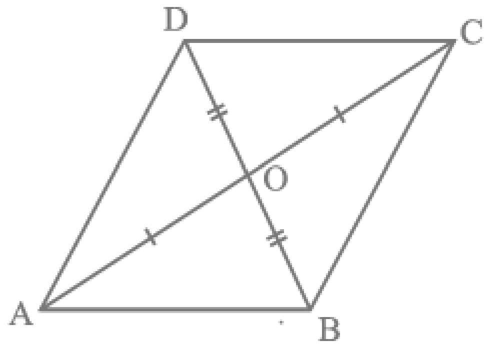
So OA=OC and OB=OD, Thus diagonal bisect each other

A quadrilateral is said to a parallelogram if any of these conditions are met

(a) Opposite sides are equal

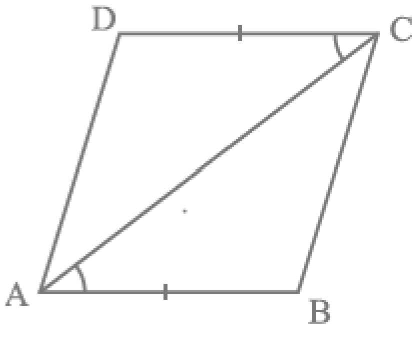
(b) Opposite angles are equal

(c) Diagonal bisects each other



We can easily show here $\Delta AOB\cong\Delta COD$and then $\angle ABO=\angle CDO$ By alternate angle theorem, AB||CD and hence parallelogram

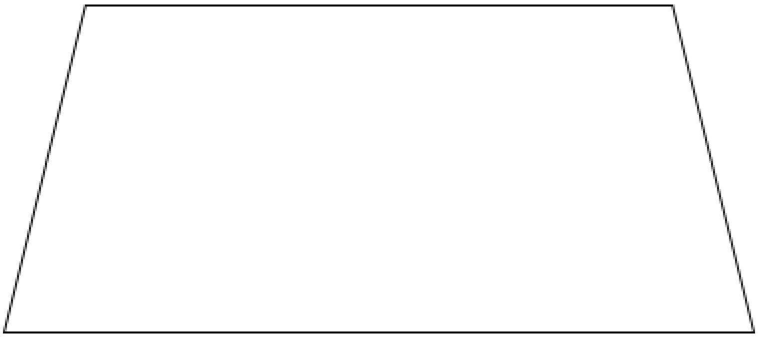
(d) A pair of opposite are parallel and equal



We can easily show here $\Delta ABC\cong\Delta CDA$ and then BC || AD and hence parallelogram

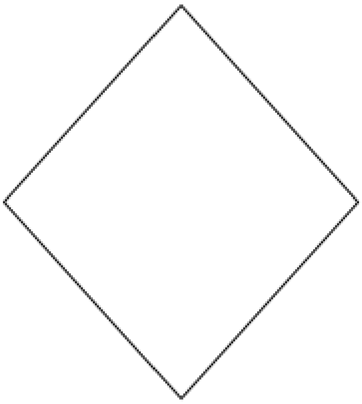
Trapezium

A quadrilateral which has one pair of opposite sides parallel is called a trapezium.



Rhombus

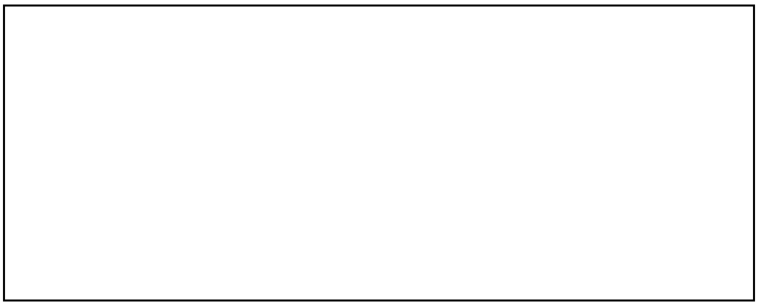
Rhombus is a parallelogram in which any pair of adjacent sides is equal.
Properties of a rhombus:
(a)All sides of a rhombus are equal
(b)The opposite angles of a rhombus are equal
(c)The diagonals of a rhombus bisect each other at right angles.



Rectangle

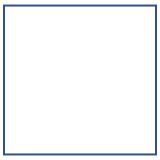
A parallelogram which has one of its angles a right angle is called a rectangle.
Properties of a rectangle are:
(a)The opposite sides of a rectangle are equal
(b) Each angle of a rectangle is a right-angle.
(c) The diagonals of a rectangle are equal.

(d) The diagonals of a rectangle bisect each other.

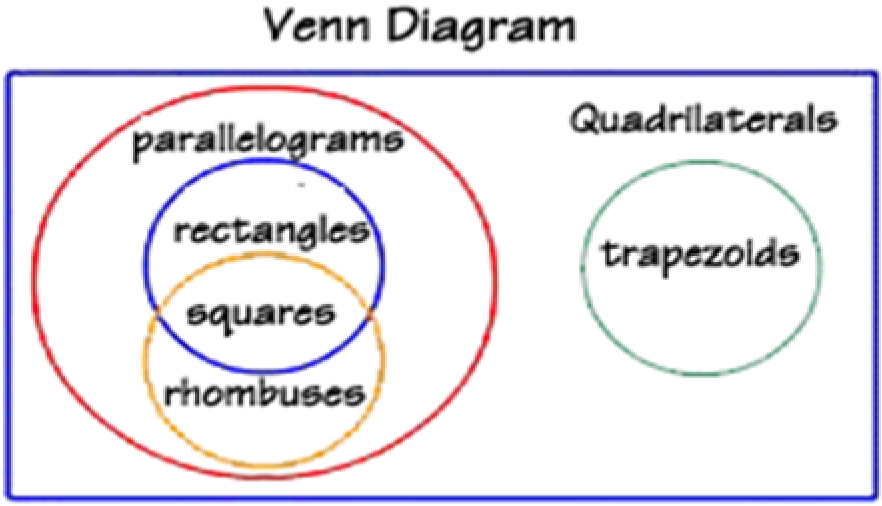


Square

A quadrilateral, all of whose sides are equal and all of whose angles are right angles.
Properties of square are:
(a)All the sides of a square are equal.
(b) Each of the angles measures $90_{0}900$.
(c) The diagonals of a square bisect each other at right angles.
(d) The diagonals of a square are equal.



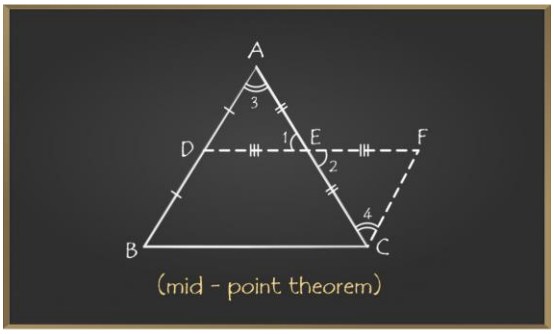
All the quadrilaterals can be shown in Venn diagram like this



Some Other observation from this
(a) A square is always a parallelogram Similarly a rectangle is always a parallelogram
(b) A square is always a rectangle,rhombus
(c) A rhombus can be square.
(d) A rectangle has four right angles.
(e) A rectangle is not always a rhombus
(f) A Trapezium is not a parallelogram

Mid-point Theorem for Triangles

Theorem-I
The line segment joining the mid points of the two sides of the triangle is parallel to the third side



Given: A triangle ABC in which D is the mid-point of AB and E is the mid-point of AC.

To Prove: DE ∥ BC and DE = 1/2(BC)

Construction

Extend the line segment joining points D and E to F such that DE = EF and join CF.

Proof

In △AED and △CEF

DE = EF (construction)

∠1 = ∠2 (vertically opposite angles)

AE = CE (E is the mid-point)

△AED ≅ △CEF by SAS criteria

Therefore,

∠3 =∠4 (c.p.c.t)

But these are alternate interior angles.

So, AB ∥ CF

AD = CF(c.p.c.t)

But AD = DB (D is the mid-point)

Therefore, BD = CF

In BCFD

BD ∥ CF (as AB ∥ CF)

BD = CF

BCFD is a parallelogram as one pair of opposite sides is parallel and equal.

Therefore,

DF ∥ BC (opposite sides of parallelogram)

DF = BC (opposite sides of parallelogram)

As DF ∥ BC, DE ∥ BC and DF = BC

But DE = EF

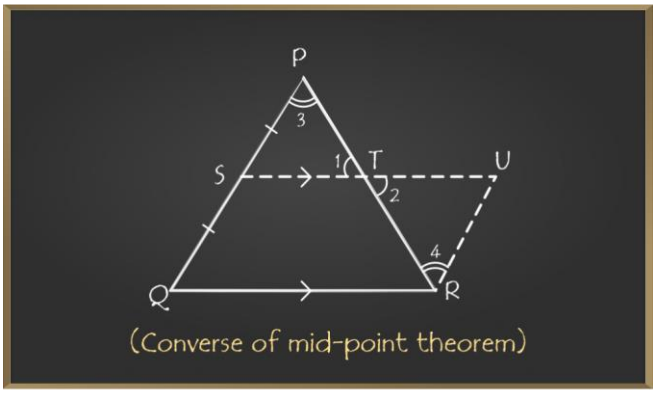
So, DF = 2(DE)

2(DE) = BC

DE = 1/2(BC)

Hence, proved that the line joining mid-points of two sides of the triangle is parallel to the third side and is half of it.

Theorem-II
A line drawn through mid point of one side of a triangle and parallel to another side bisect the third side of the triangle.



Proof of the Theorem

Given: In triangle PQR, S is the mid-point of PQ and ST ∥ QR

To Prove: T is the mid-point of PR.

Construction

Draw a line through R parallel to PQ and extend ST to U.

Proof

ST∥QR(given)

So, SU∥QR

PQ∥RU (construction)

Therefore, SURQ is a parallelogram.

SQ = RU(Opposite sides of parallelogram)

But SQ = PS (S is the mid-point of PQ)

Therefore, RU = PS

In △PST and △RUT

∠1 =∠2(vertically opposite angles)

∠3 =∠4(alternate angles)

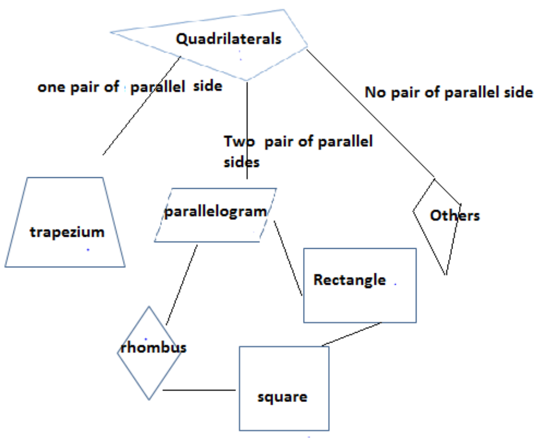
PS = RU(proved above)

△PST ≅ △RUT by AAS criteria

Therefore, PT = RT

T is the mid-point of PR.

Sides and Angles of Quadrilaterals

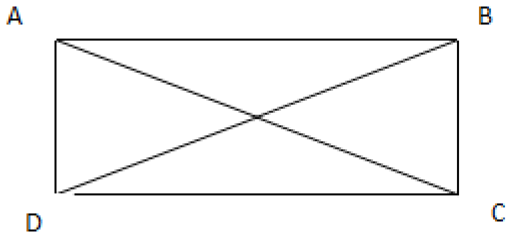


Sides and angles	**Square**	**Rectangle**	**Rhombus**	**Parallelogram**	**Trapezium**
All sides are equal	Yes	No	Yes	No	No
Opposite sides are parallel	Yes	Yes	Yes	Yes	Yes
Opposite sides are equal	Yes	Yes	Yes	Yes	No
All the angles are of the same measure	Yes	Yes	No	No	No
Opposite angles are of equal measure	Yes	Yes	Yes	Yes	No
Diagonals bisect each other	Yes	Yes	Yes	Yes	No
Two adjacent angles are supplementary	Yes	Yes	Yes	Yes	No

Quadrilaterals Exercise 8.1

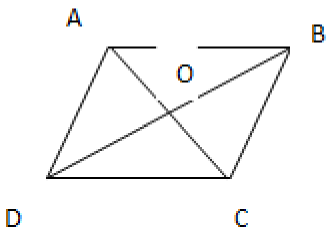
Question 1
The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the Quadrilateral
Solution:
Let the common ratio between the angles be y. Therefore, the angles will be 3y, 5y, 9y, and 13y respectively.
As the sum of all interior angles of a quadrilateral is 360^0
$3y + 5y + 9y + 13y = 360^0$
$30y= 360^0$
$y = 12^0$
Hence, the angles are
3y=36
$5y = 5 \times 12 = 60^0$
$9y = 9 \times 12 = 108^0$
$13y = 13 \times 12 = 156^0$

Question 2
If the diagonals of a parallelogram are equal, then show that it is a rectangle
Solution:
Let ABCD is a parallelogram



Given
AC=BD and AD=BC and CD=AB (in parallelogram opposite side are equal)
To Prove:
Show that it is a rectangle
Proof
In Δ ADC and Δ BCD
AD=BC
CD =CD
AC=BD
So by **SSS congruence**
ΔADC≅ΔBCD
So ∠ADC=∠BCD by CPCT (corresponding parts of the two congruent triangles)
But, we also have ∠ADC+∠BCD=180º (Co-interior angles because BC?AD)
So, 2∠ADC=180
=> ∠ADC=90
So ∠BCD=90
So all the angles A, B, C, D are 90^0 .Hence rectangles

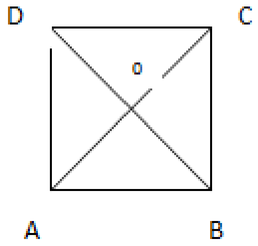
Question 3
Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.
Solution:
Given
Let ABCD be the quadrilateral and AC and BD are diagonal which bisect at right angles
OA = OC, OB = OD and AOB = BOC = COD = AOD = 90º



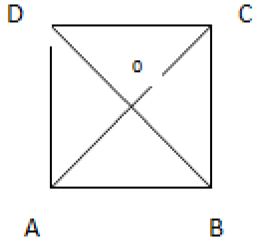
To Prove: ABCD is rhombus
Proof:
Now, in ΔAOD and Δ COD
OA = OC (Diagonal bisects each other)
∠AOD =∠COD (given)
OD = OD (common)
So by **SAS congruence** rule
ΔAOD ≅ ΔCOD AD = CD(by CPCT) (i)
Similarly we can prove that

AD = AB and CD = BC (ii)
From equations (i) and (ii), we can say that
AB = BC = CD = AD
Since all sides are equal, it is a rhombus

Question 4
Show that the diagonals of a square are equal and bisect each other at right angles.
Solution:
Let ABCD is a square whose diagonal BD and AC intersect art O



Given
AB=BC=CD=AD



To prove: Diagonal are equal and bisects each other at right angle
OA=OC
OD=OB
AC=BD

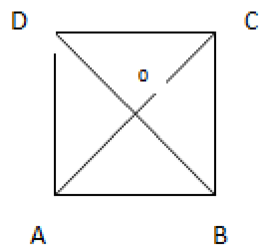
Proof:
In ΔABC andΔDCB,
AB = DC (From Sides of a square are equal to each other)
∠ABC =∠DCB (All interior angles are of 90)
BC = CB (Common side)
So by **SAS congruence**
ΔABC≅ΔDCB
AC = DB (By CPCT)
Hence, the diagonals of a square are equal in length.
In ΔAOB and ΔCOD,
∠AOB =∠COD (Vertically opposite angles)
∠ABO =∠CDO (Alternate interior angles)
AB = CD (Sides of a square are always equal)
By AAS congruence rule
ΔAOB ≅ ΔCOD
∴AO = CO and OB = OD (By CPCT)
Hence, the diagonals of a square bisect each other.

In ΔAOB and ΔCOB,
As we had proved that diagonals bisect each other, therefore,

AO = CO
AB = CB (we know that Sides of a square are equal)
BO = BO (Common)
By SSS congruency
ΔAOB ≅ ΔCOB

∴∠AOB =∠COB (By CPCT)
However,∠AOB +∠COB = 180^0 (Linear pair)
2∠AOB = 180^0
∠AOB = 90^0
Hence, the diagonals of a square bisect each other at right angles.

Question 5
Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square
Solution:
Let ABCD is a quadrilateral whose diagonal BD and AC bisect each other at right angle at O



Given
AO=OC, BO=OD, AC=BD
∠AOB=∠COD
To Prove: ABCD is a square
Proof:
In ΔAOB and ΔCOD
AO=CO (Given)
∠AOB=∠COD (Each given equal to 90^o)
BO=DO (Given)
So by **SAS congruence** rule,
ΔAOB ≅ ΔCOD.
Therefore
∠OBA=∠ODC (by CPCT)
But, these are alternate interior angles which means that AB?CD. (i)
Similarly, we can prove that BC?AD (ii)
From (1) and (2), we can say that quadrilateral ABCD is a parallelogram. Hence, we have AB=CD and BC=AD because opposite sides of a parallelogram are equal. (iii)

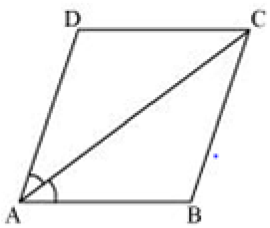
Now, in ΔAOB and ΔAOD
AO=AO (Common)
∠AOB=∠AOD (Each given equal to 90^o)
OB=OD (Given)
So by **SAS congruence** rule
ΔAOB ≅ ΔAOD
Therefore
AB=AD (by CPCT) (iv)

In ΔACD and ΔBDC
AC=BD (Given)

AD=BC (Proved above in (1))
CD=DC (Common)
So by SSS congruence rule
ΔACD ≅ ΔBDC
Therefore
∠ADC=∠BCD (by CPCT) (v)
But, we also have
∠ADC + ∠BCD= 180^o (Co-Interior angles) (vi)
From (5) and (6), we can say that
∠ADC+∠ADC=180^o
2∠ADC=180^o
∠ADC=180/2=90^o (vii)

From (iii), (iv) -> we can say that ABCD is a parallelogram having all the sides equal.
(vii) -> we have showed that it's one angle is equal to 90^owhich is enough to consider it a square. Therefore, ABCD is a square.

Question 6
Diagonal AC of a parallelogram ABCD bisects ∠A (see the given figure). Show that
(i) It bisects ∠C also,
(ii) ABCD is a rhombus.



Solution:
(i)
ABCD is a parallelogram and AC bisect angle A
∠DAC =∠BAC

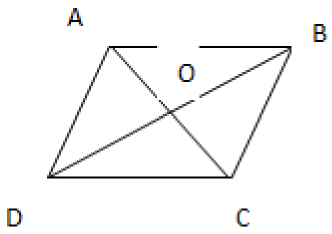
To Prove: AC bisects ∠C
Proof:
As ABCD is a parallelogram
∠DAC =∠BCA (Alternate interior angles) ... (i)
And ∠BAC =∠DCA (Alternate interior angles) ... (ii)
But it is given that AC bisects ∠A.
∠DAC =∠BAC ... (iii)
From equations (i), (ii) and (iii), we have
∠DAC =∠BCA =∠BAC =∠DCA ... (iv)
=> ∠DCA =∠BCA
Hence, AC bisects ∠C.

(ii)

To Prove: ABCD is a rhombus
Proof :

From equation (iv), we have
∠DAC =∠DCA DA = DC (we know that side opposite to equal angles are equal)
But DA = BC and AB = CD (opposite sides of parallelogram are equal)
AB = BC = CD = DA
Hence, ABCD is rhombus

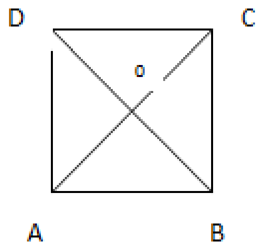
Question 7
ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.
Solution
Given: ABCD is a rhombus



To Prove: Diagonal BD bisect angle B and D
Diagonal AC bisect angle A and C
Proof:
Let us join AC.
In ΔABC,
BC = AB (Sides of a rhombus are equal to each other)
∴∠BAC =∠BCA (Angles opposite to equal sides of a triangle are equal)

However,∠BAC =∠DCA (Alternate interior angles for parallel lines AB and CD)
∠BCA=∠DCA
Therefore, AC bisects ∠C.
Also,∠BCA=∠DAC (Alternate interior angles for || lines BC and DA)
∠BAC =∠DAC
Therefore, AC bisects ∠A.
Similarly, it can be proved that BD bisects ∠B and ∠D as well.

Question 8
ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i)ABCD is a square
(ii) Diagonal BD bisects ∠ B as well as ∠ D
Solution
Given
ABCD is a rectangle and ∠DAC=∠BAC and ∠DCA=∠BCA

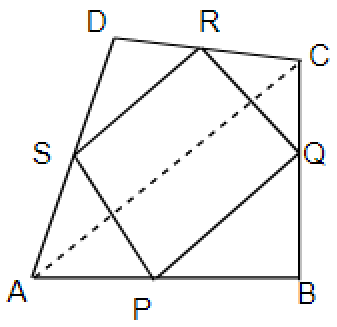


(i)
To Prove: ABCD is a square
Proof:
In ΔADC and ΔABC
∠CAD=∠CAB (Given)
AC=AC (Common)
∠DCA=∠BCA (Given)
Therefore, by ASA congruence rule
ΔADC ≅ ΔABC
AD=AB (by CPCT) (2)
From (1) and (2), we can say that ABCD is a rectangle having all the sides equal. It means that ABCD is a square.

(ii)
To prove diagonal BD bisects ∠B as well as ∠D.
In solution (i), we have showed that ABCD is a square.
Now in ΔCBD and ΔABD
BC=BA (Sides of square are equal)
BD=BD (Common)
CD=AD (Sides of square are equal)
So by **SSS congruence** rule
ΔCBD ≅ ΔABD
∠CBD=∠ABD (by CPCT) (3)
And,∠CDB=∠ADB (by CPCT) (4)
From (3) and (4), we can say that BD bisects ∠B as well as ∠D.

Quadrilaterals Exercise 8.2

Question 1.
ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that



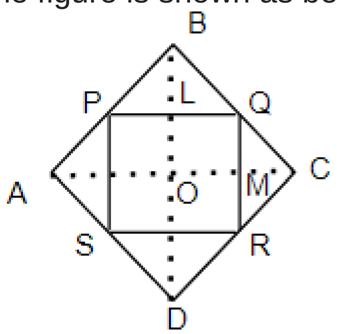
(i) SR || AC and SR =1/2 AC
(ii) PQ = SR
(iii) PQRS is a parallelogram
Solution
(i) In Δ ACD
S is the mid point of AD
R is the mid point of CD
By Mid point theorem

SR || AC and SR =1/2 AC --(X)

(ii)
In Δ ABC
P is the mid point of AB
Q is the mid point of BC
By Mid point theorem
PQ || AC and PQ =1/2 AC --(Y)
From equation X and Y
PQ = SR
and PQ || SR||AC

(iii) In Quadrilaterals PQRS
PQ=SR
and PQ||SR
Hence PQRS is a parallelogram

Question 2.
ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.
Solution
The figure is shown as below



To Prove: quadrilateral PQRS is a rectangle
Proof:
In Δ ABC
P and Q are mid points of sides AB and BC
By Mid point theorem
PQ= 1/2 AC and PQ || AC -(X)

In Δ ACD
S and R are mid points of sides AD and DC
By Mid point theorem
SR= 1/2 AC and SR || AC --(Y)
From (X) and (Y), we have
PQ=SR
PQ ||SR
Hence PQSR is a parallelogram

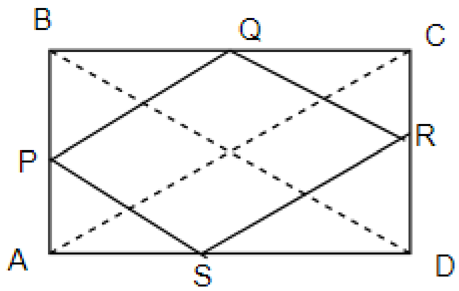
Now in Δ BCD
Q and R are mid points of sides BC and DC
By Mid point theorem

BD || QR
or
QM || OL

Also we already proved that
PQ ||SR
or LQ ||OM

In LQOM
QM || OL
LQ ||OM
Therefore LQOM is a parallelogram
Now,we know that opposite angles in the parallelogram are equal
∠ LQM = ∠ LOM
Now ∠ LOM=90 as diagonals bisects at right angle in Rhombus
Therefore
∠ LQM=90°
or
∠ PQR=90°
Now
PQRS is a parallelogram with one angle as right angle. Hence PQRS is a rectangle

Question 3.
ABCD is a rectangle and P, Q, R and S are mid points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rhombus.
Solution
The figure is shown as below



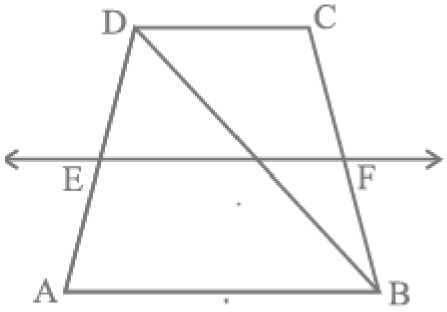
To Prove: quadrilateral PQRS is a rhombus
Proof:
In Δ ABC
P and Q are mid points of sides AB and BC
By Mid point theorem
PQ= 1/2 AC and PQ || AC -(X)

In Δ ACD
S and R are mid points of sides AD and DC
By Mid point theorem
SR= 1/2 AC and SR || AC --(Y)
From (X) and (Y), we have
PQ=SR
PQ ||SR

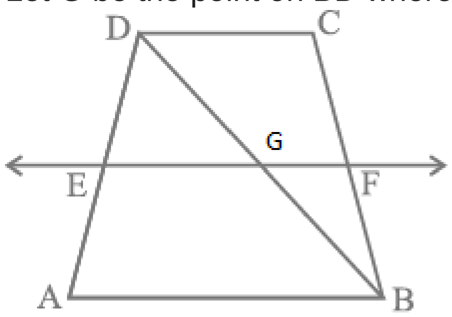
Hence PQSR is a parallelogram

Now in Δ BCD
Q and R are mid points of sides BC and DC
By Mid point theorem
QR= 1/2 BD
Nowe AC = BC
Hence
PQ=SR=QR
Now a parallelogram whose adjacent sides are equal is a rhombus.
Hence proved

Question 4.
ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F . Show that F is the mid-point of BC.



Solution
Let G be the point on BD where EF intersect

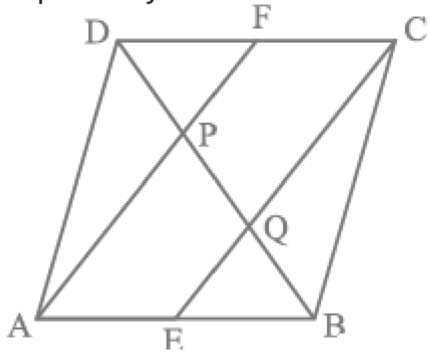


Now in Δ ABD
E is the mid-point of AD
EG ||AB
By converse of Mid-point theorem
G is the mid-point of BD

Now in Δ BDC
G is the mid-point of BD
FG || BC
By converse of Mid-point theorem
G is the mid-point of BC

Question 5.

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Solution

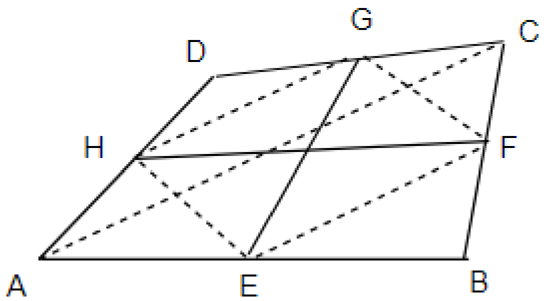
AB=CD (Opposite side of parallelogram)
Since E and F are mid-points of AB and DC respectively.
AE = 1/2 AB and CF = 1/2 DC
Therefore
AE = CF and AE || CF.
One pair of opposite sides is parallel and equal
Hence AECF is a parallelogram.
AF ||EC
In Δ BAP
E is the mid-point of AB
Now AF ||EC
So, EQ || AP
Now Converse of mid-point theorem
Q is mid-point of PB
PQ = QB ---(X)
Similarly, in ΔDQC, P is the mid-point of DQ
DP = PQ ---(Y)
From (X) and (Y), we have DP = PQ = QB

Question 6.

Show that the line segments joining the mid- point of the opposite sides of a quadrilateral bisect each other.

Solution

To Prove : EG and FH bisect each other.
Construction : Join EF, FG, GH, HE and AC



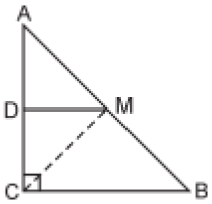
Proof:
In Δ ABC

E and F are mid-points of AB and BC respectively.
By Mid point theorem
EF = 1/2 AC and EF || AC ----(i)
In Δ ADC
H and G are mid-points of AD and CD respectively.
By Mid point theorem
HG = 1/2 AC and HG || AC ----(ii)
From (i) and (ii), we get
EF = HG
and EF || HG
Hence EFGH is a parallelogram
Now We know that Diagonals of the parallelogram bisects each other
Here EG and FH are diagonals of the parallelogram EFGH.
Hence Proved

Question 7.
ABC is a triangle right angles at C. A line through the mid -point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

1. D is mid- point of AC
2. MD ⊥ AC
3. CM=MA=1/2AB

Solution



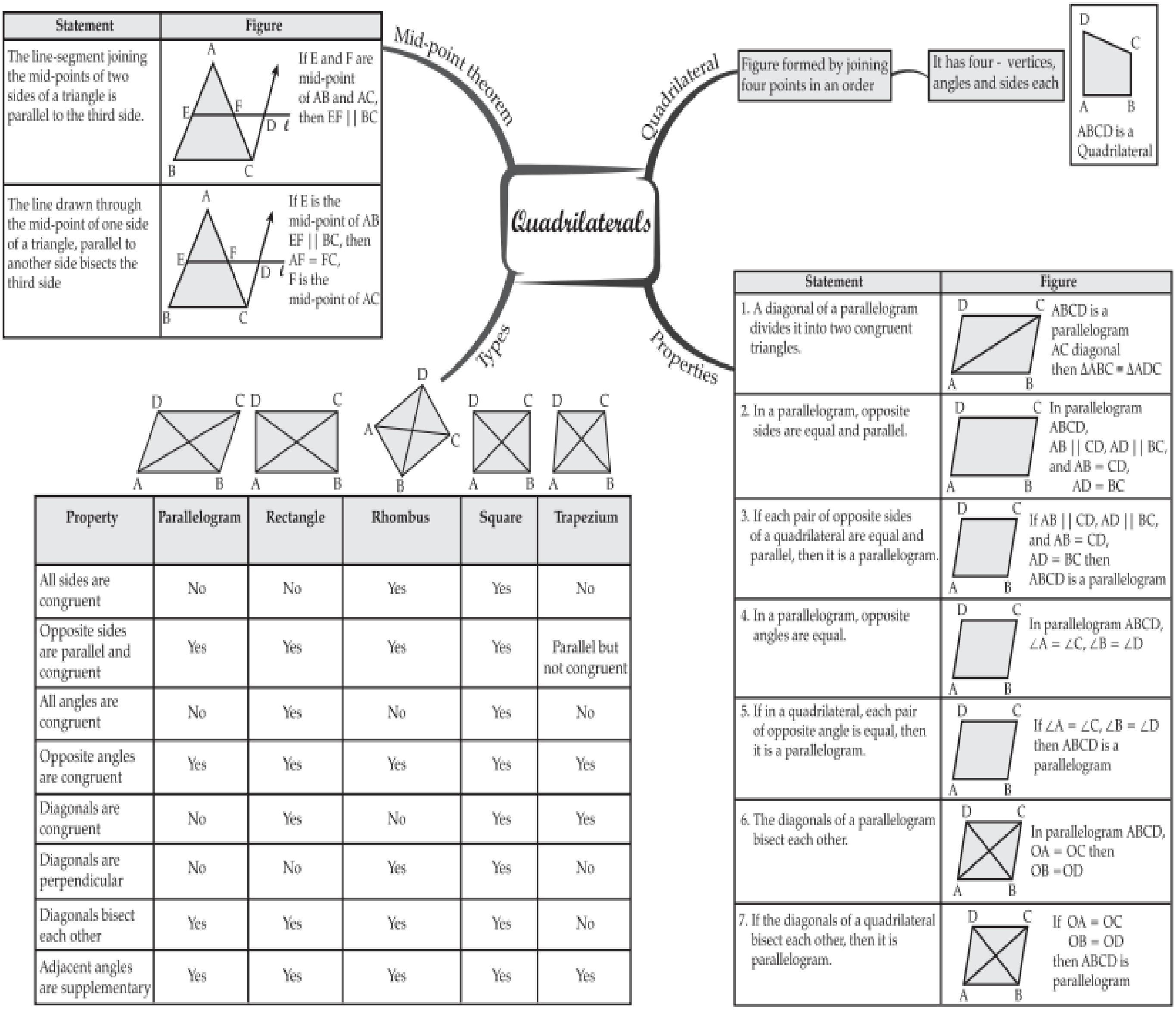
Construction : Join CM.
Proof:
(i) In Δ ABC
M is the mid-point of AB.
BC || DM
By converse of Mid-point theorem
D is the mid-point of AC

(ii)
Now BC || DM
Hence Corresponding angles]
∠ ADM = ∠ACB
Now ∠ACB=90°
Hence
∠ ADM =90°
Hence MD ⊥ AC

(iii) In Δ ADM and CMD
AD=DC
∠ ADM = ∠ CDM = 90°
DM is common

Therefore by SAS congruence
&Delta ADM ≅ &Delta CDM
Now by CPCT
CM=MA
Since M is mid-point of AB,
MA = 1/2 AB
Hence, CM=MA=1/2AB

MIND MAP



Mid-point theorem

Statement	Figure
The line-segment joining the mid-points of two sides of a triangle is parallel to the third side.	If E and F are mid-point of AB and AC, then EF || BC
The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side	If E is the mid-point of AB EF || BC, then AF = FC, F is the mid-point of AC

Types

Property	Parallelogram	Rectangle	Rhombus	Square	Trapezium
All sides are congruent	No	No	Yes	Yes	No
Opposite sides are parallel and congruent	Yes	Yes	Yes	Yes	Parallel but not congruent
All angles are congruent	No	Yes	No	Yes	No
Opposite angles are congruent	Yes	Yes	Yes	Yes	Yes
Diagonals are congruent	No	Yes	No	Yes	Yes
Diagonals are perpendicular	No	No	Yes	Yes	No
Diagonals bisect each other	Yes	Yes	Yes	Yes	No
Adjacent angles are supplementary	Yes	Yes	Yes	Yes	Yes

Properties

Statement	Figure
1. A diagonal of a parallelogram divides it into two congruent triangles.	ABCD is a parallelogram AC diagonal then ΔABC ≅ ΔADC
2. In a parallelogram, opposite sides are equal and parallel.	In parallelogram ABCD, AB || CD, AD || BC, and AB = CD, AD = BC
3. If each pair of opposite sides of a quadrilateral are equal and parallel, then it is a parallelogram.	If AB || CD, AD || BC, and AB = CD, AD = BC then ABCD is a parallelogram
4. In a parallelogram, opposite angles are equal.	In parallelogram ABCD, ∠A = ∠C, ∠B = ∠D
5. If in a quadrilateral, each pair of opposite angle is equal, then it is a parallelogram.	If ∠A = ∠C, ∠B = ∠D then ABCD is a parallelogram
6. The diagonals of a parallelogram bisect each other.	In parallelogram ABCD, OA = OC then OB = OD
7. If the diagonals of a quadrilateral bisect each other, then it is parallelogram.	If OA = OC OB = OD then ABCD is parallelogram