

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH-13 SURFACE AREAS AND VOLUMES

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE: 1 SURFACE AREA

- Q.1. A tent of height 8.25 m is in the form of a right circular cylinder with diameter of base 30 m and height 5.5 m, surmounted by a right circular cone of the same base. Find the cost of the tent at the rate of rupees 45 per m^2 .
- Q.2 A golf ball has diameter equal to 4.2 cm. Its surface has 200 dimples each of radius 2 mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.

TYPE: 2 COMPARISION OF VOLUME OF TWO SOLIDS

- Q.1. If the diameter of cross-section of a wire is decreased by 5%, how much percent will the length increased so that the volume remains the same.
- Q.2 The radii of the bases of two right circular solid cones of same height are X and Y respectively. The cones are melted and recast into a solid sphere of radius Z. Show that the height of each cone is given by $h = \frac{4Z^3}{X^2+Y^2}$.
- Q.3 A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 12 m deep. If water flows through the pipe at the rate of 3km/h, in how much time will the tank be filled?

- Q.4 A cistern, internally measuring 150 cm x 120 cm x 110 cm has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one seventeenth of its own volume of water. How many bricks can be put in without the water overflowing, each brick being $22.5 \text{ cm} \times 7.5 \text{ cm} \times 6.5 \text{ cm}$?
- Q.5 Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. If the base of the shed is 7 m by 15 m and height of the cuboidal portion is 8 m, find the volume of the air that the shed can hold. If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space

ACHIEVER'S SECTION

1. The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14cm long is 88cm^2 . If the volume of metal used in making cylinder is 176 cm^3 , find outer and inner diameters of the cylinder?
2. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.
3. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of cylinder is $\frac{14}{3} \text{ m}$ and radius of the hemisphere is 3.5m. Find the volume and internal surface area of the solid.
4. The difference between outside and inside surface areas of cylindrical metallic pipe 14cm long is 44m^2 . If pipe is made of metal of 99cm^2 . Find outer and inner radii of pipe.
5. A hemispherical depression is cut from one face of a cubical wooden block of edge 21cm such that the diameter of hemisphere is equal to the edge of cube. Determine the volume and total surface area of the remaining block.
6. A tag is in the form of a hemisphere surmounted by a right circular cone of same base radius as that of the hemisphere. If the radius of the base of cone is 21cm and its volume is $\frac{2}{3}$ of volume of hemisphere. Calculate height of cone and surface area of toy.

7. A hollow sphere of internal and external diameter 4cm and 8cm is melted into a cone of base diameter 8cm. Calculate height of cone.
8. A cylindrical jar of radius 6cm contains oil. Iron sphere each of radius 1.5cm are immersed in the oil. How many spheres are necessary to raise level of the oil by two centimeter?
9. The difference between outer and inner curved surface areas of a hollow right circular cylinder 14cm long is 88cm^2 . If the volume of metal used in making cylinder is 176cm^3 . Find the outer and inner diameters of the cylinder.
10. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 cm.
 - (i) How many students can sit in the tent if a student on an average, occupies $\frac{5}{7}\text{m}^2$ on the ground?
 - (ii) Find the Volume of the cone.

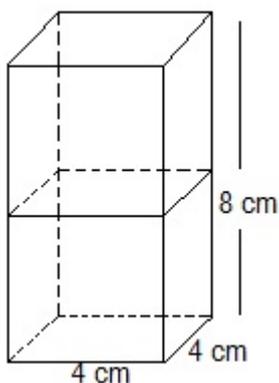
Class 10 Chapter 13 – Surface Areas and Volumes

Exercise: 13.1

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Answer:

The diagram is given as:



Given,

The Volume (V) of each cube is $= 64 \text{ cm}^3$

This implies that $a^3 = 64 \text{ cm}^3$

$\therefore a = 4 \text{ cm}$

Now, the side of the cube $= a = 4 \text{ cm}$

Also, the length and breadth of the resulting cuboid will be 4 cm each. While its height will be 8 cm.

So, the surface area of the cuboid $= 2(lb+bh+lh)$

$$= 2(8 \times 4 + 4 \times 4 + 4 \times 8) \text{ cm}^2$$

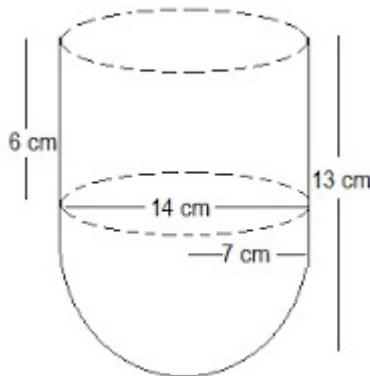
$$= 2(32 + 16 + 32) \text{ cm}^2$$

$$= (2 \times 80) \text{ cm}^2 = 160 \text{ cm}^2$$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer:

The diagram is as follows:



Now, the given parameters are:

The diameter of the hemisphere = $D = 14$ cm

The radius of the hemisphere = $r = 7$ cm

Also, the height of the cylinder = $h = (13 - 7) = 6$ cm

And, the radius of the hollow hemisphere = 7 cm

Now, the inner surface area of the vessel = CSA of the cylindrical part + CSA of hemispherical part

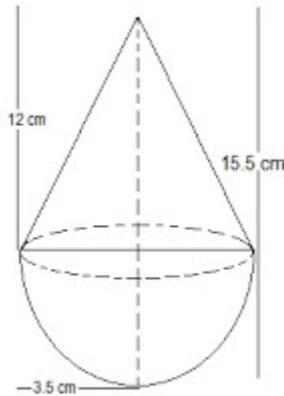
$$(2\pi rh + 2\pi r^2) \text{ cm}^2 = 2\pi r(h+r) \text{ cm}^2$$

$$2 \times (22/7) \times 7(6+7) \text{ cm}^2 = 572 \text{ cm}^2$$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer:

The diagram is as follows:



Given that the radius of the cone and the hemisphere (r) = 3.5 cm or $7/2$ cm

The total height of the toy is given as 15.5 cm.

So, the height of the cone (h) = $15.5 - 3.5 = 12$ cm

$$\text{Slant height of the cone}(l) = \sqrt{h^2 + r^2}$$

$$\Rightarrow l = \sqrt{12^2 + (3.5)^2}$$

$$\Rightarrow l = \sqrt{12^2 + (7/2)^2}$$

$$\Rightarrow l = \sqrt{144 + 49/4} = \sqrt{(576 + 49)/4} = \sqrt{625/4}$$

$$\Rightarrow l = 25/2$$

\therefore The curved surface area of cone = $\pi r l$

$$(22/7) \times (7/2) \times (25/2) = 275/2 \text{ cm}^2$$

Also, the curved surface area of the hemisphere = $2\pi r^2$

$$2 \times (22/7) \times (7/2)^2$$

$$= 77 \text{ cm}^2$$

Now, the Total surface area of the toy = CSA of cone + CSA of hemisphere

$$= (275/2) + 77 \text{ cm}^2$$

$$= (275 + 154)/2 \text{ cm}^2$$

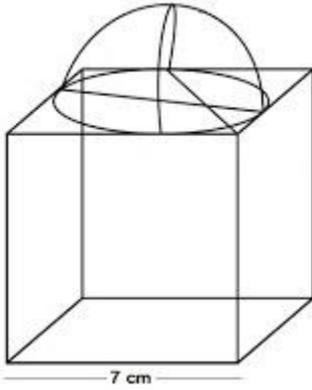
$$= 429/2 \text{ cm}^2 = 214.5 \text{ cm}^2$$

So, the total surface area (TSA) of the toy is 214.5 cm^2

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer:

It is given that each side of cube is 7 cm. So, the radius will be $7/2$ cm.



We know,

The total surface area of solid (TSA) = surface area of cubical block + CSA of hemisphere – Area of base of hemisphere

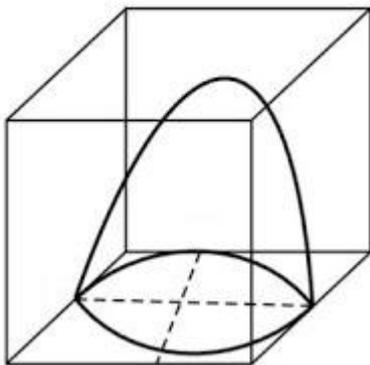
$$\begin{aligned} \therefore \text{TSA of solid} &= 6 \times (\text{side})^2 + 2\pi r^2 - \pi r^2 \\ &= 6 \times (\text{side})^2 + \pi r^2 \\ &= 6 \times (7)^2 + (22/7) \times (7/2) \times (7/2) \\ &= (6 \times 49) + (77/2) \\ &= 294 + 38.5 = 332.5 \text{ cm}^2 \end{aligned}$$

So, the surface area of the solid is 332.5 cm^2

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer:

The diagram is as follows:



Now, the diameter of hemisphere = Edge of the cube = l

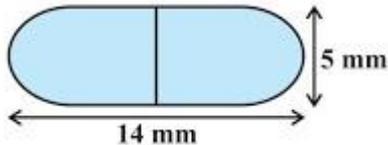
So, the radius of hemisphere = $l/2$

\therefore The total surface area of solid = surface area of cube + CSA of hemisphere – Area of base of hemisphere

$$\text{TSA of remaining solid} = 6 (\text{edge})^2 + 2\pi r^2 - \pi r^2$$

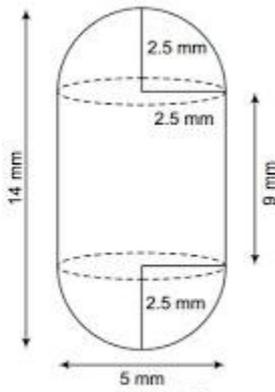
$$\begin{aligned}
&= 6l^2 \pi r^2 \\
&= 6l^2 + \pi (l/2)^2 \\
&= 6l^2 + \pi l^2/4 \\
&= l^2/4(24 + \pi) \text{ sq. units}
\end{aligned}$$

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Answer:

Two hemisphere and one cylinder are shown in the figure given below.



Here, the diameter of the capsule = 5 mm

$$\therefore \text{Radius} = 5/2 = 2.5 \text{ mm}$$

Now, the length of the capsule = 14 mm

$$\text{So, the length of the cylinder} = 14 - (2.5 + 2.5) = 9 \text{ mm}$$

$$\therefore \text{The surface area of a hemisphere} = 2\pi r^2 = 2 \times (22/7) \times 2.5 \times 2.5$$

$$= 275/7 \text{ mm}^2$$

Now, the surface area of the cylinder = $2\pi rh$

$$= 2 \times (22/7) \times 2.5 \times 9$$

$$(22/7) \times 45 = 990/7 \text{ mm}^2$$

Thus, the required surface area of medicine capsule will be

$$= 2 \times \text{surface area of hemisphere} + \text{surface area of the cylinder}$$

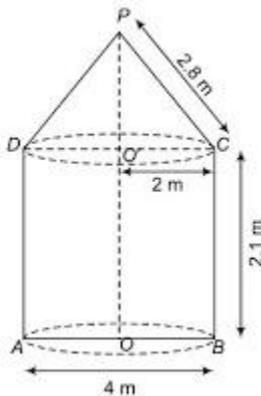
$$= (2 \times 275/7) + 990/7$$

$$(550/7) + (990/7) = 1540/7 = 220 \text{ mm}^2$$

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m². (Note that the base of the tent will not be covered with canvas.)

Answer:

It is known that a tent is a combination of cylinder and a cone.



From the question we know that

Diameter = 4 m

Slant height of the cone (l) = 2.8 m

Radius of the cone (r) = Radius of cylinder = $4/2 = 2$ m

Height of the cylinder (h) = 2.1 m

So, the required surface area of tent = surface area of cone + surface area of cylinder

$$= \pi r l + 2\pi r h$$

$$= \pi r (l + 2h)$$

$$= (22/7) \times 2 (2.8 + 2 \times 2.1)$$

$$= (44/7) (2.8 + 4.2)$$

$$= (44/7) \times 7 = 44 \text{ m}^2$$

\therefore The cost of the canvas of the tent at the rate of ₹500 per m² will be

$$= \text{Surface area} \times \text{cost per m}^2$$

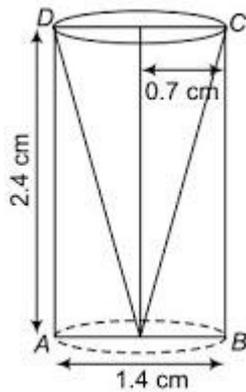
$$44 \times 500 = ₹22000$$

So, ₹22000 will be the total cost of the canvas.

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Answer:

The diagram for the question is as follows:



From the question we know the following:

The diameter of the cylinder = diameter of conical cavity = 1.4 cm

So, the radius of the cylinder = radius of the conical cavity = $1.4/2 = 0.7$

Also, the height of the cylinder = height of the conical cavity = 2.4 cm

$$\begin{aligned}
 \therefore \text{Slant height of the conical cavity } (l) &= \sqrt{h^2 + r^2} \\
 &= \sqrt{(2.4)^2 + (0.7)^2} \\
 &= \sqrt{5.76 + 0.49} = \sqrt{6.25} \\
 &= 2.5 \text{ cm}
 \end{aligned}$$

Now, the TSA of remaining solid = surface area of conical cavity + TSA of the cylinder

$$= \pi r l + (2\pi r h + \pi r^2)$$

$$= \pi r (l + 2h + r)$$

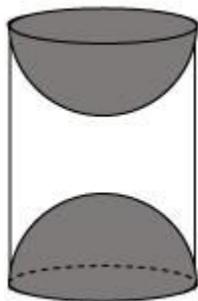
$$= (22/7) \times 0.7 (2.5 + 4.8 + 0.7)$$

$$= 2.2 \times 8 = 17.6 \text{ cm}^2$$

So, the total surface area of the remaining solid is 17.6 cm^2

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in given figure. If the height of the cylinder is 10 cm, and its base is of

radius 3.5 cm, find the total surface area of the article. $\left[\text{Use } \pi = \frac{22}{7} \right]$



ANSWER:

Given that,

Radius (r) of cylindrical part = Radius (r) of hemispherical part = 3.5 cm

Height of cylindrical part (h) = 10 cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$$\begin{aligned} &= 2\pi rh + 2 \times 2\pi r^2 \\ &= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5 \\ &= 70\pi + 49\pi \\ &= 119\pi \\ &= 17 \times 22 = 374 \text{ cm}^2 \end{aligned}$$

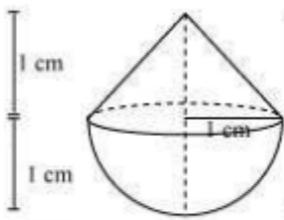
Exercise: 13.2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Solution:

Here $r = 1$ cm and $h = 1$ cm.

The diagram is as follows.



Now, Volume of solid = Volume of conical part + Volume of hemispherical part

We know the volume of cone = $\frac{1}{3} \pi r^2 h$

And,

The volume of hemisphere = $\frac{2}{3} \pi r^3$

So, volume of solid will be

$$\begin{aligned} &= \frac{1}{3} \pi (1)^2 [1 + 2(1)] \text{ cm}^3 = \frac{1}{3} \pi \times 1 \times [3] \text{ cm}^3 \\ &= \pi \text{ cm}^3 \end{aligned}$$

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Solution:

Given,

Height of cylinder = $12 - 4 = 8$ cm

Radius = 1.5 cm

Height of cone = 2 cm

Now, the total volume of the air contained will be = Volume of cylinder + $2 \times$ (Volume of cone)

$$\therefore \text{Total volume} = \pi r^2 h + [2 \times (\frac{1}{3} \pi r^2 h)]$$

$$= 18\pi + 2(1.5\pi)$$

$$= 66 \text{ cm}^3.$$

3. A Gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure).

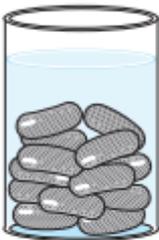


Fig. 13.15

Solution:

It is known that the gulab jamuns are similar to a cylinder with two hemispherical ends.

So, the total height of a gulab jamun = 5 cm.

Diameter = 2.8 cm

So, radius = 1.4 cm

\therefore The height of the cylindrical part = $5 \text{ cm} - (1.4 + 1.4) \text{ cm}$

$$= 2.2 \text{ cm}$$

Now, total volume of One Gulab Jamun = Volume of Cylinder + Volume of two hemispheres

$$= \pi r^2 h + (4/3)\pi r^3$$

$$= 4.312\pi + (10.976/3)\pi$$

$$= 25.05 \text{ cm}^3$$

We know that the volume of sugar syrup = 30% of total volume

So, volume of sugar syrup in 45 gulab jamuns = $45 \times 30\% (25.05 \text{ cm}^3)$
 $= 45 \times 7.515 = 338.184 \text{ cm}^3$

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig.).

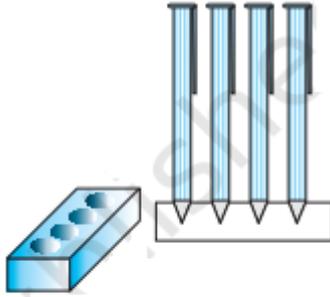


Fig. 13.16

Solution:

Volume of cuboid = length \times width \times height

We know the cuboid's dimensions as 15 cm \times 10 cm \times 3.5 cm

So, the volume of the cuboid = $15 \times 10 \times 3.5 = 525 \text{ cm}^3$

Here, depressions are like cones and we know,

Volume of cone = $(\frac{1}{3})\pi r^2 h$

Given, radius (r) = 0.5 cm and depth (h) = 1.4 cm

\therefore Volume of 4 cones = $4 \times (\frac{1}{3})\pi r^2 h$

$= 1.46 \text{ cm}^2$

Now, volume of wood = Volume of cuboid – 4 \times volume of cone

$= 525 - 1.46 = 523.54 \text{ cm}^2$

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

For the cone,

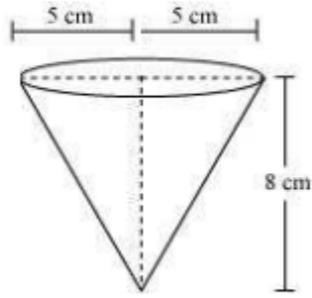
Radius = 5 cm,

Height = 8 cm

Also,

Radius of sphere = 0.5 cm

The diagram will be like



It is known that,

Volume of cone = volume of water in the cone

$$= \frac{1}{3}\pi r^2 h = \frac{(200/3)\pi \text{ cm}^3}{3}$$

Now,

$$\text{Total volume of water overflow} = \frac{1}{4} \times \frac{(200/3)\pi}{3} = \frac{(50/3)\pi}{3}$$

Volume of lead shot

$$= \frac{(4/3)\pi r^3}{3}$$

$$= \frac{(1/6)\pi}{3}$$

Now,

The number of lead shots = Total Volume of Water over flow / Volume of Lead shot

$$= \frac{(50/3)\pi / 3}{(1/6)\pi / 3}$$

$$= \frac{(50/3) \times 6}{3} = 100$$

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass.

Solution:

Given, the height of the big cylinder (H) = 220 cm

Radius of the base (R) = 24/2 = 12 cm

So, the volume of the big cylinder = $\pi R^2 H$

$$= \pi (12)^2 \times 220 \text{ cm}^3$$

$$= 99565.8 \text{ cm}^3$$

Now, the height of smaller cylinder (h) = 60 cm

Radius of the base (r) = 8 cm

So, the volume of the smaller cylinder = $\pi r^2 h$

$$= \pi (8)^2 \times 60 \text{ cm}^3$$

$$= 12068.5 \text{ cm}^3$$

\therefore Volume of iron = Volume of the big cylinder + Volume of the small cylinder

$$= 99565.8 + 12068.5$$

$$= 111634.5 \text{ cm}^3$$

We know,

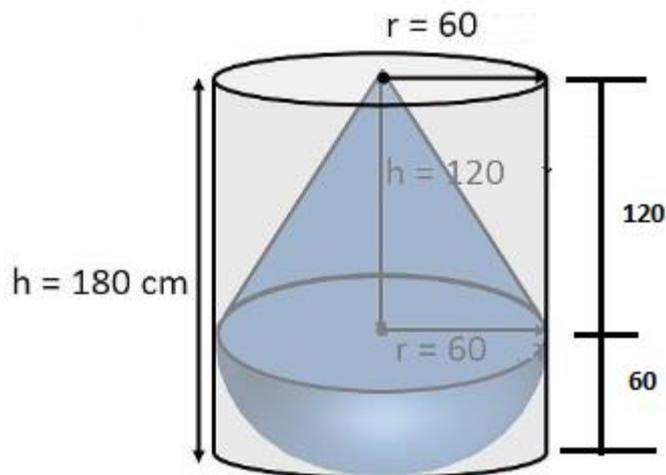
$$\text{Mass} = \text{Density} \times \text{volume}$$

$$\text{So, mass of the pole} = 8 \times 111634.5$$

$$= 893 \text{ Kg (approx.)}$$

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Solution:



Here, the volume of water left will be = Volume of cylinder – Volume of solid

Given,

Radius of cone = 60 cm,

Height of cone = 120 cm

Radius of cylinder = 60 cm

Height of cylinder = 180 cm

Radius of hemisphere = 60 cm

Now,

Total volume of solid = Volume of Cone + Volume of hemisphere

$$\text{Volume of cone} = \pi \times 60^2 \times 120 \text{ cm}^3 = 144 \times 10^3 \pi \text{ cm}^3$$

$$\text{So, Total volume of solid} = 144 \times 10^3 \pi \text{ cm}^3 - \left(\frac{2}{3}\right) \times \pi \times 60^3 \text{ cm}^3$$

$$\text{Volume of hemisphere} = \left(\frac{2}{3}\right) \times \pi \times 60^3 \text{ cm}^3$$

$$\text{Volume of cylinder} = \pi \times 60^2 \times 180 = 648000 = 648 \times 10^3 \pi \text{ cm}^3$$

Now, volume of water left will be = Volume of cylinder – Volume of solid
 $= (648 - 288) \times 10^3 \times \pi = 1.131 \text{ m}^3$

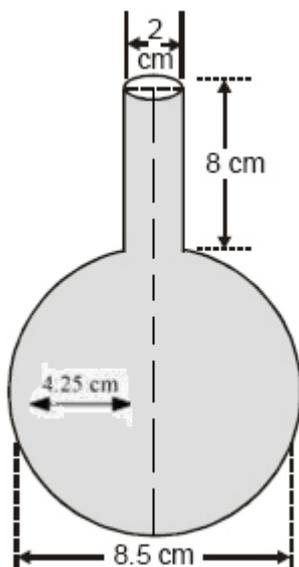
8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3 . Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Solution:

Given,

For the cylinder part, Height (h) = 8 cm and Radius (R) = (2/2) cm = 1 cm

For the spherical part, Radius (r) = (8.5/2) = 4.25 cm



Now, volume of this vessel = Volume of cylinder + Volume of sphere
 $= \pi \times (1)^2 \times 8 + (4/3)\pi(1)^3$
 $= 346.51 \text{ cm}^3$

Exercise: 13.3

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution:

It is given that radius of the sphere (R) = 4.2 cm

Also, Radius of cylinder (r) = 6 cm

Now, let height of cylinder = h

It is given that the sphere is melted into a cylinder.

So, Volume of Sphere = Volume of Cylinder

$$\therefore (4/3) \times \pi \times R^3 = \pi \times r^2 \times h.$$

$$h = 2.74 \text{ cm}$$

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

For Sphere 1:

$$\text{Radius } (r_1) = 6 \text{ cm}$$

$$\therefore \text{Volume } (V_1) = (4/3) \times \pi \times r_1^3$$

For Sphere 2:

$$\text{Radius } (r_2) = 8 \text{ cm}$$

$$\therefore \text{Volume } (V_2) = (4/3) \times \pi \times r_2^3$$

For Sphere 3:

$$\text{Radius } (r_3) = 10 \text{ cm}$$

$$\therefore \text{Volume } (V_3) = (4/3) \times \pi \times r_3^3$$

Also, let the radius of the resulting sphere be “r”

Now,

$$\text{Volume of resulting sphere} = V_1 + V_2 + V_3$$

$$(4/3) \times \pi \times r^3 = (4/3) \times \pi \times r_1^3 + (4/3) \times \pi \times r_2^3 + (4/3) \times \pi \times r_3^3$$

$$r^3 = 6^3 + 8^3 + 10^3$$

$$r^3 = 1728$$

$$r = 12 \text{ cm}$$

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Solution:

It is given that the shape of the well is in the shape of a cylinder with a diameter of 7 m

$$\text{So, radius} = 7/2 \text{ m}$$

$$\text{Also, Depth } (h) = 20 \text{ m}$$

Volume of the earth dug out will be equal to the volume of the cylinder

$$\therefore \text{Volume of Cylinder} = \pi \times r^2 \times h$$

$$= 22 \times 7 \times 5 \text{ m}^3$$

Let the height of the platform = H

Volume of soil from well (cylinder) = Volume of soil used to make such platform

$$\pi \times r^2 \times h = \text{Area of platform} \times \text{Height of the platform}$$

We know that the dimension of the platform is = 22×14

$$\text{So, Area of platform} = 22 \times 14 \text{ m}^2$$

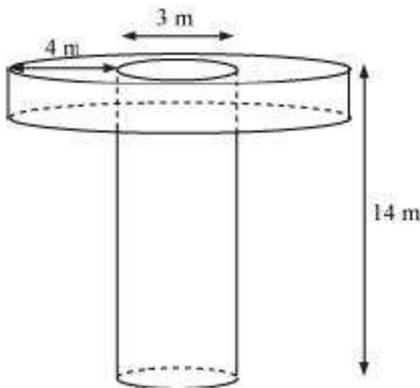
$$\therefore \pi \times r^2 \times h = 22 \times 14 \times H$$

$$\Rightarrow H = 2.5 \text{ m}$$

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution:

The shape of the well will be cylindrical as given below.



Given, Depth (h_1) of well = 14 m

Diameter of the circular end of the well = 3 m

So, Radius (r_1) = $3/2$ m

Width of the embankment = 4 m

From the figure, it can be said that the embankment will be a cylinder having an outer radius (r_2) as $4 + (3/2) = 11/2$ m and inner radius (r_1) as $3/2$ m

Now, let the height of embankment be h_2

\therefore Volume of soil dug from well = Volume of earth used to form embankment

$$\pi \times r_1^2 \times h = \pi \times (r_2^2 - r_1^2) \times h_2$$

Solving this, we get,

The height of the embankment (h_2) as 1.125 m.

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Solution:

Number of cones will be = Volume of cylinder / Volume of ice cream cone

For the cylinder part,

$$\text{Radius} = 12/2 = 6 \text{ cm}$$

$$\text{Height} = 15 \text{ cm}$$

$$\therefore \text{Volume of cylinder} = \pi \times r^2 \times h = 540\pi$$

For the ice cone part,

$$\text{Radius of conical part} = 6/2 = 3 \text{ cm}$$

$$\text{Height} = 12 \text{ cm}$$

$$\text{Radius of hemispherical part} = 6/2 = 3 \text{ cm}$$

Now,

Volume of ice cream cone = Volume of conical part + Volume of hemispherical part

$$= \left(\frac{1}{3}\right) \times \pi \times r^2 \times h + \left(\frac{2}{3}\right) \times \pi \times r^3$$

$$= 36\pi + 18\pi$$

$$= 54\pi$$

$$\therefore \text{Number of cones} = (540\pi/54\pi)$$

$$= 10$$

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Solution:

It is known that the coins are cylindrical in shape.

$$\text{So, height (h}_1\text{) of the cylinder} = 2 \text{ mm} = 0.2 \text{ cm}$$

$$\text{Radius (r) of circular end of coins} = 1.75/2 = 0.875 \text{ cm}$$

Now, the number of coins to be melted to form the required cuboids be “n”

So, Volume of n coins = Volume of cuboids

$$n \times \pi \times r^2 \times h_1 = l \times b \times h$$

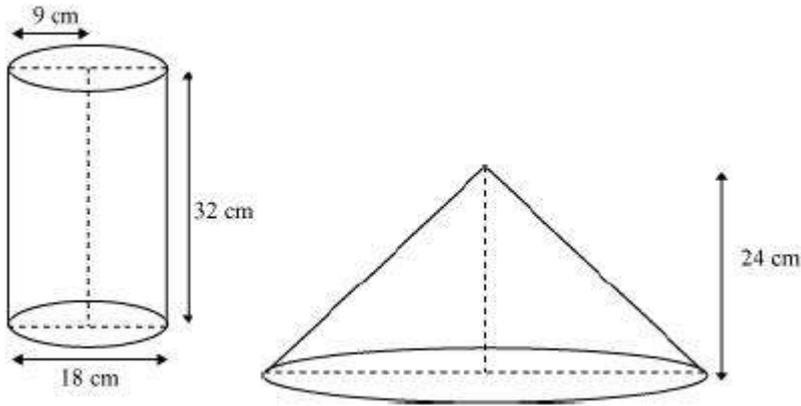
$$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$$

$$\text{Or, } n = 400$$

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

The diagram will be as-



Given,

Height (h_1) of cylindrical part of the bucket = 32 cm

Radius (r_1) of circular end of the bucket = 18 cm

Height of the conical heap (h_2) = 24 cm

Now, let " r_2 " be the radius of the circular end of the conical heap.

We know that volume of the sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

\therefore Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \left(\frac{1}{3}\right) \times \pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \left(\frac{1}{3}\right) \times \pi \times r_2^2 \times 24$$

$$\text{Or, } r_2 = 36 \text{ cm}$$

And,

$$\text{Slant height (l)} = \sqrt{(36^2 + 24^2)} = 12\sqrt{13} \text{ cm.}$$

8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Solution:

It is given that the canal is the shape of a cuboid with dimensions as:

Breadth (b) = 6 m and Height (h) = 1.5 m

It is also given that

The speed of canal = 10 km/hr

Length of canal covered in 1 hour = 10 km

Length of canal covered in 60 minutes = 10 km

Length of canal covered in 1 min = $(1/60) \times 10$ km

Length of canal covered in 30 min (l) = $(30/60) \times 10 = 5 \text{ km} = 5000 \text{ m}$

We know that the canal is cuboidal in shape. So,

Volume of canal = $l \times b \times h$

$$= 5000 \times 6 \times 1.5 \text{ m}^3$$

$$= 45000 \text{ m}^3$$

Now, Volume of water in canal = Volume of area irrigated

= Area irrigated \times Height

So, Area irrigated = 56.25 hectares

\therefore Volume of canal = $l \times b \times h$

$$45000 = \text{Area irrigated} \times 8 \text{ cm}$$

$$45000 = \text{Area irrigated} \times (8/100) \text{ m}$$

Or, Area irrigated = $562500 \text{ m}^2 = 56.25$ hectares.

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Solution:

Consider the following diagram-



$$\text{Radius } (r_1) \text{ of circular end of pipe} = \frac{20}{200} = 0.1 \text{ m}$$

$$\text{Area of cross-section} = \pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$$

$$\text{Speed of water} = 3 \text{ km/h} = \frac{3000}{60} = 50 \text{ metre/min}$$

$$\text{Volume of water that flows in 1 minute from pipe} = 50 \times 0.01\pi = 0.5\pi \text{ m}^3$$

$$\text{Volume of water that flows in } t \text{ minutes from pipe} = t \times 0.5\pi \text{ m}^3$$

$$\text{Volume of water that flows in } t \text{ minutes from pipe} = t \times 0.5\pi \text{ m}^3$$

$$\text{Radius } (r_2) \text{ of circular end of cylindrical tank} = 10/2 = 5 \text{ m}$$

$$\text{Depth } (h_2) \text{ of cylindrical tank} = 2 \text{ m}$$

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$$t \times 0.5\pi = \pi \times r_2^2 \times h_2$$

$$\text{Or, } t = 100 \text{ minutes}$$

Exercise: 13.5 (Optional)

1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm³.

Solution:

Given that,

Diameter of cylinder = 10 cm

So, radius of the cylinder (r) = 10/2 cm = 5 cm

∴ Length of wire in completely one round = $2\pi r = 3.14 \times 5 \text{ cm} = 31.4 \text{ cm}$

It is given that diameter of wire = 3 mm = 3/10 cm

∴ The thickness of cylinder covered in one round = 3/10 m

Hence, the number of turns (rounds) of the wire to cover 12 cm will be-

$$= \frac{12}{3/10} = 12 \times \frac{10}{3} = 40$$

Now, the length of wire required to cover the whole surface = length of wire required to complete 40 rounds

$$40 \times 31.4 \text{ cm} = 1256 \text{ cm}$$

Radius of the wire = 0.3/2 = 0.15 cm

Volume of wire = Area of cross-section of wire × Length of wire

$$= \pi(0.15)^2 \times 1257.14$$

$$= 88.898 \text{ cm}^3$$

We know,

Mass = Volume × Density

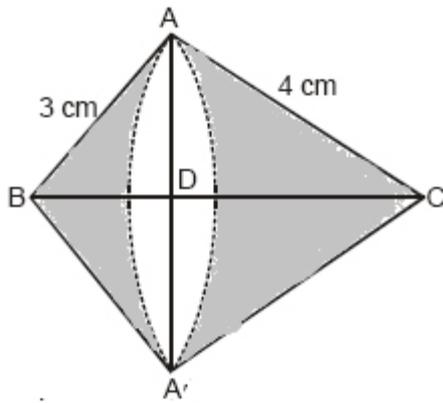
$$= 88.898 \times 8.88$$

$$= 789.41 \text{ gm}$$

2. A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)

Solution:

Draw the diagram as follows:



Let us consider the ABA

Here,

$$AS = 3 \text{ cm}, AC = 4 \text{ cm}$$

$$\text{So, Hypotenuse } BC = 5 \text{ cm}$$

We have got 2 cones on the same base AA' where the radius = DA or DA'

$$\text{Now, } AD/CA = AB/CB$$

By putting the value of CA , AB and CB we get,

$$AD = 2/5 \text{ cm}$$

We also know,

$$DB/AB = AB/CB$$

$$\text{So, } DB = 9/5 \text{ cm}$$

$$\text{As, } CD = BC - DB,$$

$$CD = 16/5 \text{ cm}$$

Now, volume of double cone will be

$$= \left[\frac{1}{3} \pi \times \left(\frac{12}{5} \right)^2 \frac{9}{5} + \frac{1}{3} \pi \times \left(\frac{12}{5} \right)^2 \times \frac{16}{5} \right] \text{cm}^3$$

Solving this we get,

$$V = 30.14 \text{ cm}^3$$

The surface area of the double cone will be

$$= \left(\pi \times \frac{12}{5} \times 3 \right) + \left(\pi \times \frac{12}{5} \times 4 \right) \text{cm}^2 = \pi \times \frac{12}{5} [3 + 4] \text{cm}^2$$

$$= 52.75 \text{ cm}^2$$

3. A cistern, internally measuring $150 \text{ cm} \times 120 \text{ cm} \times 100 \text{ cm}$, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each being $22.5 \text{ cm} \times 7.5 \text{ cm} \times 6.5 \text{ cm}$?

Solution:

Given that the dimension of the cistern = $150 \times 120 \times 110$

So, volume = 1980000 cm^3

Volume to be filled in cistern = $1980000 - 129600$

= 1850400 cm^3

Now, let the number of bricks placed be “n”

So, volume of n bricks will be = $n \times 22.5 \times 7.5 \times 6.5$

Now as each brick absorbs one-seventeenth of its volume, the volume will be

= $n/(17) \times (22.5 \times 7.5 \times 6.5)$

For the condition given in the question,

The volume of n bricks has to be equal to volume absorbed by n bricks + Volume to be filled in cistern

Or, $n \times 22.5 \times 7.5 \times 6.5 = 1850400 + n/(17) \times (22.5 \times 7.5 \times 6.5)$

Solving this we get,

$n = 1792.41$

DELHI PUBLIC SCHOOL, GANDHINAGAR
CHAPTER – 13 SURFACE AREAS AND VOLUMES

MIND MAP

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE: 1 SURFACE AREA

- Q.1. A tent of height 8.25 m is in the form of a right circular cylinder with diameter of base 30 m and height 5.5 m, surmounted by a right circular cone of the same base. Find the cost of the tent at the rate of rupees 45 per m^2 .
- Q.2 A golf ball has diameter equal to 4.2 cm. Its surface has 200 dimples each of radius 2 mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.

TYPE: 2 COMPARISION OF VOLUME OF TWO SOLIDS

- Q.1. If the diameter of cross-section of a wire is decreased by 5%, how much percent will the length increased so that the volume remains the same.
- Q.2 The radii of the bases of two right circular solid cones of same height are X and Y respectively. The cones are melted and recast into a solid sphere of radius Z. Show that the height of each cone is given by $h = \frac{4Z^3}{X^2+Y^2}$.
- Q.3 A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 12 m deep. If water flows through the pipe at the rate of 3km/h, in how much time will the tank be filled?

- Q.4 A cistern, internally measuring 150 cm x 120 cm x 110 cm has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one seventeenth of its own volume of water. How many bricks can be put in without the water overflowing, each brick being 22.5 cm x 7.5 cm x 6.5 cm?
- Q.5 Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. If the base of the shed is 7 m by 15 m and height of the cuboidal portion is 8 m, find the volume of the air that the shed can hold. If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working?

TYPE: 3 FRUSTUM OF A CONE

- Q.1. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?
- Q.2. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle point of its axis by a plane parallel to the base. If the frustum so obtained be drawn into a wire of diameter 0.2 cm, find the length of the wire.
- Q.3. A bucket in the form of a frustum of a cone with a capacity of 12308.8 m^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. ($\pi = 3.14$)

CHAPTER 11: CONSTRUCTIONS

Exercise 11.1

Q1 : Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts. Give the justification of the construction.

Answer :

A line segment of length 7.6 cm can be divided in the ratio of 5:8 as follows.

Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with line segment AB.

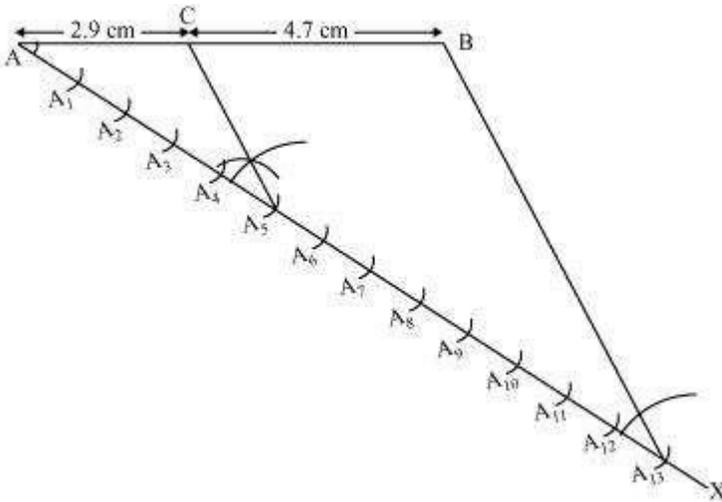
Step 2 Locate 13 (= 5 + 8) points, $A_1, A_2, A_3, A_4, \dots, A_{13}$, on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.

Step 3 Join BA_{13} .

Step 4 Through the point A_5 , draw a line parallel to BA_{13} (by making an angle equal to $\angle AA_{13}B$) at A_5 intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5:8.

The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.



Justification

The construction can be justified by proving that

$$\frac{AC}{CB} = \frac{5}{8}$$

By construction, we have $A_5C \parallel A_{13}B$. By applying Basic proportionality theorem for the triangle $AA_{13}B$, we obtain

$$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}} \dots (1)$$

From the figure, it can be observed that AA_5 and A_5A_{13} contain 5 and 8 equal divisions of line segments respectively.

$$\therefore \frac{AA_5}{A_5A_{13}} = \frac{5}{8} \dots (2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AC}{CB} = \frac{5}{8}$$

This justifies the construction.

Exercise 11.2

Q1 : Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

Answer :

A pair of tangents to the given circle can be constructed as follows.

Step 1

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

Step 2

Bisect OP. Let M be the mid-point of PO.

Step 3

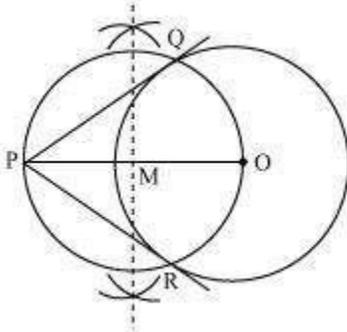
Taking M as centre and MO as radius, draw a circle.

Step 4

Let this circle intersect the previous circle at point Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



The lengths of tangents PQ and PR are 8 cm each.

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.

$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$$\therefore \angle PQO = 90^\circ$$

$$\Rightarrow OQ \perp PQ$$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

Q2 : Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

Answer :

Tangents on the given circle can be drawn as follows.

Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3

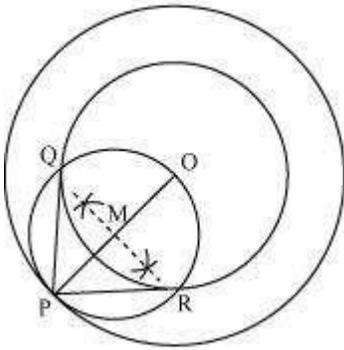
Bisect OP. Let M be the mid-point of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO ,

Since PQ is a tangent,

$$\angle P Q O = 90^\circ$$

$$P O = 6 \text{ cm}$$

$$Q O = 4 \text{ cm}$$

Applying Pythagoras theorem in ΔPQO , we obtain

$$P Q^2 + Q O^2 = P O^2$$

$$P Q^2 + (4)^2 = (6)^2$$

$$P Q^2 + 16 = 36$$

$$P Q^2 = 36 - 16$$

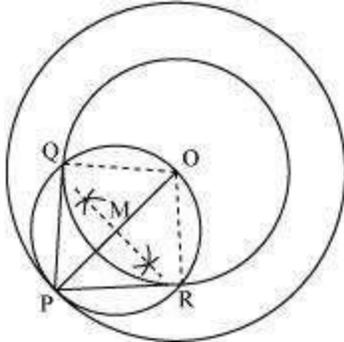
$$P Q^2 = 20$$

$$P Q = 2\sqrt{5}$$

$$P Q = 4.47 \text{ cm}$$

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$$\therefore \angle PQO = 90^\circ$$

$$\Rightarrow OQ \perp PQ$$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

Q3 : Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. Give the justification of the construction.

Answer :

The tangent can be constructed on the given circle as follows.

Step 1

Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

Step 2

Take one of its diameters, PQ, and extend it on both sides. Locate two points on this diameter such that $OR = OS = 7$ cm

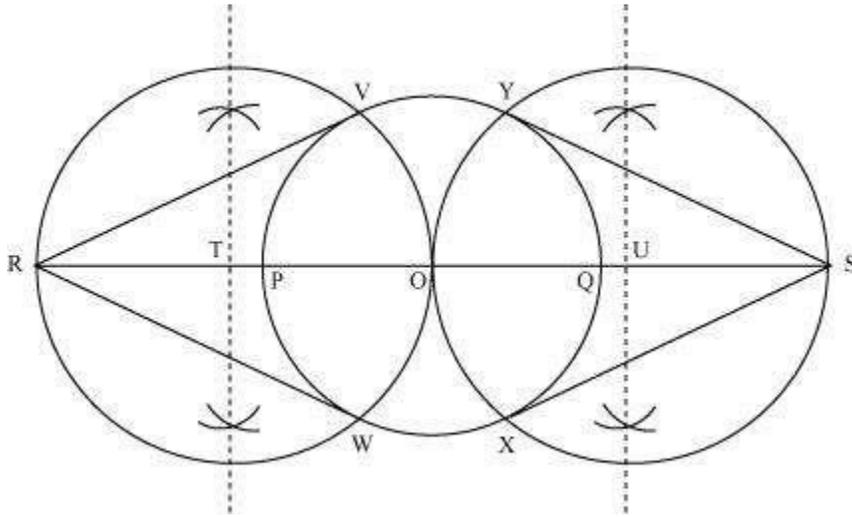
Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

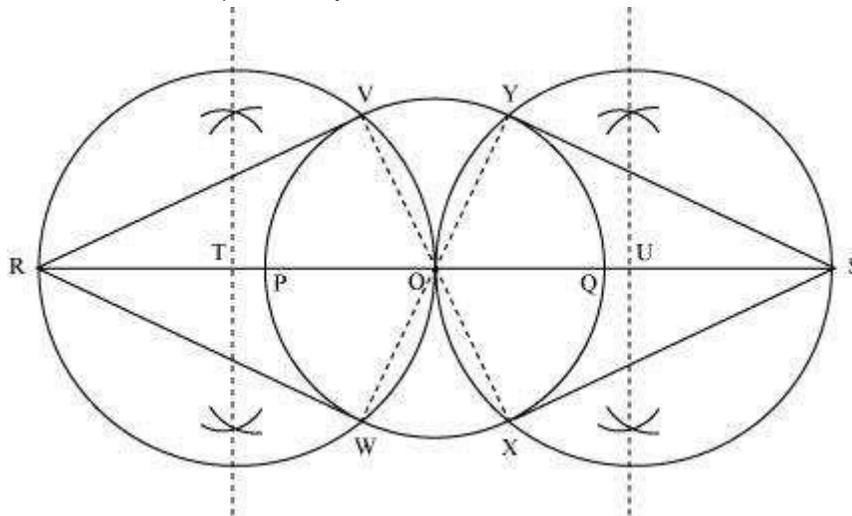
Step 4

Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.

Justification



The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OY.



$\angle RVO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$$\therefore \angle RVO = 90^\circ$$

$$\Rightarrow OV \perp RV$$

Since OV is the radius of the circle, RV has to be a tangent to the circle.

Similarly, it can be shown that RW, SX, and SY are the tangents of the circle.

Q4 : Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60° . Give the justification of the construction.

Answer :

The tangents can be constructed in the following manner:

Step 1

Draw a circle of radius 5 cm and with centre as O.

Step 2

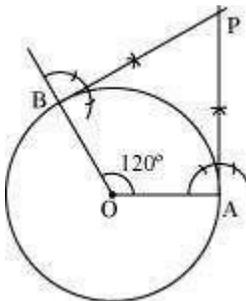
Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

Draw a radius OB, making an angle of 120° ($180^\circ - 60^\circ$) with OA.

Step 4

Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60° .



Justification

The construction can be justified by proving that $\angle APB = 60^\circ$

By our construction

$$\angle OAP = 90^\circ$$

$$\angle OBP = 90^\circ$$

$$\text{And } \angle AOB = 120^\circ$$

We know that the sum of all interior angles of a quadrilateral = 360°

$$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360^\circ$$

$$90^\circ + 120^\circ + 90^\circ + \angle APB = 360^\circ$$

$$\angle APB = 60^\circ$$

This justifies the construction.

Q5 : Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

Answer :

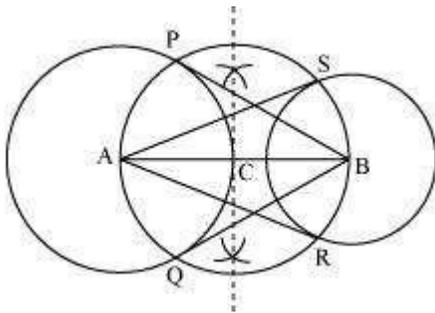
The tangents can be constructed on the given circles as follows.

Step 1

Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

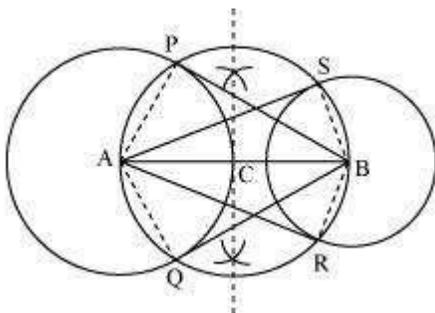
Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents.



Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm) and BP and BQ are the tangents of the circle (whose centre is A and radius is 4 cm). For this, join AP, AQ, BS, and BR.



$\angle ASB$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$$\therefore \angle ASB = 90^\circ$$

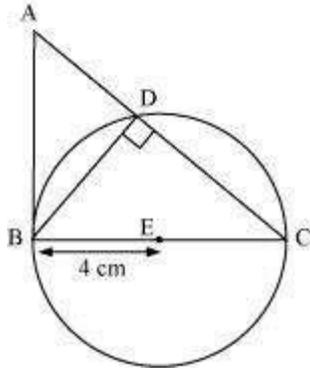
$$\Rightarrow BS \perp AS$$

Since BS is the radius of the circle, AS has to be a tangent of the circle. Similarly, AR, BP, and BQ are the tangents.

Q6 : Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C, and D is drawn. Construct the tangents from A to this circle. Give the justification of the construction.

Answer :

Consider the following situation. If a circle is drawn through B, D, and C, BC will be its diameter as $\angle BDC$ is of measure 90° . The centre E of this circle will be the mid-point of BC.



The required tangents can be constructed on the given circle as follows.

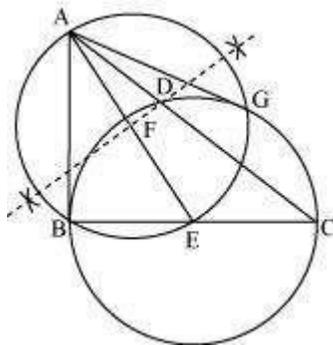
Step 1

Join AE and bisect it. Let F be the mid-point of AE.

Step 2

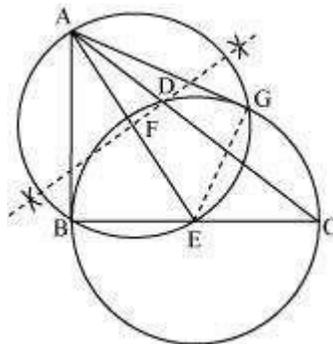
Taking F as centre and FE as its radius, draw a circle which will intersect the circle at point B and G. Join AG.

AB and AG are the required tangents.



Justification

The construction can be justified by proving that AG and AB are the tangents to the circle. For this, join EG.



$\angle AGE$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$$\therefore \angle AGE = 90^\circ$$

$$\Rightarrow EG \perp AG$$

Since EG is the radius of the circle, AG has to be a tangent of the circle.

Already, $\angle B = 90^\circ$

$$\Rightarrow AB \perp BE$$

Since BE is the radius of the circle, AB has to be a tangent of the circle.

Q6 :

Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

Answer :

The required tangents can be constructed on the given circle as follows.

Step 1

Draw a circle with the help of a bangle.

Step 2

Take a point P outside this circle and take two chords QR and ST.

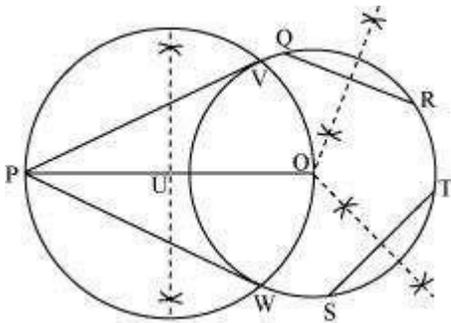
Step 3

Draw perpendicular bisectors of these chords. Let them intersect each other at point O.

Step 4

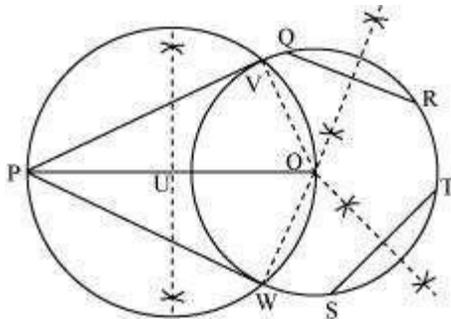
Join PO and bisect it. Let U be the mid-point of PO. Taking U as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.

PV and PW are the required tangents.



Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW.



We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. $\angle PVO$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$$\therefore \angle PVO = 90^\circ$$

$$\Rightarrow OV \perp PV$$

Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.

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CHAPTER – 11 CONSTRUCTIONS

MIND MAP

This chapter consists of three different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 LINE – SEGMENT

- Q.1. Determine a point which divides a line segment of length 12 cm internally in the ratio 2: 3. Also, justify the constructions.

TYPE: 2 PAIR OF TANGENTS TO A CIRCLE

- Q.1. Construct a pair of tangents PQ and PR to a circle of radius 4 cm from a point P outside the circle 8 cm away from the centre. Measure PQ and PR.
- Q.2. Draw a circle of radius 6 cm and draw a tangent to this circle making an angle of 30° with a line passing through the centre.
- Q.3. Draw a circle of radius 5 cm and draw two tangents to this circle if the angle between them is 30° .

NCERT Solutions for Class 10 Maths Unit 10

Circles Class 10

Unit 10 Circles Exercise 10.1, 10.2 Solutions

Exercise 10.1 : Solutions of Questions on Page Number : 204

Q1 :

How many tangents can a circle have?

Answer :

A circle can have infinite tangents.

Q2 :

Fill in the blanks:

- (i) A tangent to a circle intersects it in _____ point (s).
- (ii) A line intersecting a circle in two points is called a _____.
- (iii) A circle can have _____ parallel tangents at the most.
- (iv) The common point of a tangent to a circle and the circle is called _____.

Answer :

- (i) One
- (ii) Secant
- (iii) Two
- (iv) Point of contact

Q3 :

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

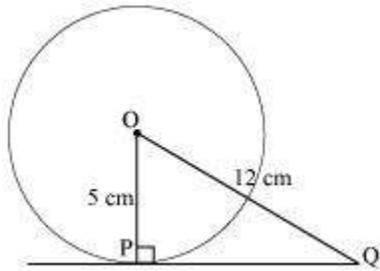
- (A) 12 cm. (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Answer :

We know that the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

$$\therefore OP \perp PQ$$

By applying Pythagoras theorem in $\triangle OPQ$,



$$\therefore OP^2 + PQ^2 = OQ^2$$

$$5^2 + PQ^2 = 12^2$$

$$PQ^2 = 144 - 25$$

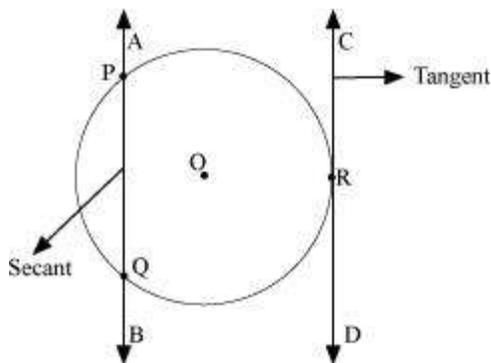
$$PQ = \sqrt{119} \text{ cm.}$$

Hence, the correct answer is (D).

Q4 :

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer :



It can be observed that AB and CD are two parallel lines. Line AB is intersecting the circle at exactly two points, P and Q. Therefore, line AB is the secant of this circle. Since line CD is intersecting the circle at exactly one point, R, line CD is the tangent to the circle.

Exercise 10.2 : Solutions of Questions on Page Number : 213

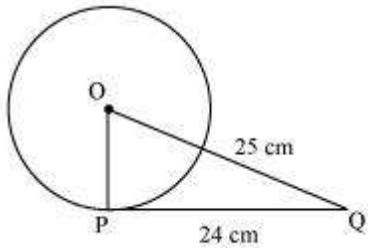
Q1 :

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

Answer :



Let O be the centre of the circle.

Given that,

$OQ = 25\text{cm}$ and $PQ = 24\text{ cm}$

As the radius is perpendicular to the tangent at the point of contact,

Therefore, $OP \perp PQ$

Applying Pythagoras theorem in ΔOPQ , we obtain

$$OP^2 + PQ^2 = OQ^2$$

$$OP^2 + 24^2 = 25^2$$

$$OP^2 = 625 - 576$$

$$OP^2 = 49$$

$$OP = 7$$

Therefore, the radius of the circle is 7 cm.

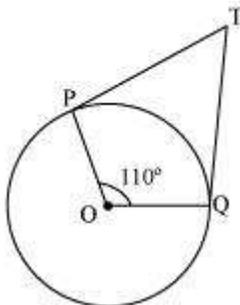
Hence, alternative (A) is correct.

Q2 :

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



Answer :

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

$$\angle OPT = 90^\circ$$

$$\angle OQT = 90^\circ$$

In quadrilateral POQT,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$$

$$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

Hence, alternative (B) is correct.

Q3 :

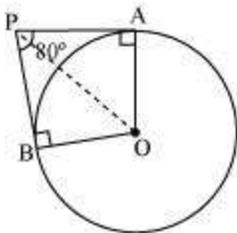
If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80° , then $\angle POA$ is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Answer :

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp PA$ and $OB \perp PB$

$$\angle OBP = 90^\circ$$

$$\angle OAP = 90^\circ$$

In AOBP,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

$$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$$

$$\angle BOA = 100^\circ$$

In $\triangle OPB$ and $\triangle OPA$,

$AP = BP$ (Tangents from a point)

$OA = OB$ (Radii of the circle)

$OP = OP$ (Common side)

Therefore, $\triangle OPB \cong \triangle OPA$ (SSS congruence criterion)

$A \leftrightarrow B, P \leftrightarrow P, O \leftrightarrow O$

And thus, $\angle POB = \angle POA$

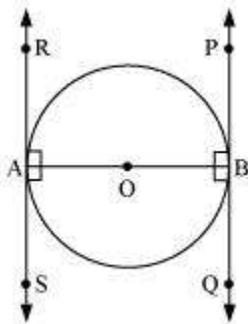
$$\angle POA = \frac{1}{2} \angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Q4 :

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer :



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp RS$ and $OB \perp PQ$

$$\angle OAR = 90^\circ$$

$$\angle OAS = 90^\circ$$

$$\angle OBP = 90^\circ$$

$$\angle OBQ = 90^\circ$$

It can be observed that

$\angle OAR = \angle OBQ$ (Alternate interior angles)

$\angle OAS = \angle OBP$ (Alternate interior angles)

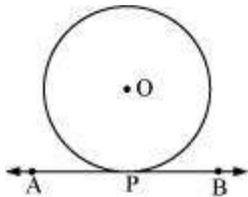
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Q5 :

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

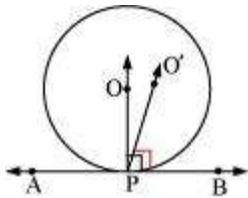
Answer :

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$$\angle O'PB = 90^\circ \dots (1)$$

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$$\therefore \angle OPB = 90^\circ \dots (2)$$

Comparing equations (1) and (2), we obtain

$$\angle O'PB = \angle OPB \dots (3)$$

From the figure, it can be observed that,

$$\angle O'PB < \angle OPB \dots (4)$$

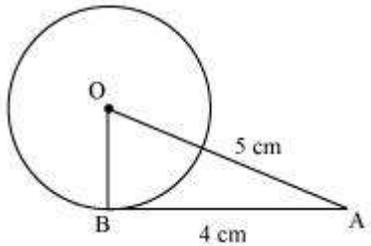
Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.

Q6 :

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer :



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5 cm and AB = 4 cm

In $\triangle ABO$,

$OB \perp AB$ (Radius \perp tangent at the point of contact)

Applying Pythagoras theorem in $\triangle ABO$, we obtain

$$AB^2 + BO^2 = OA^2$$

$$4^2 + BO^2 = 5^2$$

$$16 + BO^2 = 25$$

$$BO^2 = 9$$

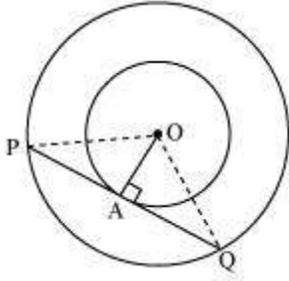
$$BO = 3$$

Hence, the radius of the circle is 3 cm.

Q7 :

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer :



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

$OA \perp PQ$ (As OA is the radius of the circle)

Applying Pythagoras theorem in $\triangle OAP$, we obtain

$$OA^2 + AP^2 = OP^2$$

$$3^2 + AP^2 = 5^2$$

$$9 + AP^2 = 25$$

$$AP^2 = 16$$

$$AP = 4$$

In $\triangle OPQ$,

Since $OA \perp PQ$,

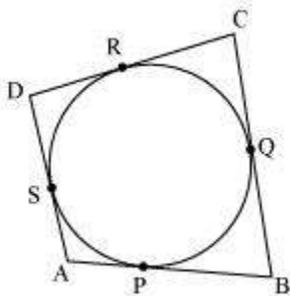
$AP = AQ$ (Perpendicular from the center of the circle bisects the chord)

$$\therefore PQ = 2AP = 2 \times 4 = 8$$

Therefore, the length of the chord of the larger circle is 8 cm.

Q8 :

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that $AB + CD = AD + BC$



Answer :

It can be observed that

$$DR = DS \text{ (Tangents on the circle from point D) ... (1)}$$

$CR = CQ$ (Tangents on the circle from point C) ... (2)

$BP = BQ$ (Tangents on the circle from point B) ... (3)

$AP = AS$ (Tangents on the circle from point A) ... (4)

Adding all these equations, we obtain

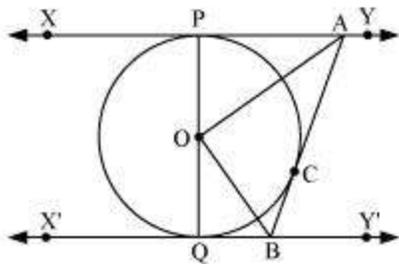
$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

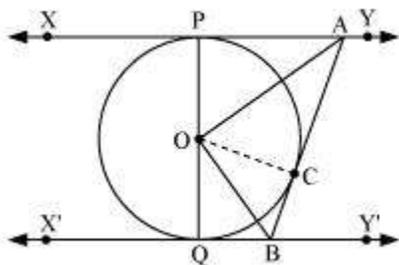
Q9 :

In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$.



Answer :

Let us join point O to C .



In $\triangle OPA$ and $\triangle OCA$,

$OP = OC$ (Radii of the same circle)

$AP = AC$ (Tangents from point A)

$AO = AO$ (Common side)

$\triangle OPA \cong \triangle OCA$ (SSS congruence criterion)

Therefore, $\angle POA = \angle COA$... (i)

Similarly, $\triangle OQB \cong \triangle OCB$

$$\angle QOB = \angle COB \dots (ii)$$

Since POQ is a diameter of the circle, it is a straight line.

$$\text{Therefore, } \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$$

From equations (i) and (ii), it can be observed that

$$2\angle COA + 2\angle COB = 180^\circ$$

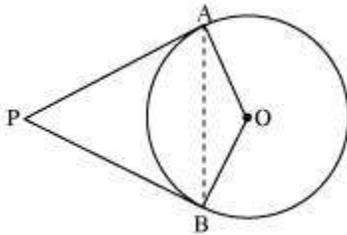
$$\angle COA + \angle COB = 90^\circ$$

$$\angle AOB = 90^\circ$$

Q10 :

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer :



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of the circle.

It can be observed that

OA (radius) \perp PA (tangent)

$$\text{Therefore, } \angle OAP = 90^\circ$$

Similarly, OB (radius) \perp PB (tangent)

$$\angle OBP = 90^\circ$$

In quadrilateral OAPB,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

$$90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$$

$$\angle APB + \angle BOA = 180^\circ$$

Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Q11 :

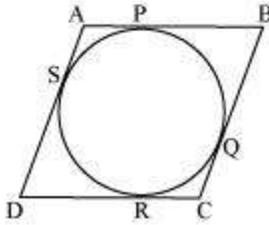
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer :

Since ABCD is a parallelogram,

$$AB = CD \dots(1)$$

$$BC = AD \dots(2)$$



It can be observed that

$$DR = DS \text{ (Tangents on the circle from point D)}$$

$$CR = CQ \text{ (Tangents on the circle from point C)}$$

$$BP = BQ \text{ (Tangents on the circle from point B)}$$

$$AP = AS \text{ (Tangents on the circle from point A)}$$

Adding all these equations, we obtain

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

On putting the values of equations (1) and (2) in this equation, we obtain

$$2AB = 2BC$$

$$AB = BC \dots(3)$$

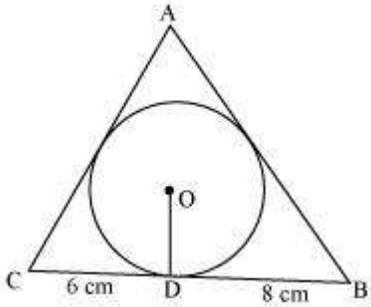
Comparing equations (1), (2), and (3), we obtain

$$AB = BC = CD = DA$$

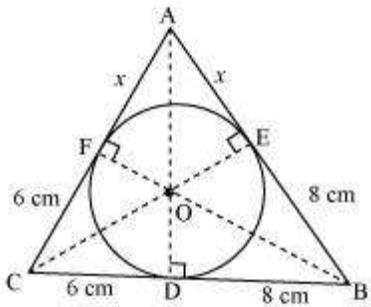
Hence, ABCD is a rhombus.

Q12 :

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Answer :



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be x .

In $\triangle ABC$,

$CF = CD = 6\text{ cm}$ (Tangents on the circle from point C)

$BE = BD = 8\text{ cm}$ (Tangents on the circle from point B)

$AE = AF = x$ (Tangents on the circle from point A)

$AB = AE + EB = x + 8$

$BC = BD + DC = 8 + 6 = 14$

$CA = CF + FA = 6 + x$

$2s = AB + BC + CA$

$= x + 8 + 14 + 6 + x$

$= 28 + 2x$

$s = 14 + x$

$$\begin{aligned}
 \text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}} \\
 &= \sqrt{(14+x)(x)(8)(6)} \\
 &= 4\sqrt{3(14x+x^2)}
 \end{aligned}$$

$$\text{Area of } \triangle OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$$

$$\text{Area of } \triangle OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6 + x) = 12 + 2x$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8 + x) = 16 + 2x$$

Area of $\triangle ABC$ = Area of $\triangle OBC$ + Area of $\triangle OCA$ + Area of $\triangle OAB$

$$4\sqrt{3(14x + x^2)} = 28 + 12 + 2x + 16 + 2x$$

$$\Rightarrow 4\sqrt{3(14x + x^2)} = 56 + 4x$$

$$\Rightarrow \sqrt{3(14x + x^2)} = 14 + x$$

$$\Rightarrow 3(14x + x^2) = (14 + x)^2$$

$$\Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$$

$$\Rightarrow 2x^2 + 14x - 196 = 0$$

$$\Rightarrow x^2 + 7x - 98 = 0$$

$$\Rightarrow x^2 + 14x - 7x - 98 = 0$$

$$\Rightarrow x(x + 14) - 7(x + 14) = 0$$

$$\Rightarrow (x + 14)(x - 7) = 0$$

Either $x + 14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7

However, $x = -14$ is not possible as the length of the sides will be negative.

Therefore, $x = 7$

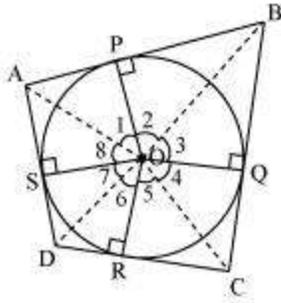
Hence, $AB = x + 8 = 7 + 8 = 15$ cm

$CA = 6 + x = 6 + 7 = 13$ cm

Q13 :

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer :



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider $\triangle OAP$ and $\triangle OAS$,

$AP = AS$ (Tangents from the same point)

$OP = OS$ (Radii of the same circle)

$OA = OA$ (Common side)

$\triangle OAP \cong \triangle OAS$ (SSS congruence criterion)

Therefore, $\angle POA = \angle AOS$, $\angle SOB = \angle BOQ$, $\angle QOC = \angle COR$, $\angle ROD = \angle DOS$

And thus, $\angle POA = \angle AOS$

$$\angle 1 = \angle 8$$

Similarly,

$$\angle 2 = \angle 3$$

$$\angle 4 = \angle 5$$

$$\angle 6 = \angle 7$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$$

$$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$$

$$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$$

$$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

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CHAPTER – 10 CIRCLES

MIND MAP

This chapter consists of two different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 TANGENT TO A CIRCLE

- Q.1. If PT is a tangent at T to a circle whose centre is O and $OP=17$ cm, $OT= 8$ cm, find the length of the tangent segment PT.
- Q.2. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.
- Q.3. If PA and PB are tangents from an outside point P, such that $PA=10$ cm and $\angle APB = 60^\circ$, find the length of chord AB.
- Q.4. Two tangents PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.
- Q.5. In two concentric circles, prove that all chords of the outer circle which touch the inner are of equal length.
-

TYPE: 2 IN-CIRCLE

- Q.1. ABC is a right triangle right-angled at B such that $BC = 6$ cm and $AB = 8$ cm. Find the radius of its incircle.
- Q.2. If ΔABC is isosceles with $AB = AC$ and C (O, r) is the incircle of the ΔABC touching BC at L, prove that L bisects BC.

NCERT Solutions for Class 10 Maths Unit 9

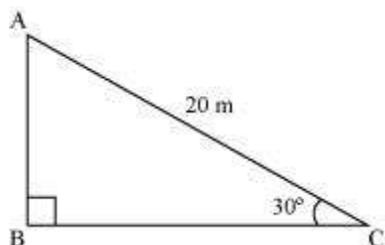
Some Applications of Trigonometry Class 10

Unit 9 Some Applications of Trigonometry Exercise 9.1 Solutions

Exercise 9.1 : Solutions of Questions on Page Number : 203

Q1 :

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° .



Answer :

It can be observed from the figure that AB is the pole.

In $\triangle ABC$,

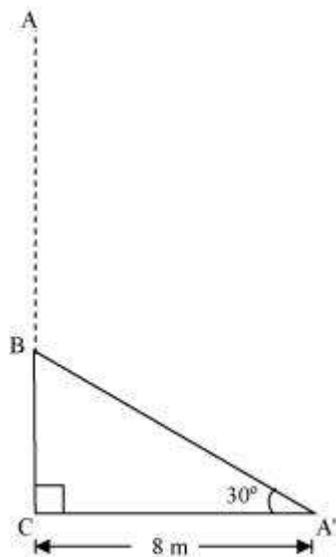
$$\frac{AB}{AC} = \sin 30^\circ$$
$$\frac{AB}{20} = \frac{1}{2}$$
$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Q2 :

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer :



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30^\circ$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right) \text{m}$$

$$\frac{A'C}{A'B} = \cos 30^\circ$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right) \text{m}$$

Height of tree = $A'B + BC$

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) \text{m} = \frac{24}{\sqrt{3}} \text{m}$$

$$= 8\sqrt{3} \text{m}$$

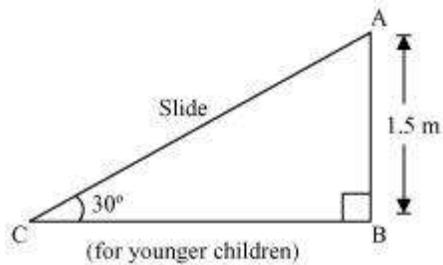
Hence, the height of the tree is $8\sqrt{3} \text{m}$.

Q3 :

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Answer :

It can be observed that AC and PR are the slides for younger and elder children respectively.

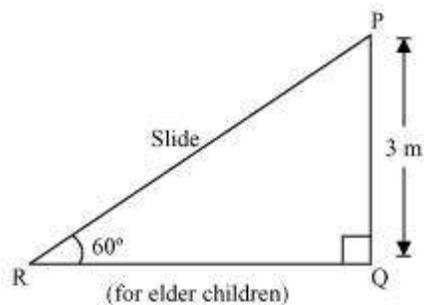


In $\triangle ABC$,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$



In $\triangle PQR$,

$$\frac{PQ}{PR} = \sin 60^\circ$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

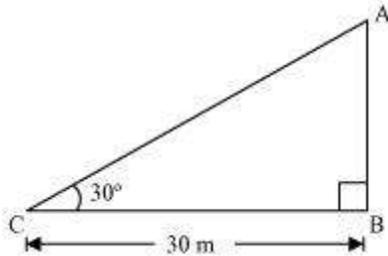
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m.

Q4 :

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30° . Find the height of the tower.

Answer :



Let AB be the tower and the angle of elevation from point C (on ground) is 30° .

In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

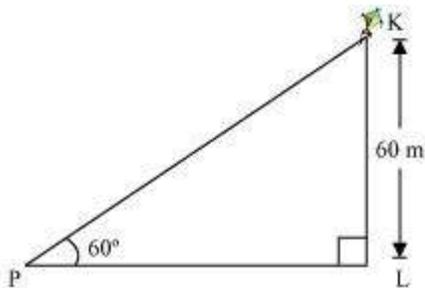
$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3} \text{ m}$.

Q5 :

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

Answer :



Let K be the kite and the string is tied to point P on the ground.

In $\triangle KLP$,

$$\frac{KL}{KP} = \sin 60^\circ$$

$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

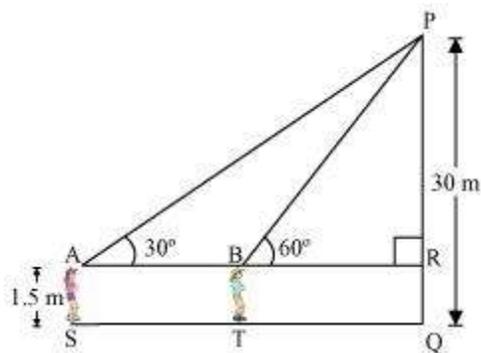
$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3} \text{ m}$.

Q6 :

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer :



Let the boy was standing at point S initially. He walked towards the building and reached at point T.

It can be observed that

$$PR = PQ - RQ$$

$$= (30 - 1.5) \text{ m} = 28.5 \text{ m} = \frac{57}{2} \text{ m}$$

In $\triangle PAR$,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2} \sqrt{3} \right) \text{ m}$$

In $\triangle PRB$,

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{57}{2 BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) \text{ m}$$

$$ST = AB$$

$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) \text{ m}$$

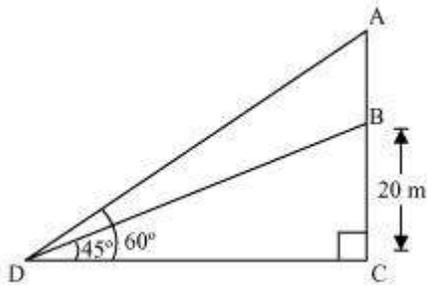
$$= \left(\frac{38\sqrt{3}}{2}\right) \text{ m} = 19\sqrt{3} \text{ m}$$

Hence, he walked $19\sqrt{3} \text{ m}$ towards the building.

Q7 :

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer :



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In $\triangle BCD$,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In $\triangle ACD$,

$$\frac{AC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3}$$

$$AB = (20\sqrt{3} - 20)\text{m}$$

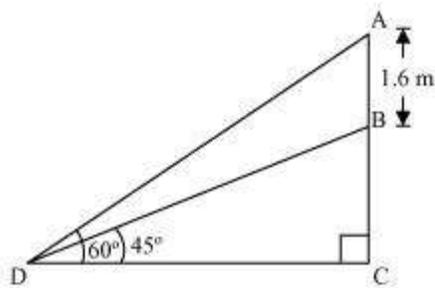
$$= 20(\sqrt{3} - 1)\text{m}$$

Therefore, the height of the transmission tower is $20(\sqrt{3} - 1)$ m.

Q8 :

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

Answer :



Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In $\triangle BCD$,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{BC}{CD} = 1$$

$$BC = CD$$

In $\triangle ACD$,

$$\frac{AB + BC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{BC} = \sqrt{3}$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC(\sqrt{3} - 1) = 1.6$$

$$BC = \frac{(1.6)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{1.6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1)$$

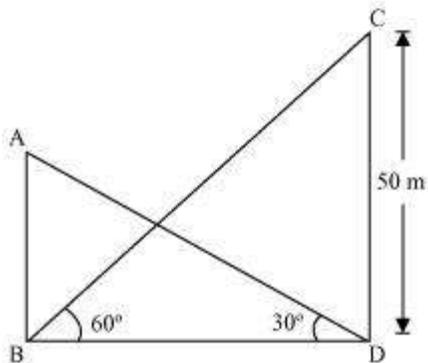
Therefore, the height of the pedestal is $0.8(\sqrt{3} + 1)$ m.

Q9 :

The angle of elevation of the top of a building from the foot of the tower is

30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

Answer :



Let AB be the building and CD be the tower.

In $\triangle CDB$,

$$\frac{CD}{BD} = \tan 60^\circ$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

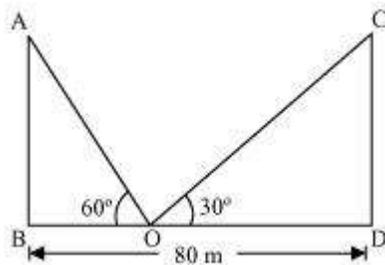
$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Q10 :

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of poles and the distance of the point from the poles.

Answer :



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In $\triangle ABO$,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In $\triangle CDO$,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

$$CD = AB$$

$$CD \left[\sqrt{3} + \frac{1}{\sqrt{3}} \right] = 80$$

$$CD \left(\frac{3+1}{\sqrt{3}} \right) = 80$$

$$CD = 20\sqrt{3} \text{ m}$$

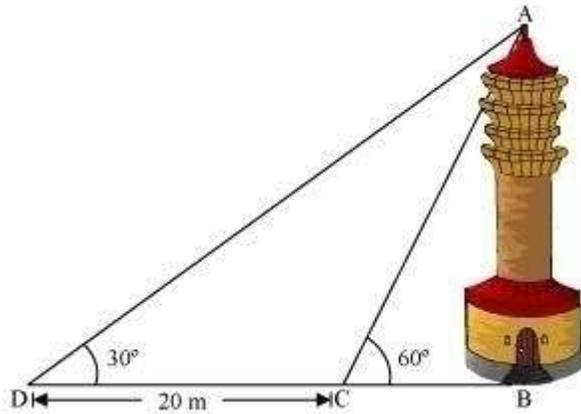
$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \left(\frac{20\sqrt{3}}{\sqrt{3}} \right) \text{ m} = 20 \text{ m}$$

$$DO = BD - BO = (80 - 20) \text{ m} = 60 \text{ m}$$

Therefore, the height of poles is $20\sqrt{3} \text{ m}$ and the point is 20 m and 60 m far from these poles.

Q11 :

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.



Answer :

In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right) \text{ m} = 10 \text{ m}$$

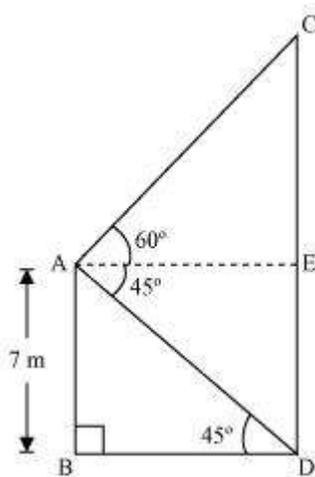
Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is

10 m.

Q12 :

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

Answer :



Let AB be a building and CD be a cable tower.

In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In $\triangle ACE$,

$$AE = BD = 7 \text{ m}$$

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

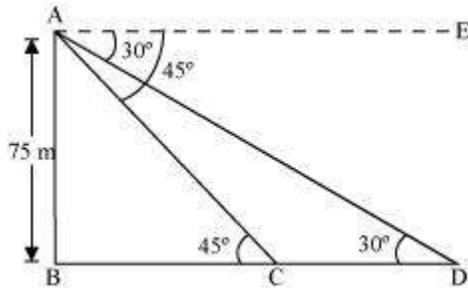
$$\begin{aligned} CD &= CE + ED = (7\sqrt{3} + 7) \text{ m} \\ &= 7(\sqrt{3} + 1) \text{ m} \end{aligned}$$

Therefore, the height of the cable tower is $7(\sqrt{3} + 1) \text{ m}$.

Q13 :

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer :



Let AB be the lighthouse and the two ships be at point C and D respectively.

In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 45^\circ$$
$$\frac{75}{BC} = 1$$
$$BC = 75 \text{ m}$$

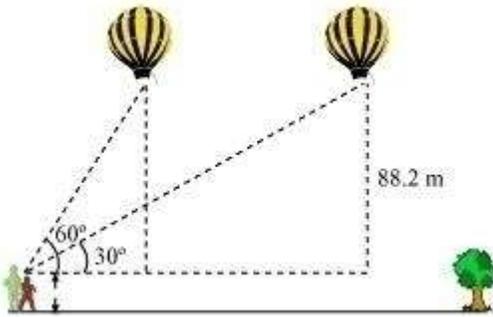
In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$
$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$
$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$
$$75\sqrt{3} = 75 + CD$$
$$75(\sqrt{3} - 1) \text{ m} = CD$$

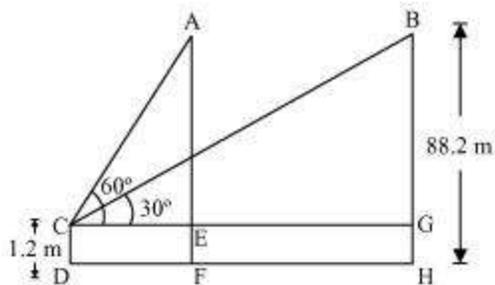
Therefore, the distance between the two ships is $75(\sqrt{3} - 1)$ m.

Q14 :

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.



Answer :



Let the initial position A of balloon change to B after some time and CD be the girl.

In $\triangle ACE$,

$$\frac{AE}{CE} = \tan 60^\circ$$

$$\frac{AF - EF}{CE} = \tan 60^\circ$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In $\triangle BCG$,

$$\frac{BG}{CG} = \tan 30^\circ$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon = EG = CG - CE

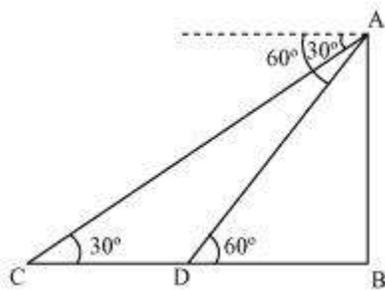
$$= (87\sqrt{3} - 29\sqrt{3})\text{m}$$

$$= 58\sqrt{3}\text{ m}$$

Q15 :

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.

Answer :



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In $\triangle ADB$,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right) = 6$ seconds

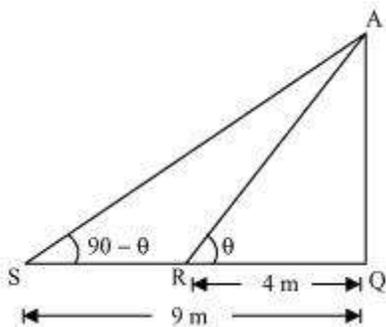
Time taken by the car to travel distance DB $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{2AB} \times \frac{AB}{\sqrt{3}}$

$$= \frac{6}{2} = 3 \text{ seconds}$$

Q16 :

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer :



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ , the other will be $90 - \theta$.

In ΔAQR ,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \quad \dots(i)$$

In ΔAQS ,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot \theta \quad \dots(ii)$$

On multiplying equations (i) and (ii), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan \theta) \cdot (\cot \theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.

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CHAPTER 9 SOME APPLICATIONS OF TRIGONOMETRY

MIND MAP

This chapter consists of 3 different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 ANGLE OF ELEVATION

- Q.1 From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45° . Find the length of the flagstaff and the distance of the building from the point P.
- Q.2 The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60° . Find the height of the tower.
- Q.3 Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles.

TYPE: 2 ANGLE OF DEPRESSION

- Q.1 The angles of depression of the top and the bottom of an 10 m tall building from the top of a multi-storeyed building are 30° and 45° , respectively. Find the height of the multi-storeyed building and the distance between the two buildings.
- Q.2 From the top of a 10 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.
- Q.3 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 45° , which is approaching the foot of the tower with a uniform speed. 8 seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.

TYPE: 3 SOME SPECIAL TYPE QUESTIONS

- Q.1 The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.
- Q.2 From an aeroplane vertically above the straight road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β . Show that the height in miles of aeroplane above the road is given by $\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$.

NCERT Solutions for Class 10 Maths Unit 5

Arithmetic Progressions Class 10

Unit 5 Arithmetic Progressions Exercise 5.1, 5.2, 5.3, 5.4 Solutions

Exercise 5.1 : Solutions of Questions on Page Number : 99

Q1 :

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer :

(i) It can be observed that

Taxi fare for 1st km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Clearly 15, 23, 31, 39 ... forms an A.P. because every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be V lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in

the cylinder at a time. In other words, after every stroke, only $1 - \frac{1}{4} = \frac{3}{4}$ th part of air will remain.

Therefore, volumes will be $V, \frac{3}{4}V, \left(\frac{3}{4}\right)^2V, \left(\frac{3}{4}\right)^3V, \dots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 ... forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at $r\%$ compound interest per annum for n years, our money will

be $P\left(1 + \frac{r}{100}\right)^n$ after n years.

Therefore, after every year, our money will be

$$10000\left(1 + \frac{8}{100}\right), 10000\left(1 + \frac{8}{100}\right)^2, 10000\left(1 + \frac{8}{100}\right)^3, 10000\left(1 + \frac{8}{100}\right)^4, \dots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Q2 :

Write first four terms of the A.P. when the first term a and the common difference d are given as follows

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Answer :

(i) $a = 10, d = 10$

Let the series be $a_1, a_2, a_3, a_4, a_5 \dots$

$$a_1 = a = 10$$

$$a_2 = a_1 + d = 10 + 10 = 20$$

$$a_3 = a_2 + d = 20 + 10 = 30$$

$$a_4 = a_3 + d = 30 + 10 = 40$$

$$a_5 = a_4 + d = 40 + 10 = 50$$

Therefore, the series will be 10, 20, 30, 40, 50 ...

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2, d = 0$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = -2$$

$$a_2 = a_1 + d = -2 + 0 = -2$$

$$a_3 = a_2 + d = -2 + 0 = -2$$

$$a_4 = a_3 + d = -2 + 0 = -2$$

Therefore, the series will be - 2, - 2, - 2, - 2 ...

First four terms of this A.P. will be - 2, - 2, - 2 and - 2.

$$(iii) a = 4, d = - 3$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = 4$$

$$a_2 = a_1 + d = 4 - 3 = 1$$

$$a_3 = a_2 + d = 1 - 3 = - 2$$

$$a_4 = a_3 + d = - 2 - 3 = - 5$$

Therefore, the series will be 4, 1, - 2 - 5 ...

First four terms of this A.P. will be 4, 1, - 2 and - 5.

$$(iv) a = - 1, d = \frac{1}{2}$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = -1$$

$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Clearly, the series will be

$$-1, -\frac{1}{2}, 0, \frac{1}{2}, \dots$$

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$.

$$(v) a = - 1.25, d = - 0.25$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = - 1.25$$

$$a_2 = a_1 + d = - 1.25 - 0.25 = - 1.50$$

$$a_3 = a_2 + d = - 1.50 - 0.25 = - 1.75$$

$$a_4 = a_3 + d = - 1.75 - 0.25 = - 2.00$$

Clearly, the series will be 1.25, - 1.50, - 1.75, - 2.00

First four terms of this A.P. will be - 1.25, - 1.50, - 1.75 and - 2.00.

Q3 :

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, - 1, - 3 ...

(ii) - 5, - 1, 3, 7 ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

(iv) 0.6, 1.7, 2.8, 3.9 ...

Answer :

(i) 3, 1, - 1, - 3 ...

Here, first term, $a = 3$

Common difference, $d = \text{Second term} - \text{First term}$

$$= 1 - 3 = - 2$$

(ii) - 5, - 1, 3, 7 ...

Here, first term, $a = - 5$

Common difference, $d = \text{Second term} - \text{First term}$

$$= (- 1) - (- 5) = - 1 + 5 = 4$$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

Here, first term, $a = \frac{1}{3}$

Common difference, $d = \text{Second term} - \text{First term}$

$$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$

(iv) 0.6, 1.7, 2.8, 3.9 ...

Here, first term, $a = 0.6$

Common difference, $d = \text{Second term} - \text{First term}$

$$= 1.7 - 0.6$$

$$= 1.1$$

Q4 :

Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2, 4, 8, 16 ...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \dots$

(iii) - 1.2, - 3.2, - 5.2, - 7.2 ...

(iv) - 10, - 6, - 2, 2 ...

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2} \dots$

(vi) 0.2, 0.22, 0.222, 0.2222

(vii) 0, - 4, - 8, - 12 ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \dots$

(ix) 1, 3, 9, 27 ...

(x) $a, 2a, 3a, 4a \dots$

(xi) $a, a^2, a^3, a^4 \dots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \dots$

(xiv) $1^2, 3^2, 5^2, 7^2 \dots$

(xv) $1^2, 5^2, 7^2, 73 \dots$

Answer :

(i) 2, 4, 8, 16 ...

It can be observed that

$$a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$a_4 - a_3 = 16 - 8 = 8$$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not forming an A.P.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \dots$

It can be observed that

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

$$d = \frac{1}{2}$$

Therefore, and the given numbers are in A.P.

Three more terms are

$$a_5 = \frac{7}{2} + \frac{1}{2} = 4$$

$$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$$

$$a_7 = \frac{9}{2} + \frac{1}{2} = 5$$

(iii) - 1.2, - 3.2, - 5.2, - 7.2 ...

It can be observed that

$$a_2 - a_1 = (-3.2) - (-1.2) = -2$$

$$a_3 - a_2 = (-5.2) - (-3.2) = -2$$

$$a_4 - a_3 = (-7.2) - (-5.2) = -2$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -2$

The given numbers are in A.P.

Three more terms are

$$a_5 = -7.2 - 2 = -9.2$$

$$a_6 = -9.2 - 2 = -11.2$$

$$a_7 = -11.2 - 2 = -13.2$$

(iv) - 10, - 6, - 2, 2 ...

It can be observed that

$$a_2 - a_1 = (-6) - (-10) = 4$$

$$a_3 - a_2 = (-2) - (-6) = 4$$

$$a_4 - a_3 = (2) - (-2) = 4$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 4$

The given numbers are in A.P.

Three more terms are

$$a_5 = 2 + 4 = 6$$

$$a_6 = 6 + 4 = 10$$

$$a_7 = 10 + 4 = 14$$

$$(v) \quad 3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$$

It can be observed that

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

$$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in A.P.

Three more terms are

$$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$$

$$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$

$$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

$$(vi) \quad 0.2, 0.22, 0.222, 0.2222 \dots$$

It can be observed that

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$

$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$

$$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

$$(vii) \quad 0, -4, -8, -12 \dots$$

It can be observed that

$$a_2 - a_1 = (-4) - 0 = -4$$

$$a_3 - a_2 = (-8) - (-4) = -4$$

$$a_4 - a_3 = (-12) - (-8) = -4$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -4$

The given numbers are in A.P.

Three more terms are

$$a_5 = -$$

Q1 :

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8
II	- 18	10	0
III	- 3	18	- 5
IV	- 18.9	2.5	3.6
V	3.5	0	105

Answer :

I. $a = 7, d = 3, n = 8, a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1) d$

$$= 7 + (8 - 1) 3$$

$$= 7 + (7) 3$$

$$= 7 + 21 = 28$$

Hence, $a_n = 28$

II. Given that

$a = - 18, n = 10, a_n = 0, d = ?$

We know that,

$$a_n = a + (n - 1) d$$

$$0 = - 18 + (10 - 1) d$$

$$18 = 9d$$

$$d = \frac{18}{9} = 2$$

Hence, common difference, $d = 2$

III. Given that

$d = - 3, n = 18, a_n = - 5$

We know that,

$$a_n = a + (n - 1) d$$

$$- 5 = a + (18 - 1) (- 3)$$

$$- 5 = a + (17) (- 3)$$

$$-5 = a - 51$$

$$a = 51 - 5 = 46$$

Hence, $a = 46$

IV. $a = -18.9, d = 2.5, a_n = 3.6, n = ?$

We know that,

$$a_n = a + (n - 1) d$$

$$3.6 = -18.9 + (n - 1) 2.5$$

$$3.6 + 18.9 = (n - 1) 2.5$$

$$22.5 = (n - 1) 2.5$$

$$(n - 1) = \frac{22.5}{2.5}$$

$$n - 1 = 9$$

$$n = 10$$

Hence, $n = 10$

V. $a = 3.5, d = 0, n = 105, a_n = ?$

We know that,

$$a_n = a + (n - 1) d$$

$$a_n = 3.5 + (105 - 1) 0$$

$$a_n = 3.5 + 104 \times 0$$

$$a_n = 3.5$$

Hence, $a_n = 3.5$

Q2 :

Choose the correct choice in the following and justify

I. 30th term of the A.P: 10, 7, 4, ..., is

A. 97 B. 77 C. - 77 D. - 87

II 11th term of the A.P. $-3, -\frac{1}{2}, 2, \dots$ is

A. 28 B. 22 C. - 38 D. $-48\frac{1}{2}$

Answer :

I. Given that

A.P. 10, 7, 4, ...

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10$

$$= -3$$

We know that, $a_n = a + (n - 1) d$

$$a_{30} = 10 + (30 - 1) (-3)$$

$$a_{30} = 10 + (29) (-3)$$

$$a_{30} = 10 - 87 = -77$$

Hence, the correct answer is **C**.

II. Given that, A.P. $-3, -\frac{1}{2}, 2, \dots$

First term $a = -3$

Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$

$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that,

$$a_n = a + (n - 1) d$$

$$a_{11} = -3 + (11 - 1) \left(\frac{5}{2} \right)$$

$$a_{11} = -3 + (10) \left(\frac{5}{2} \right)$$

$$a_{11} = -3 + 25$$

$$a_{11} = 22$$

Hence, the answer is **B**.

Q3 :

In the following APs find the missing term in the boxes

I. $2, \square, 26$

II. $\square, 13, \square, 3$

III. $5, \square, \square, 9\frac{1}{2}$

IV. $-4, \square, \square, \square, \square, 6$

V. $\square, 38, \square, \square, \square, -22$

Answer :

I. $2, \square, 26$

For this A.P.,

$$a = 2$$

$$a_3 = 26$$

We know that, $a_n = a + (n - 1) d$

$$a_3 = 2 + (3 - 1) d$$

$$26 = 2 + 2d$$

$$24 = 2d$$

$$d = 12$$

$$a_2 = 2 + (2 - 1) 12$$

$$= 14$$

Therefore, 14 is the missing term.

II. $\square, 13, \square, 3$

For this A.P.,

$$a_2 = 13 \text{ and}$$

$$a_4 = 3$$

We know that, $a_n = a + (n - 1) d$

$$a_2 = a + (2 - 1) d$$

$$13 = a + d \text{ (I)}$$

$$a_4 = a + (4 - 1) d$$

$$3 = a + 3d \text{ (II)}$$

On subtracting (I) from (II), we obtain

$$-10 = 2d$$

$$d = -5$$

From equation (I), we obtain

$$13 = a + (-5)$$

$$a = 18$$

$$a_3 = 18 + (3 - 1) (-5)$$

$$= 18 + 2 (-5) = 18 - 10 = 8$$

Therefore, the missing terms are 18 and 8 respectively.

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$$a = 5$$

$$a_4 = 9\frac{1}{2} = \frac{19}{2}$$

We know that,

$$a_n = a + (n-1)d$$

$$a_4 = a + (4-1)d$$

$$\frac{19}{2} = 5 + 3d$$

$$\frac{19}{2} - 5 = 3d$$

$$\frac{9}{2} = 3d$$

$$d = \frac{3}{2}$$

$$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$

$$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. $-4, \square, \square, \square, \square, 6$

For this A.P.,

$$a = -4 \text{ and}$$

$$a_6 = 6$$

We know that,

$$a_n = a + (n-1)d$$

$$a_6 = a + (6-1)d$$

$$6 = -4 + 5d$$

$$10 = 5d$$

$$d = 2$$

$$a_2 = a + d = -4 + 2 = -2$$

$$a_3 = a + 2d = -4 + 2(2) = 0$$

$$a_4 = a + 3d = -4 + 3(2) = 2$$

$$a_5 = a + 4d = -4 + 4(2) = 4$$

Therefore, the missing terms are - 2, 0, 2, and 4 respectively.

v. $\square, 38, \square, \square, \square, -22$

For this A.P.,

$$a_2 = 38$$

$$a_6 = -22$$

We know that

$$a_n = a + (n - 1) d$$

$$a_2 = a + (2 - 1) d$$

$$38 = a + d(1)$$

$$a_6 = a + (6 - 1) d$$

$$-22 = a + 5d(2)$$

On subtracting equation (1) from (2), we obtain

$$-22 - 38 = 4d$$

$$-60 = 4d$$

$$d = -15$$

$$a = a_2 - d = 38 - (-15) = 53$$

$$a_3 = a + 2d = 53 + 2(-15) = 23$$

$$a_4 = a + 3d = 53 + 3(-15) = 8$$

$$a_5 = a + 4d = 53 + 4(-15) = -7$$

Therefore, the missing terms are 53, 23, 8, and - 7 respectively.

Q4 :

Which term of the A.P. 3, 8, 13, 18, ... is 78?

Answer :

3, 8, 13, 18, ...

For this A.P.,

$$a = 3$$

$$d = a_2 - a_1 = 8 - 3 = 5$$

Let n^{th} term of this A.P. be 78.

$$a_n = a + (n - 1) d$$

$$78 = 3 + (n - 1) 5$$

$$75 = (n - 1) 5$$

$$(n - 1) = 15$$

$$n = 16$$

Hence, 16th term of this A.P. is 78.

Q5 :

Find the number of terms in each of the following A.P.

I. 7, 13, 19, ..., 205

II. $18, 15\frac{1}{2}, 13, \dots, -47$

Answer :

I. 7, 13, 19, ..., 205

For this A.P.,

$$a = 7$$

$$d = a_2 - a_1 = 13 - 7 = 6$$

Let there are n terms in this A.P.

$$a_n = 205$$

We know that

$$a_n = a + (n - 1) d$$

$$\text{Therefore, } 205 = 7 + (n - 1) 6$$

$$198 = (n - 1) 6$$

$$33 = (n - 1)$$

$$n = 34$$

Therefore, this given series has 34 terms in it.

II. $18, 15\frac{1}{2}, 13, \dots, -47$

For this A.P.,

$$a = 18$$

$$d = a_2 - a_1 = 15\frac{1}{2} - 18$$

$$d = \frac{31 - 36}{2} = -\frac{5}{2}$$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$

$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$

$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$

$$-65 = (n-1)\left(-\frac{5}{2}\right)$$

$$(n-1) = \frac{-130}{-5}$$

$$(n-1) = 26$$

$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Q6 :

Check whether - 150 is a term of the A.P. 11, 8, 5, 2, ...

Answer :

For this A.P.,

$$a = 11$$

$$d = a_2 - a_1 = 8 - 11 = -3$$

Let - 150 be the n^{th} term of this A.P.

We know that,

$$\begin{aligned}
 a_n &= a + (n-1)d \\
 -150 &= 11 + (n-1)(-3) \\
 -150 &= 11 - 3n + 3 \\
 -164 &= -3n \\
 n &= \frac{164}{3}
 \end{aligned}$$

Clearly, n is not an integer.

Therefore, - 150 is not a term of this A.P.

Q7 :

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Answer :

Given that,

$$a_{11} = 38$$

$$a_{16} = 73$$

We know that,

$$a_n = a + (n - 1) d$$

$$a_{11} = a + (11 - 1) d$$

$$38 = a + 10d \quad (1)$$

Similarly,

$$a_{16} = a + (16 - 1) d$$

$$73 = a + 15d \quad (2)$$

On subtracting (1) from (2), we obtain

$$35 = 5d$$

$$d = 7$$

From equation (1),

$$38 = a + 10 \times (7)$$

$$38 - 70 = a$$

$$a = -32$$

$$a_{31} = a + (31 - 1) d$$

$$= - 32 + 30 (7)$$

$$= - 32 + 210$$

$$= 178$$

Hence, 31st term is 178.

Q8 :

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Answer :

Given that,

$$a_3 = 12$$

$$a_{50} = 106$$

We know that,

$$a_n = a + (n - 1) d$$

$$a_3 = a + (3 - 1) d$$

$$12 = a + 2d \text{ (I)}$$

$$\text{Similarly, } a_{50} = a + (50 - 1) d$$

$$106 = a + 49d \text{ (II)}$$

On subtracting (I) from (II), we obtain

$$94 = 47d$$

$$d = 2$$

From equation (I), we obtain

$$12 = a + 2(2)$$

$$a = 12 - 4 = 8$$

$$a_{29} = a + (29 - 1) d$$

$$a_{29} = 8 + (28)2$$

$$a_{29} = 8 + 56 = 64$$

Therefore, 29th term is 64.

Q9 :

If the 3rd and the 9th terms of an A.P. are 4 and - 8 respectively. Which term of this A.P. is zero.

Answer :

Given that,

$$a_3 = 4$$

$$a_9 = -8$$

We know that,

$$a_n = a + (n - 1) d$$

$$a_3 = a + (3 - 1) d$$

$$4 = a + 2d \text{ (I)}$$

$$a_9 = a + (9 - 1) d$$

$$-8 = a + 8d \text{ (II)}$$

On subtracting equation (I) from (II), we obtain

$$-12 = 6d$$

$$d = -2$$

From equation (I), we obtain

$$4 = a + 2(-2)$$

$$4 = a - 4$$

$$a = 8$$

Let n^{th} term of this A.P. be zero.

$$a_n = a + (n - 1) d$$

$$0 = 8 + (n - 1) (-2)$$

$$0 = 8 - 2n + 2$$

$$2n = 10$$

$$n = 5$$

Hence, 5th term of this A.P. is 0.

Q10 :

If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.

Answer :

We know that,

For an A.P., $a_n = a + (n - 1) d$

$$a_{17} = a + (17 - 1) d$$

$$a_{17} = a + 16d$$

Similarly, $a_{10} = a + 9d$

It is given that

$$a_{17} - a_{10} = 7$$

$$(a + 16d) - (a + 9d) = 7$$

$$7d = 7$$

$$d = 1$$

Therefore, the common difference is 1.

Q11 :

Which term of the A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term?

Answer :

Given A.P. is 3, 15, 27, 39, ...

$$a = 3$$

$$d = a_2 - a_1 = 15 - 3 = 12$$

$$a_{54} = a + (54 - 1) d$$

$$= 3 + (53) (12)$$

$$= 3 + 636 = 639$$

$$132 + 639 = 771$$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$$a_n = a + (n - 1) d$$

$$771 = 3 + (n - 1) 12$$

$$768 = (n - 1) 12$$

$$(n - 1) = 64$$

$$n = 65$$

Therefore, 65th term was 132 more than 54th term.

Alternatively,

Let n^{th} term be 132 more than 54th term.

$$n = 54 + \frac{132}{12}$$

$$= 54 + 11 = 65^{\text{th}} \text{ term}$$

Q12 :

Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Answer :

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d .

For first A.P.,

$$a_{100} = a_1 + (100 - 1) d$$

$$= a_1 + 99d$$

$$a_{1000} = a_1 + (1000 - 1) d$$

$$a_{1000} = a_1 + 999d$$

For second A.P.,

$$a_{100} = a_2 + (100 - 1) d$$

$$= a_2 + 99d$$

$$a_{1000} = a_2 + (1000 - 1) d$$

$$= a_2 + 999d$$

Given that, difference between

100th term of these A.P.s = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$$a_1 - a_2 = 100 \quad (1)$$

Difference between 1000th terms of these A.P.s

$$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000th terms of these A.P. will be 100.

Q13 :

How many three digit numbers are divisible by 7

Answer :

First three-digit number that is divisible by 7 = 105

Next number = $105 + 7 = 112$

Therefore, 105, 112, 119, ...

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, $999 - 5 = 994$ is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, ..., 994

Let 994 be the n th term of this A.P.

$$a = 105$$

$$d = 7$$

$$a_n = 994$$

$$n = ?$$

$$a_n = a + (n - 1) d$$

$$994 = 105 + (n - 1) 7$$

$$889 = (n - 1) 7$$

$$(n - 1) = 127$$

$$n = 128$$

Therefore, 128 three-digit numbers are divisible by 7.

Q14 :

How many multiples of 4 lie between 10 and 250?

Answer :

First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, ...

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, $250 - 2 = 248$ is divisible by 4.

The series is as follows.

12, 16, 20, 24, ..., 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$

$$d = 4$$

$$a_n = 248$$

$$a_n = a + (n - 1) d$$

$$248 = 12 + (n - 1) 4$$

$$\frac{236}{4} = n - 1$$

$$59 = n - 1$$

$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Q15 :

For what value of n , are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal

Answer :

63, 65, 67, ...

$$a = 63$$

$$d = a_2 - a_1 = 65 - 63 = 2$$

$$n^{\text{th}} \text{ term of this A.P.} = a_n = a + (n - 1) d$$

$$a_n = 63 + (n - 1) 2 = 63 + 2n - 2$$

$$a_n = 61 + 2n \quad (1)$$

3, 10, 17, ...

$$a = 3$$

$$d = a_2 - a_1 = 10 - 3 = 7$$

$$n^{\text{th}} \text{ term of this A.P.} = 3 + (n - 1) 7$$

$$a_n = 3 + 7n - 7$$

$$a_n = 7n - 4 \quad (2)$$

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating both these equations, we obtain

$$61 + 2n = 7n - 4$$

$$61 + 4 = 5n$$

$$5n = 65$$

$$n = 13$$

Therefore, 13th terms of both these A.P.s are equal to each other.

Q16 :

Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Answer :

$$a_3 = 16$$

$$a + (3 - 1) d = 16$$

$$a + 2d = 16 \quad (1)$$

$$a_7 - a_5 = 12$$

$$[a + (7 - 1)d] - [a + (5 - 1)d] = 12$$

$$(a + 6d) - (a + 4d) = 12$$

$$2d = 12$$

$$d = 6$$

From equation (1), we obtain

$$a + 2(6) = 16$$

$$a + 12 = 16$$

$$a = 4$$

Therefore, A.P. will be

4, 10, 16, 22, ...

Q17 :

Find the 20th term from the last term of the A.P. 3, 8, 13, ..., 253

Answer :

Given A.P. is

3, 8, 13, ..., 253

Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as

253, 248, 243, ..., 13, 8, 5

For this A.P.,

$$a = 253$$

$$d = 248 - 253 = -5$$

$$n = 20$$

$$a_{20} = a + (20 - 1)d$$

$$a_{20} = 253 + (19)(-5)$$

$$a_{20} = 253 - 95$$

$$a = 158$$

Therefore, 20th term from the last term is 158.

Q18 :

The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer :

We know that,

$$a_n = a + (n - 1) d$$

$$a_4 = a + (4 - 1) d$$

$$a_4 = a + 3d$$

Similarly,

$$a_8 = a + 7d$$

$$a_6 = a + 5d$$

$$a_{10} = a + 9d$$

Given that, $a_4 + a_8 = 24$

$$a + 3d + a + 7d = 24$$

$$2a + 10d = 24$$

$$a + 5d = 12 \text{ (1)}$$

$$a_6 + a_{10} = 44$$

$$a + 5d + a + 9d = 44$$

$$2a + 14d = 44$$

$$a + 7d = 22 \text{ (2)}$$

On subtracting equation (1) from (2), we obtain

$$2d = 22 - 12$$

$$2d = 10$$

$$d = 5$$

From equation (1), we obtain

$$a + 5d = 12$$

$$a + 5(5) = 12$$

$$a + 25 = 12$$

$$a = -13$$

$$a_2 = a + d = -13 + 5 = -8$$

$$a_3 = a_2 + d = -8 + 5 = -3$$

Therefore, the first three terms of this A.P. are -13, -8, and -3.

Q19 :

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her weekly savings become Rs 20.75, find n .

Answer :

Given that,

$$a = 5$$

$$d = 1.75$$

$$a_n = 20.75$$

$$n = ?$$

$$a_n = a + (n - 1) d$$

$$20.75 = 5 + (n - 1)1.75$$

$$15.75 = (n - 1)1.75$$

$$(n - 1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$$n - 1 = 9$$

$$n = 10$$

Hence, n is 10.

Exercise 5.3 : Solutions of Questions on Page Number : 112

Q1 :

Find the sum of the following APs.

(i) 2, 7, 12,, to 10 terms.

(ii) - 37, - 33, - 29,, to 12 terms

(iii) 0.6, 1.7, 2.8,, to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms

Answer :

(i) 2, 7, 12, ..., to 10 terms

For this A.P.,

$$a = 2$$

$$d = a_2 - a_1 = 7 - 2 = 5$$

$$n = 10$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{10} &= \frac{10}{2} [2(2) + (10-1)5] \\ &= 5 [4 + (9) \times (5)] \\ &= 5 \times 49 = 245 \end{aligned}$$

(ii) - 37, - 33, - 29, ..., to 12 terms

For this A.P.,

$$a = - 37$$

$$d = a_2 - a_1 = (- 33) - (- 37)$$

$$= - 33 + 37 = 4$$

$$n = 12$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{12} &= \frac{12}{2} [2(-37) + (12-1)4] \\ &= 6 [-74 + 11 \times 4] \\ &= 6 [-74 + 44] \\ &= 6(-30) = -180 \end{aligned}$$

(iii) 0.6, 1.7, 2.8, ..., to 100 terms

For this A.P.,

$$a = 0.6$$

$$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$$

$$n = 100$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{100} &= \frac{100}{2} [2(0.6) + (100-1)1.1] \\ &= 50 [1.2 + (99) \times (1.1)] \\ &= 50 [1.2 + 108.9] \\ &= 50 [110.1] \\ &= 5505 \end{aligned}$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms

For this A.P.,

$$a = \frac{1}{15}$$

$$n = 11$$

$$\begin{aligned} d &= a_2 - a_1 = \frac{1}{12} - \frac{1}{15} \\ &= \frac{5-4}{60} = \frac{1}{60} \end{aligned}$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{11} &= \frac{11}{2} \left[2 \left(\frac{1}{15} \right) + (11-1) \frac{1}{60} \right] \\ &= \frac{11}{2} \left[\frac{2}{15} + \frac{10}{60} \right] \\ &= \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right] = \frac{11}{2} \left[\frac{4+5}{30} \right] \\ &= \left(\frac{11}{2} \right) \left(\frac{9}{30} \right) = \frac{33}{20} \end{aligned}$$

Q2 :

Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

(ii) $34 + 32 + 30 + \dots + 10$

(iii) $-5 + (-8) + (-11) + \dots + (-230)$

Answer :

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

For this A.P.,

$$a = 7$$

$$l = 84$$

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

$$l = a + (n - 1)d$$

$$84 = 7 + (n - 1)\frac{7}{2}$$

$$77 = (n - 1)\frac{7}{2}$$

$$22 = n - 1$$

$$n = 23$$

We know that,

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{23}{2}[7 + 84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) $34 + 32 + 30 + \dots + 10$

For this A.P.,

$$a = 34$$

$$d = a_2 - a_1 = 32 - 34 = -2$$

$$l = 10$$

Let 10 be the n^{th} term of this A.P.

$$l = a + (n - 1)d$$

$$10 = 34 + (n - 1)(-2)$$

$$-24 = (n - 1)(-2)$$

$$12 = n - 1$$

$$n = 13$$

$$\begin{aligned} S_n &= \frac{n}{2}(a+l) \\ &= \frac{13}{2}(34+10) \\ &= \frac{13 \times 44}{2} = 13 \times 22 \\ &= 286 \end{aligned}$$

(iii) $(-5) + (-8) + (-11) + \dots + (-230)$

For this A.P.,

$$a = -5$$

$$l = -230$$

$$d = a_2 - a_1 = (-8) - (-5)$$

$$= -8 + 5 = -3$$

Let -230 be the n^{th} term of this A.P.

$$l = a + (n - 1)d$$

$$-230 = -5 + (n - 1)(-3)$$

$$-225 = (n - 1)(-3)$$

$$(n - 1) = 75$$

$$n = 76$$

And,
$$S_n = \frac{n}{2}(a+l)$$

$$\begin{aligned}
&= \frac{76}{2} [(-5) + (-230)] \\
&= 38(-235) \\
&= -8930
\end{aligned}$$

Q3 :

In an AP

- (i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n .
- (ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13} .
- (iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12} .
- (iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} .
- (v) Given $d = 5$, $S_9 = 75$, find a and a_9 .
- (vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n .
- (vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d .
- (viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a .
- (ix) Given $a = 3$, $n = 8$, $S = 192$, find d .
- (x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a .

Answer :

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

$$\text{As } a_n = a + (n - 1)d,$$

$$\therefore 50 = 5 + (n - 1)3$$

$$45 = (n - 1)3$$

$$15 = n - 1$$

$$n = 16$$

$$S_n = \frac{n}{2} [a + a_n]$$

$$\begin{aligned}
S_{16} &= \frac{16}{2} [5 + 50] \\
&= 8 \times 55 \\
&= 440
\end{aligned}$$

(ii) Given that, $a = 7$, $a_{13} = 35$

$$\text{As } a_n = a + (n - 1) d,$$

$$\therefore a_{13} = a + (13 - 1) d$$

$$35 = 7 + 12 d$$

$$35 - 7 = 12d$$

$$28 = 12d$$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\begin{aligned} S_{13} &= \frac{n}{2}[a + a_{13}] \\ &= \frac{13}{2}[7 + 35] \\ &= \frac{13 \times 42}{2} = 13 \times 21 \\ &= 273 \end{aligned}$$

(iii) Given that, $a_{12} = 37$, $d = 3$

$$\text{As } a_n = a + (n - 1)d,$$

$$a_{12} = a + (12 - 1)3$$

$$37 = a + 33$$

$$a = 4$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_n = \frac{12}{2}[4 + 37]$$

$$S_n = 6(41)$$

$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

$$\text{As } a_n = a + (n - 1)d,$$

$$a_3 = a + (3 - 1)d$$

$$15 = a + 2d \text{ (i)}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2a + (10-1)d]$$

$$125 = 5(2a + 9d)$$

$$25 = 2a + 9d \quad (\text{ii})$$

On multiplying equation (1) by 2, we obtain

$$30 = 2a + 4d \quad (\text{iii})$$

On subtracting equation (iii) from (ii), we obtain

$$-5 = 5d$$

$$d = -1$$

From equation (i),

$$15 = a + 2(-1)$$

$$15 = a - 2$$

$$a = 17$$

$$a_{10} = a + (10 - 1)d$$

$$a_{10} = 17 + (9)(-1)$$

$$a_{10} = 17 - 9 = 8$$

(v) Given that, $d = 5$, $S_9 = 75$

$$\text{As } S_n = \frac{n}{2} [2a + (n-1)d],$$

$$S_9 = \frac{9}{2} [2a + (9-1)5]$$

$$75 = \frac{9}{2} (2a + 40)$$

$$25 = 3(a + 20)$$

$$25 = 3a + 60$$

$$3a = 25 - 60$$

$$a = \frac{-35}{3}$$

$$a_n = a + (n - 1)d$$

$$a_9 = a + (9 - 1)(5)$$

$$\begin{aligned}
&= \frac{-35}{3} + 8(5) \\
&= \frac{-35}{3} + 40 \\
&= \frac{-35 + 120}{3} = \frac{85}{3}
\end{aligned}$$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

As

$$90 = \frac{n}{2} [4 + (n-1)8]$$

$$90 = n [2 + (n-1)4]$$

$$90 = n [2 + 4n - 4]$$

$$90 = n (4n - 2) = 4n^2 - 2n$$

$$4n^2 - 2n - 90 = 0$$

$$4n^2 - 20n + 18n - 90 = 0$$

$$4n(n-5) + 18(n-5) = 0$$

$$(n-5)(4n+18) = 0$$

Either $n-5 = 0$ or $4n+18 = 0$

$$n = 5 \text{ or } n = -\frac{18}{4} = -\frac{9}{2}$$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

a

Q4 :

How many terms of the AP. 9, 17, 25 ... must be taken to give a sum of 636?

Answer :

Let there be n terms of this A.P.

For this A.P., $a = 9$

$$d = a_2 - a_1 = 17 - 9 = 8$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$636 = \frac{n}{2} [2 \times a + (n-1)8]$$

$$636 = \frac{n}{2} [18 + (n-1)8]$$

$$636 = n [9 + 4n - 4]$$

$$636 = n (4n + 5)$$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n + 53) - 12(4n + 53) = 0$$

$$(4n + 53)(n - 12) = 0$$

Either $4n + 53 = 0$ or $n - 12 = 0$

$$n = \frac{-53}{4} \text{ or } n = 12$$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Q5 :

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer :

Given that,

$$a = 5$$

$$l = 45$$

$$S_n = 400$$

$$S_n = \frac{n}{2}(a+l)$$

$$400 = \frac{n}{2}(5+45)$$

$$400 = \frac{n}{2}(50)$$

$$n = 16$$

$$l = a + (n - 1) d$$

$$45 = 5 + (16 - 1) d$$

$$40 = 15d$$

$$d = \frac{40}{15} = \frac{8}{3}$$

Q6 :

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer :

Given that,

$$a = 17$$

$$l = 350$$

$$d = 9$$

Let there be n terms in the A.P.

$$l = a + (n - 1) d$$

$$350 = 17 + (n - 1)9$$

$$333 = (n - 1)9$$

$$(n - 1) = 37$$

$$n = 38$$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Q7 :

Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Answer :

$$d = 7$$

$$a_{22} = 149$$

$$S_{22} = ?$$

$$a_n = a + (n - 1)d$$

$$a_{22} = a + (22 - 1)d$$

$$149 = a + 21 \times 7$$

$$149 = a + 147$$

$$a = 2$$

$$\begin{aligned} S_n &= \frac{n}{2}(a + a_n) \\ &= \frac{22}{2}(2 + 149) \\ &= 11(151) = 1661 \end{aligned}$$

Q8 :

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer :

Given that,

$$a_2 = 14$$

$$a_3 = 18$$

$$d = a_3 - a_2 = 18 - 14 = 4$$

$$a_2 = a + d$$

$$14 = a + 4$$

$$a = 10$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{51} = \frac{51}{2}[2 \times 10 + (51-1)4]$$

$$= \frac{51}{2}[20 + (50)(4)]$$

$$= \frac{51(220)}{2} = 51(110)$$

$$= 5610$$

Q9 :

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer :

Given that,

$$S_7 = 49$$

$$S_{17} = 289$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[2a + (7-1)d]$$

$$49 = \frac{7}{2}(2a + 6d)$$

$$7 = (a + 3d)$$

$$a + 3d = 7 \text{ (i)}$$

Similarly,
$$S_{17} = \frac{17}{2}[2a + (17-1)d]$$

$$289 = \frac{17}{2}[2a + 16d]$$

$$17 = (a + 8d)$$

$$a + 8d = 17 \text{ (ii)}$$

Subtracting equation (i) from equation (ii),

$$5d = 10$$

$$d = 2$$

From equation (i),

$$a + 3(2) = 7$$

$$a + 6 = 7$$

$$a = 1$$

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{n}{2} [2(1) + (n-1)(2)] \\ &= \frac{n}{2} (2 + 2n - 2) \\ &= \frac{n}{2} (2n) \end{aligned}$$

$$= n^2$$

Q10 :

Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer :

(i) $a_n = 3 + 4n$

$$a_1 = 3 + 4(1) = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$a_3 = 3 + 4(3) = 3 + 12 = 15$$

$$a_4 = 3 + 4(4) = 3 + 16 = 19$$

It can be observed that

$$a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4$$

$$a_4 - a_3 = 19 - 15 = 4$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{15} = \frac{15}{2} [2(7) + (15-1)4]$$

$$= \frac{15}{2} [(14) + 56]$$

$$= \frac{15}{2} (70)$$

$$= 15 \times 35$$

$$= 525$$

(ii) $a_n = 9 - 5n$

$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$

$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$

$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$

$$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$$

It can be observed that

$$a_2 - a_1 = -1 - 4 = -5$$

$$a_3 - a_2 = -6 - (-1) = -5$$

$$a_4 - a_3 = -11 - (-6) = -5$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as -5 and first term as 4 .

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{15} = \frac{15}{2} [2(4) + (15-1)(-5)]$$

$$= \frac{15}{2} [8 + 14(-5)]$$

$$= \frac{15}{2} (8 - 70)$$

$$= \frac{15}{2} (-62) = 15(-31)$$

$$= -465$$

Q11 :

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the 10th and the n^{th} terms.

Answer :

Given that,

$$S_n = 4n - n^2$$

$$\text{First term, } a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$$

$$\text{Sum of first two terms} = S_2$$

$$= 4(2) - (2)^2 = 8 - 4 = 4$$

$$\text{Second term, } a_2 = S_2 - S_1 = 4 - 3 = 1$$

$$d = a_2 - a = 1 - 3 = -2$$

$$a_n = a + (n - 1)d$$

$$= 3 + (n - 1)(-2)$$

$$= 3 - 2n + 2$$

$$= 5 - 2n$$

$$\text{Therefore, } a_3 = 5 - 2(3) = 5 - 6 = -1$$

$$a_{10} = 5 - 2(10) = 5 - 20 = -15$$

Hence, the sum of first two terms is 4. The second term is 1. 3rd, 10th, and n^{th} terms are -1, -15, and $5 - 2n$ respectively.

Q12 :

Find the sum of first 40 positive integers divisible by 6.

Answer :

The positive integers that are divisible by 6 are

6, 12, 18, 24 ...

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$$a = 6$$

$$d = 6$$

$$S_{40}=?$$

$$\begin{aligned}
 S_n &= \frac{n}{2} [2a + (n-1)d] \\
 S_{40} &= \frac{40}{2} [2(6) + (40-1)6] \\
 &= 20[12 + (39)(6)] \\
 &= 20(12 + 234) \\
 &= 20 \times 246 \\
 &= 4920
 \end{aligned}$$

Q13 :

Find the sum of first 15 multiples of 8.

Answer :

The multiples of 8 are

8, 16, 24, 32...

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, $a = 8$

$d = 8$

$S_{15} = ?$

$$\begin{aligned}
 S_n &= \frac{n}{2} [2a + (n-1)d] \\
 &= \frac{15}{2} [2(8) + (15-1)8] \\
 &= \frac{15}{2} [16 + 14(8)] \\
 &= \frac{15}{2} (16 + 112) \\
 &= \frac{15(128)}{2} = 15 \times 64
 \end{aligned}$$

= 960

Q14 :

Find the sum of the odd numbers between 0 and 50.

Answer :

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 ... 49

Therefore, it can be observed that these odd numbers are in an A.P.

$$a = 1$$

$$d = 2$$

$$l = 49$$

$$l = a + (n - 1) d$$

$$49 = 1 + (n - 1)2$$

$$48 = 2(n - 1)$$

$$n - 1 = 24$$

$$n = 25$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$$= 625$$

Q15 :

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer :

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$$a = 200$$

$$d = 50$$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$$= \frac{30}{2} [2(200) + (30-1)50]$$

$$= 15 [400 + 1450]$$

$$= 15 (1850)$$

$$= 27750$$

Therefore, the contractor has to pay Rs 27750 as penalty.

Q16 :

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer :

Let the cost of 1st prize be P .

Cost of 2nd prize = $P - 20$

And cost of 3rd prize = $P - 40$

It can be observed that the cost of these prizes are in an A.P. having common difference as $- 20$ and first term as P .

$$a = P$$

$$d = - 20$$

Given that, $S_7 = 700$

$$\frac{7}{2} [2a + (7-1)d] = 700$$

$$\frac{[2a + (6)(-20)]}{2} = 100$$

$$a + 3(-20) = 100$$

$$a - 60 = 100$$

$$a = 160$$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Q17 :

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in

which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer :

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5.....12

First term, $a = 1$

Common difference, $d = 2 - 1 = 1$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{12} = \frac{12}{2} [2(1) + (12-1)(1)]$$

$$= 6 (2 + 11)$$

$$= 6 (13)$$

$$= 78$$

Therefore, number of trees planted by 1 section of the classes = 78

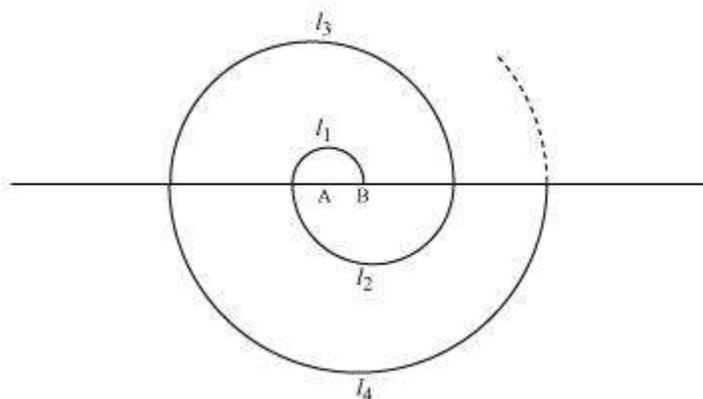
Number of trees planted by 3 sections of the classes = $3 \times 78 = 234$

Therefore, 234 trees will be planted by the students.

Q18 :

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, as shown in figure. What is the total length of such a spiral made up

of thirteen consecutive semicircles? $\left(\text{Take } \pi = \frac{22}{7} \right)$



Answer :

Semi-perimeter of circle = πr

$$l_1 = \pi(0.5) = \frac{\pi}{2} \text{ cm}$$

$$l_2 = \pi(1) = \pi \text{ cm}$$

$$l_3 = \pi(1.5) = \frac{3\pi}{2} \text{ cm}$$

Therefore, l_1, l_2, l_3 , i.e. the lengths of the semi-circles are in an A.P.,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \dots$$

$$a = \frac{\pi}{2}$$

$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$S_{13} = ?$

We know that the sum of n terms of an A.P. is given by

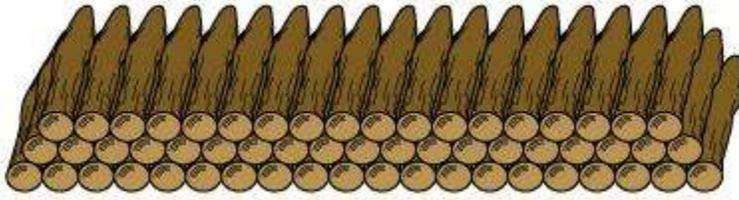
$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{13}{2} \left[2 \left(\frac{\pi}{2} \right) + (13-1) \left(\frac{\pi}{2} \right) \right] \\ &= \frac{13}{2} \left[\pi + \frac{12\pi}{2} \right] \\ &= \left(\frac{13}{2} \right) (7\pi) \\ &= \frac{91\pi}{2} \\ &= \frac{91 \times 22}{2 \times 7} = 13 \times 11 \end{aligned}$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Q19 :

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer :

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18...

For this A.P.,

$$a = 20$$

$$d = a_2 - a_1 = 19 - 20 = -1$$

Let a total of 200 logs be placed in n rows.

$$S_n = 200$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$200 = \frac{n}{2} [2(20) + (n-1)(-1)]$$

$$400 = n(40 - n + 1)$$

$$400 = n(41 - n)$$

$$400 = 41n - n^2$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 16n - 25n + 400 = 0$$

$$n(n - 16) - 25(n - 16) = 0$$

$$(n - 16)(n - 25) = 0$$

Either $(n - 16) = 0$ or $n - 25 = 0$

$$n = 16 \text{ or } n = 25$$

$$a_n = a + (n - 1)d$$

$$a_{16} = 20 + (16 - 1)(-1)$$

$$a_{16} = 20 - 15$$

$$a_{16} = 5$$

Similarly,

$$a_{25} = 20 + (25 - 1) (- 1)$$

$$a_{25} = 20 - 24$$

$$= - 4$$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

Q20 :

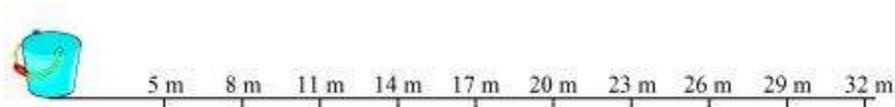
In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Answer :



The distances of potatoes are as follows.

5, 8, 11, 14...

It can be observed that these distances are in A.P.

$$a = 5$$

$$d = 8 - 5 = 3$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(5) + (10-1)3]$$

$$= 5[10 + 9 \times 3]$$

$$= 5(10 + 27) = 5(37)$$

$$= 185$$

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

$$\text{Therefore, total distance that the competitor will run} = 2 \times 185$$

$$= 370 \text{ m}$$

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14...

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,.....

$$a = 10$$

$$d = 16 - 10 = 6$$

$$S_{10} = ?$$

$$S_{10} = \frac{10}{2} [2 \times 10 + (10 - 1)6]$$

$$= 5[20 + 54]$$

$$= 5(74)$$

$$= 370$$

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4 : Solutions of Questions on Page Number : 115

Q1 :

Which term of the A.P. 121, 117, 113 ... is its first negative term?

[Hint: Find n for $a_n < 0$]

Answer :

Given A.P. is 121, 117, 113 ...

$$a = 121$$

$$d = 117 - 121 = -4$$

$$a_n = a + (n - 1) d$$

$$= 121 + (n - 1) (- 4)$$

$$= 121 - 4n + 4$$

$$= 125 - 4n$$

We have to find the first negative term of this A.P.

Therefore, $a_n < 0$

$$125 - 4n < 0$$

$$125 < 4n$$

$$n > \frac{125}{4}$$

$$n > 31.25$$

Therefore, 32nd term will be the first negative term of this A.P.

Q2 :

The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Answer :

We know that,

$$a_n = a + (n - 1) d$$

$$a_3 = a + (3 - 1) d$$

$$a_3 = a + 2d$$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$$(a + 2d) + (a + 6d) = 6$$

$$2a + 8d = 6$$

$$a + 4d = 3$$

$$a = 3 - 4d \text{ (i)}$$

Also, it is given that $(a_3) \times (a_7) = 8$

$$(a + 2d) \times (a + 6d) = 8$$

From equation (i),

$$(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$$

$$(3 - 2d) \times (3 + 2d) = 8$$

$$9 - 4d^2 = 8$$

$$4d^2 = 9 - 8 = 1$$

$$d^2 = \frac{1}{4}$$

$$d = \pm \frac{1}{2}$$

$$d = \frac{1}{2} \text{ or } -\frac{1}{2}$$

From equation (i),

$$\left(\text{When } d \text{ is } \frac{1}{2} \right)$$

$$a = 3 - 4d$$

$$a = 3 - 4\left(\frac{1}{2}\right)$$

$$= 3 - 2 = 1$$

$$\left(\text{When } d \text{ is } -\frac{1}{2} \right)$$

$$a = 3 - 4\left(-\frac{1}{2}\right)$$

$$a = 3 + 2 = 5$$

$$S_n = \frac{n}{2} [2a(n-1)d]$$

$$\left(\text{When } a \text{ is } 1 \text{ and } d \text{ is } \frac{1}{2} \right)$$

$$S_{16} = \frac{16}{2} \left[2(1) + (16-1)\left(\frac{1}{2}\right) \right]$$

$$= 8 \left[2 + \frac{15}{2} \right]$$

$$= 4(19) = 76$$

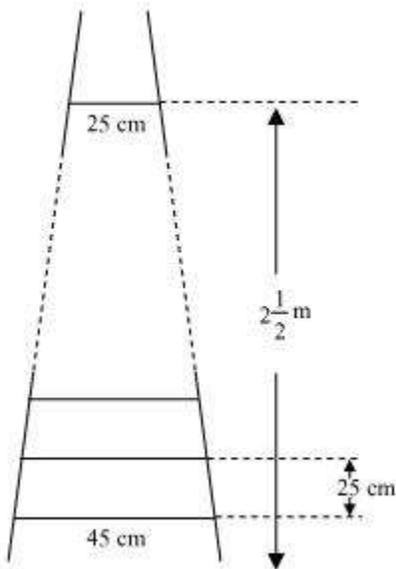
$$\left(\text{When } a \text{ is } 5 \text{ and } d \text{ is } -\frac{1}{2} \right)$$

$$\begin{aligned}
 S_{16} &= \frac{16}{2} \left[2(5) + (16-1) \left(-\frac{1}{2} \right) \right] \\
 &= 8 \left[10 + (15) \left(-\frac{1}{2} \right) \right] \\
 &= 8 \left(\frac{5}{2} \right) \\
 &= 20
 \end{aligned}$$

Q3 :

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint: number of rungs = $\frac{250}{25}$]



Answer :

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$$\begin{aligned}
 \therefore \text{Total number of rungs} &= \frac{2\frac{1}{2} \times 100}{25} + 1 = \frac{250}{25} + 1 = 11
 \end{aligned}$$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Q4 :

The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it.

Find this value of x .

[Hint $S_{x-1} = S_{49} - S_x$]

Answer :

The number of houses was

1, 2, 3 ... 49

It can be observed that the number of houses are in an A.P. having a as 1 and d also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

$$\text{Sum of } n \text{ terms in an A.P.} = \frac{n}{2}[2a + (n-1)d]$$

Sum of number of houses preceding x^{th} house = S_{x-1}

$$\begin{aligned}
&= \frac{(x-1)}{2} [2a + (x-1-1)d] \\
&= \frac{x-1}{2} [2(1) + (x-2)(1)] \\
&= \frac{x-1}{2} [2 + x - 2] \\
&= \frac{(x)(x-1)}{2}
\end{aligned}$$

Sum of number of houses following x^{th} house = $S_{49} - S_x$

$$\begin{aligned}
&= \frac{49}{2} [2(1) + (49-1)(1)] - \frac{x}{2} [2(1) + (x-1)(1)] \\
&= \frac{49}{2} (2 + 49 - 1) - \frac{x}{2} (2 + x - 1) \\
&= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x+1) \\
&= 25(49) - \frac{x(x+1)}{2}
\end{aligned}$$

It is given that these sums are equal to each other.

$$\begin{aligned}
\frac{x(x-1)}{2} &= 25(49) - x\left(\frac{x+1}{2}\right) \\
\frac{x^2}{2} - \frac{x}{2} &= 1225 - \frac{x^2}{2} - \frac{x}{2} \\
x^2 &= 1225 \\
x &= \pm 35
\end{aligned}$$

However, the house numbers are positive integers.

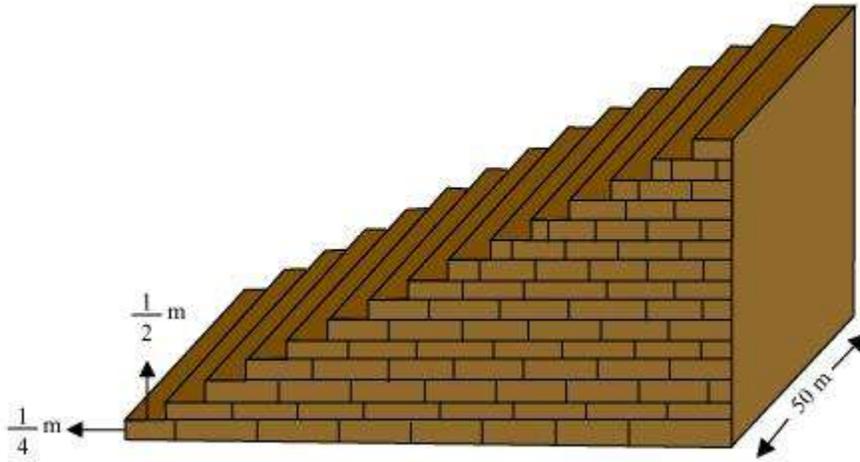
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

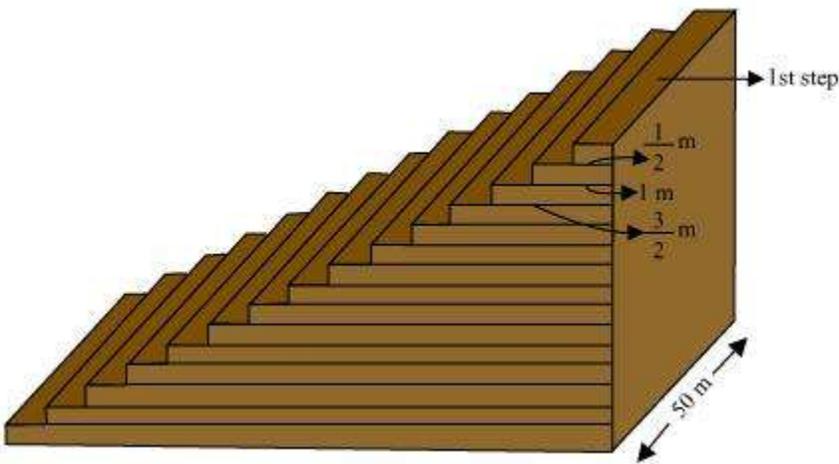
Q5 :

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.



Answer :



From the figure, it can be observed that

1st step is $\frac{1}{2}$ m wide,

2nd step is 1 m wide,

3rd step is $\frac{3}{2}$ m wide.

Therefore, the width of each step is increasing by $\frac{1}{2}$ m each time whereas their height $\frac{1}{4}$ m and length 50 m remains the same.

Therefore, the widths of these steps are

$$\frac{1}{2}, 1, \frac{3}{2}, 2, \dots$$

$$\text{Volume of concrete in 1}^{\text{st}} \text{ step} = \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$$

$$\text{Volume of concrete in 2}^{\text{nd}} \text{ step} = \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$$

$$\text{Volume of concrete in 3}^{\text{rd}} \text{ step} = \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$$

It can be observed that the volumes of concrete in these steps are in an A.P.

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \dots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

$$\text{and } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{15} = \frac{15}{2} \left[2 \left(\frac{25}{4} \right) + (15-1) \frac{25}{4} \right]$$

$$= \frac{15}{2} \left[\frac{25}{2} + \frac{(14)25}{4} \right]$$

$$= \frac{15}{2} \left[\frac{25}{2} + \frac{175}{2} \right]$$

$$= \frac{15}{2} (100) = 750$$

Volume of concrete required to build the terrace is 750 m^3 .

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH.5 ARITHMETIC PROGRESSIONS

MIND MAP

This chapter consists of two different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 TO FIND THE n^{th} TERM OF AN A.P.

1. Write fourth term of an A.P. if its n^{th} term is $3n+2$.
2. Find whether 0 is a term of the A.P: 40,37,34,31, . . .
3. Write the next four terms of an A.P. $\sqrt{2}, \sqrt{18} \dots$
4. If $\frac{4}{5}, a, 2, \dots$ three consecutive term of an A.P then find 'a'.
5. Find the middle term of A.P. 6,13,20,.....216.
6. Find A.P which fifth term is 5 and common difference is -3 .
7. Determine the 15th from the end of the A.P: 4,9,14,.....254 .
8. The n^{th} term of an A.P is $3n+5$ find its common difference.

TYPE: 2 TO FIND THE SUM OF n^{th} TERM OF AN A.P.

1. The sum of first 6 term of A.P is 42.The ratio its 10th term to 30th term is 1:3. Calculate the first and 13th term of the A.P.
2. Find the sum of first 24 term of A.P. 5, 8, 11, 14,....
3. The sum of three numbers in A.P is 21 and their product is 231. Find the numbers.
4. Find the sum of 25 terms of an A.P. whose n^{th} term is given by $7 - 3n$.
5. Find the sum of all two digit odd positive numbers.
6. Find the sum of three digits numbers that are divisible by 17.
7. How many term of the A.P: 17, 15, 13... must be added to get the sum 72? Explain the double answer.
8. The sums of $n, 2n, 3n$ term of an A.P are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Chapter 6: Triangles

Unit 6 Triangles Exercise 6.1, 6.2, 6.3, 6.4, 6.5, 6.6 Solutions

Exercise 6.1 : Solutions of Questions on Page Number : 122

Q1 :

Fill in the blanks using correct word given in the brackets:-

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Answer :

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
- (b) Proportional

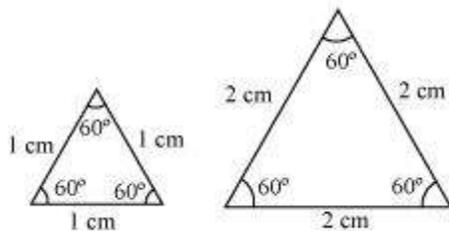
Q2 :

Give two different examples of pair of

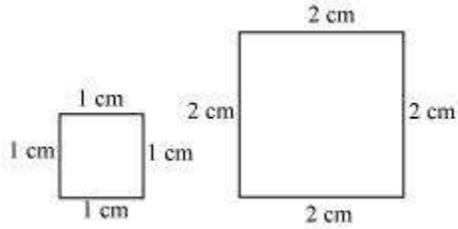
- (i) Similar figures
- (ii) Non-similar figures

Answer :

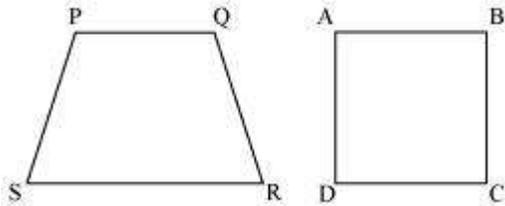
- (i) Two equilateral triangles with sides 1 cm and 2 cm



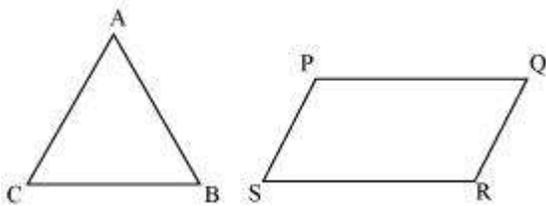
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

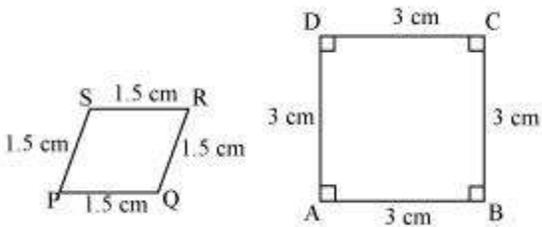


Triangle and parallelogram



Q3 :

State whether the following quadrilaterals are similar or not:



Answer :

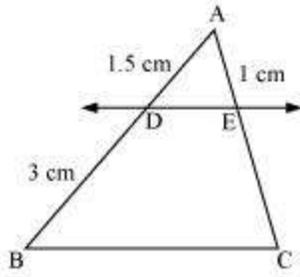
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2 : Solutions of Questions on Page Number : 128

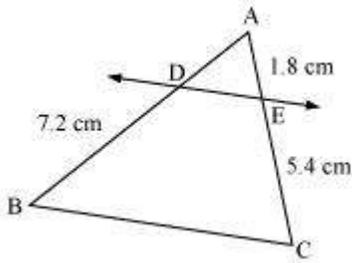
Q1 :

In figure.6.17. (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).

(i)

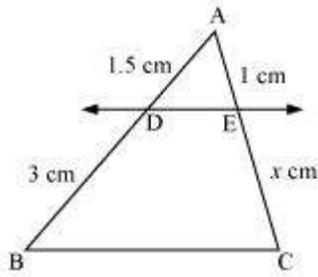


(ii)



Answer :

(i)



Let $EC = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

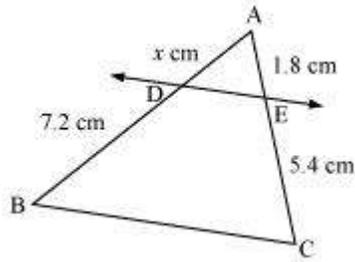
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let $AD = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$\therefore AD = 2.4$ cm

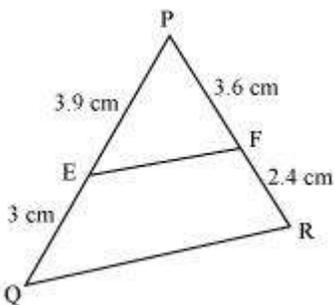
Q2 :

E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$.

- (i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm
- (ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm
- (iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.63$ cm

Answer :

(i)



Given that, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm

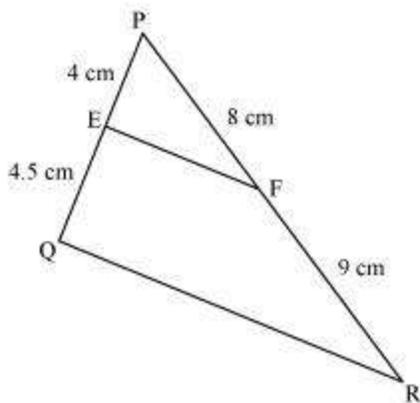
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

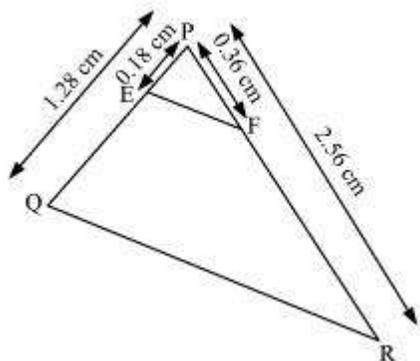
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

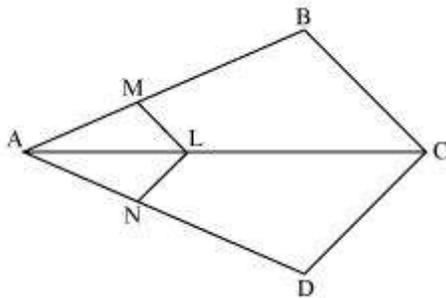
Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

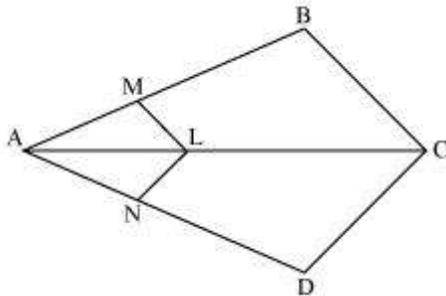
Q3 :

In the following figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer :



In the given figure, $LM \parallel CB$

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly, $LN \parallel CD$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

From (i) and (ii), we obtain

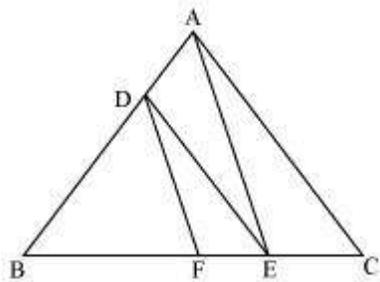
$$\frac{AM}{AB} = \frac{AN}{AD}$$



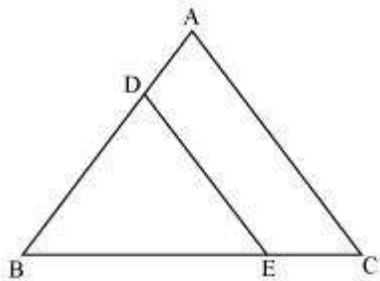
Q4 :

In the following figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

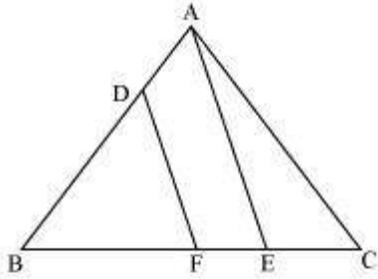


Answer :



In $\triangle ABC$, $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad (\text{Basic Proportionality Theorem}) \quad (i)$$



In $\triangle BAE$, $DF \parallel AE$

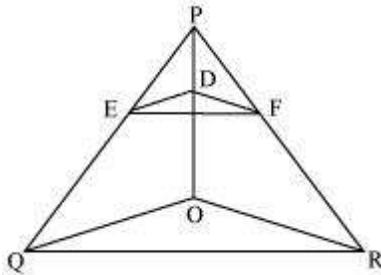
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad (\text{Basic Proportionality Theorem}) \quad (ii)$$

From (i) and (ii), we obtain

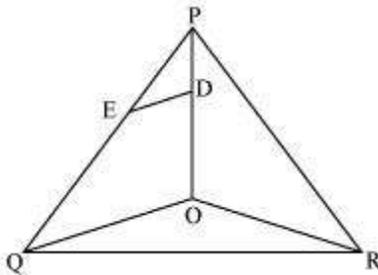
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Q5 :

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.

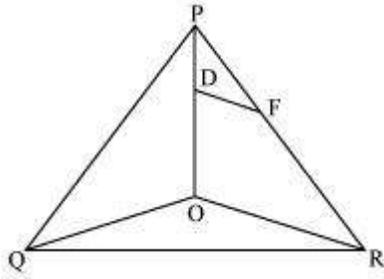


Answer :



In $\triangle POQ$, $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (i)$$



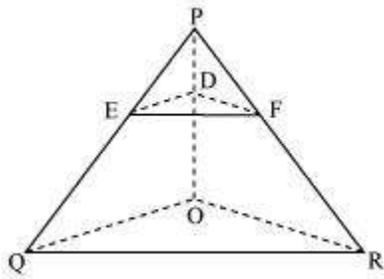
In $\triangle POR$, $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

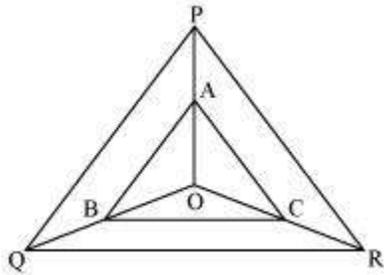
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

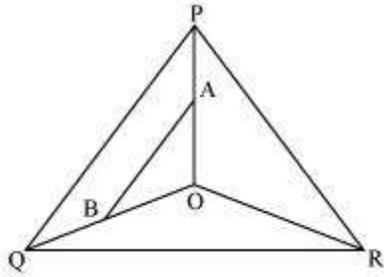


Q6 :

In the following figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

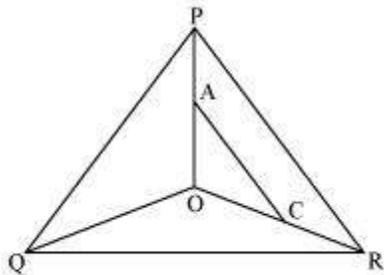


Answer :



In ΔPOQ , $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad (\text{Basic proportionality theorem}) \quad (i)$$



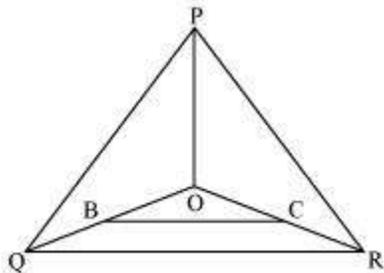
In ΔPOR , $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

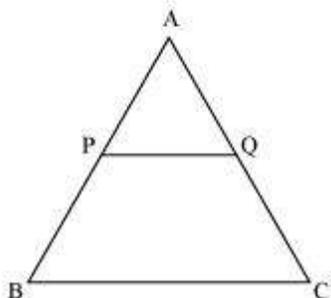
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Q7 :

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

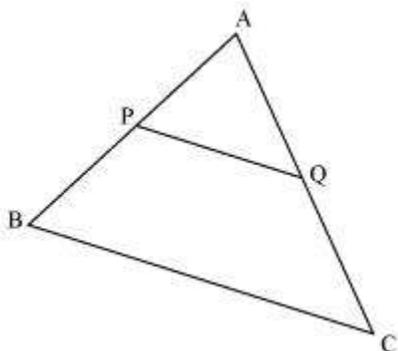
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Q8 :

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., $AP = PB$ and $AQ = QC$

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

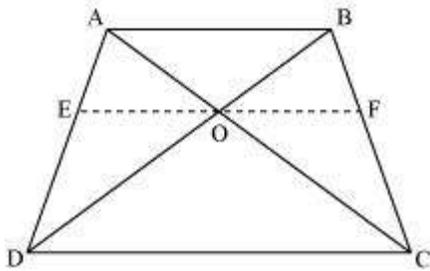
$$PQ \parallel BC$$

Q9 :

ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show

that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer :



Draw a line EF through point O, such that $EF \parallel CD$

In $\triangle ADC$, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In $\triangle ABD$, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

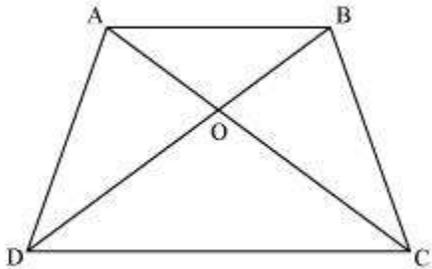
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Q10 :

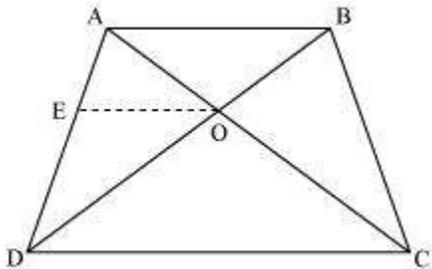
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer :

Let us consider the following figure for the given question.



Draw a line OE || AB



In $\triangle ABD$, $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

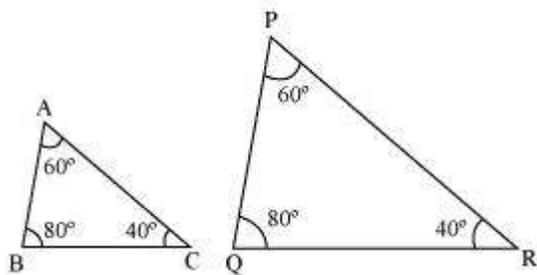
∴ ABCD is a trapezium.

Exercise 6.3 : Solutions of Questions on Page Number : 138

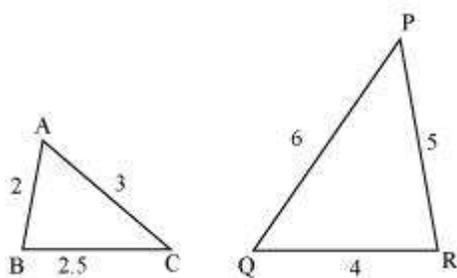
Q1 :

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

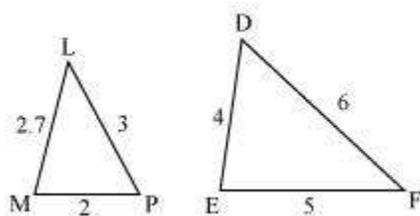
(i)



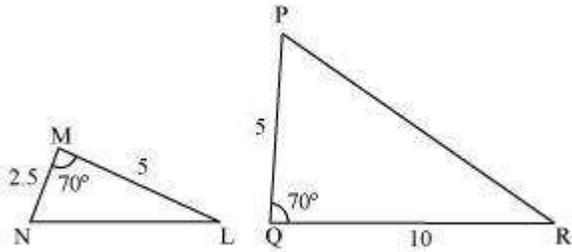
(ii)



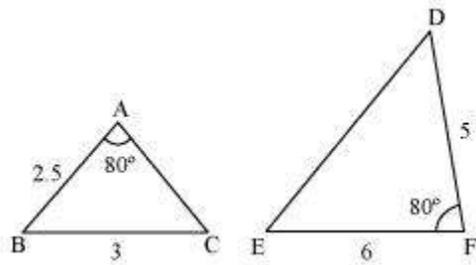
(iii)



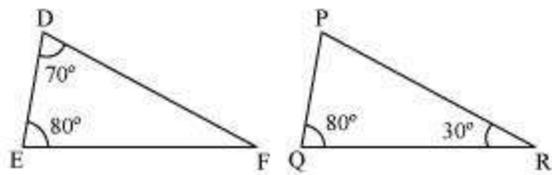
(iv)



(v)



(vi)



Answer :

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore, $\triangle ABC \sim \triangle PQR$ [By AAA similarity criterion]

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

(ii)

$\therefore \triangle ABC \sim \triangle PQR$ [By SSS similarity criterion]

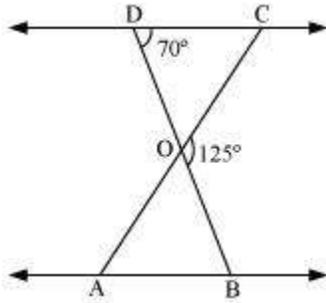
(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) In $\triangle MNL$ and $\triangle PQR$, we observe that,

$$MN \cdot QR = ML \cdot PQ = 12$$

Q2 :

In the following figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer :

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ$$

$$= 55^\circ$$

In $\triangle DOC$,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that $\triangle ODC \sim \triangle OBA$.

$\therefore \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

$$\Rightarrow \angle OAB = 55^\circ$$

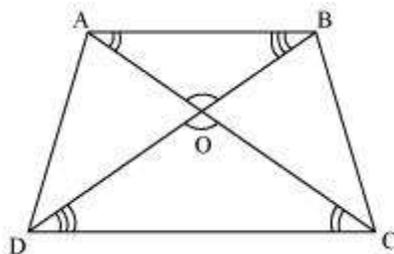
Q3 :

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a

$$\frac{AO}{OC} = \frac{OB}{OD}$$

similarity criterion for two triangles, show that

Answer :



In $\triangle ODC$ and $\triangle OBA$,

$\angle CDO = \angle ABO$ [Alternate interior angles as $AB \parallel CD$]

$\angle DCO = \angle BAO$ [Alternate interior angles as $AB \parallel CD$]

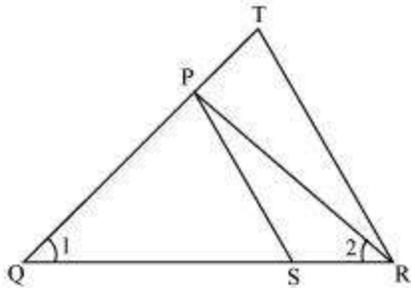
$\angle DOC = \angle BOA$ [Vertically opposite angles]

$\therefore \triangle DOC \sim \triangle BOA$ [AAA similarity criterion]

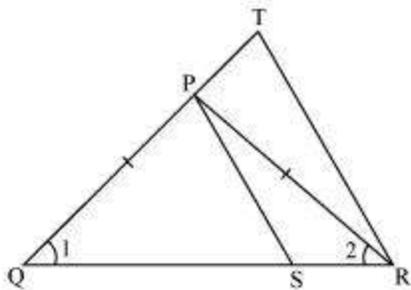
$$\begin{aligned} \therefore \frac{DO}{BO} &= \frac{OC}{OA} && \text{[Corresponding sides are proportional]} \\ \Rightarrow \frac{OA}{OC} &= \frac{OB}{OD} \end{aligned}$$

Q4 :

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$



Answer :



In $\triangle PQR$, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In ΔPQS and ΔTQR ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

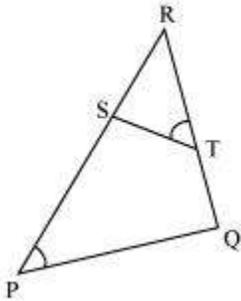
$$\angle Q = \angle Q$$

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity criterion]

Q5 :

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer :



In ΔRPQ and ΔRTS ,

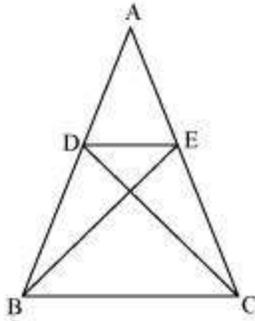
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$\therefore \Delta RPQ \sim \Delta RTS$ (By AA similarity criterion)

Q6 :

In the following figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Answer :

It is given that $\triangle ABE \cong \triangle ACD$.

$\therefore AB = AC$ [By CPCT] (1)

And, $AD = AE$ [By CPCT] (2)

In $\triangle ADE$ and $\triangle ABC$,

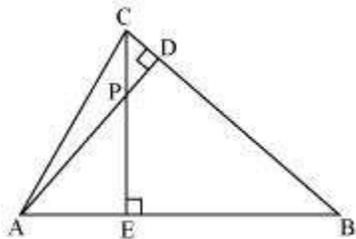
$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{[Dividing equation (2) by (1)]}$$

$\angle A = \angle A$ [Common angle]

$\therefore \triangle ADE \sim \triangle ABC$ [By SAS similarity criterion]

Q7 :

In the following figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:



(i) $\triangle AEP \sim \triangle CDP$

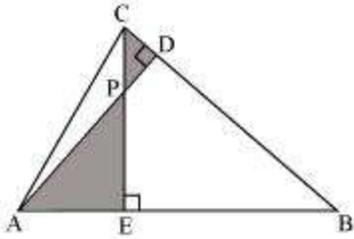
(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(v) $\triangle PDC \sim \triangle BEC$

Answer :

(i)



In $\triangle AEP$ and $\triangle CDP$,

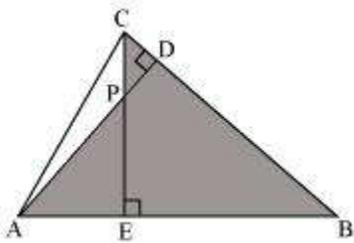
$$\angle AEP = \angle CDP \text{ (Each } 90^\circ\text{)}$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle CDP$$

(ii)



In $\triangle ABD$ and $\triangle CBE$,

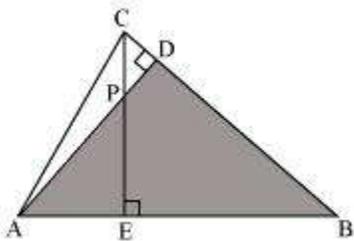
$$\angle ADB = \angle CEB \text{ (Each } 90^\circ\text{)}$$

$$\angle ABD = \angle CBE \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle ABD \sim \triangle CBE$$

(iii)



In $\triangle AEP$ and $\triangle ADB$,

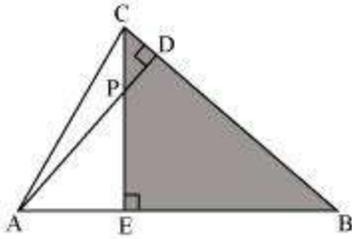
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ\text{)}$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In $\triangle PDC$ and $\triangle BEC$,

$\angle PDC = \angle BEC$ (Each 90°)

$\angle PCD = \angle BCE$ (Common angle)

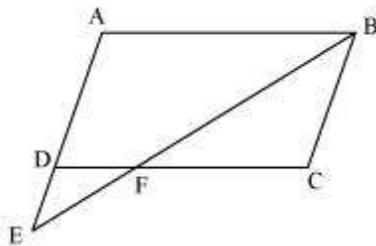
Hence, by using AA similarity criterion,

$\triangle PDC \sim \triangle BEC$

Q8 :

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$

Answer :



In $\triangle ABE$ and $\triangle CFB$,

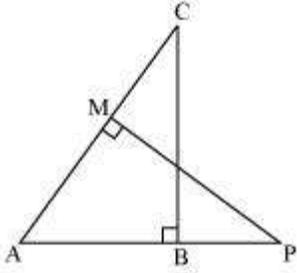
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as $AE \parallel BC$)

$\therefore \triangle ABE \sim \triangle CFB$ (By AA similarity criterion)

Q9 :

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\triangle ABC \sim \triangle AMP$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

Answer :

In $\triangle ABC$ and $\triangle AMP$,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle A = \angle A$ (Common)

$\therefore \triangle ABC \sim \triangle AMP$ (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad \text{(Corresponding sides of similar triangles are proportional)}$$

Q10 :

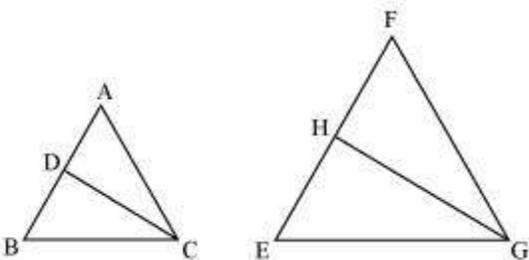
CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle FEG$ respectively. If $\triangle ABC \sim \triangle FEG$, Show that:

$$(i) \frac{CD}{GH} = \frac{AC}{FG}$$

(ii) $\triangle DCB \sim \triangle HGE$

(iii) $\triangle DCA \sim \triangle HGF$

Answer :



It is given that $\triangle ABC \sim \triangle FEG$.

$\therefore \angle A = \angle F$, $\angle B = \angle E$, and $\angle ACB = \angle FGE$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

And, $\angle DCB = \angle HGE$ (Angle bisector)

In $\triangle ACD$ and $\triangle FGH$,

$$\angle A = \angle F \text{ (Proved above)}$$

$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$\therefore \triangle ACD \sim \triangle FGH$ (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In $\triangle DCB$ and $\triangle HGE$,

$$\angle DCB = \angle HGE \text{ (Proved above)}$$

$$\angle B = \angle E \text{ (Proved above)}$$

$\therefore \triangle DCB \sim \triangle HGE$ (By AA similarity criterion)

In $\triangle DCA$ and $\triangle HGF$,

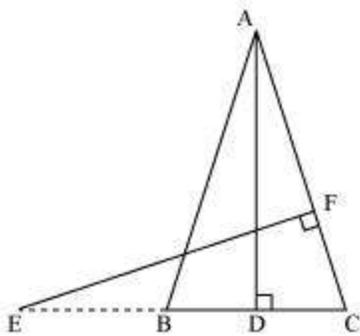
$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$$\angle A = \angle F \text{ (Proved above)}$$

$\therefore \triangle DCA \sim \triangle HGF$ (By AA similarity criterion)

Q11 :

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$



Answer :

It is given that ABC is an isosceles triangle.

$$\therefore AB = AC$$

$$\Rightarrow \angle ABD = \angle ECF$$

In $\triangle ABD$ and $\triangle ECF$,

$$\angle ADB = \angle EFC \text{ (Each } 90^\circ\text{)}$$

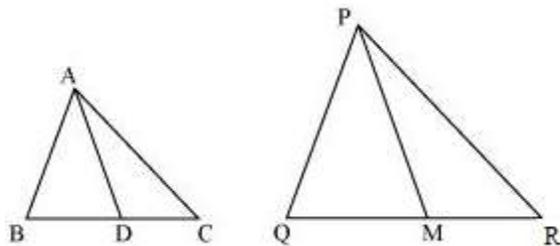
$$\angle BAD = \angle CEF \text{ (Proved above)}$$

$\therefore \triangle ABD \sim \triangle ECF$ (By using AA similarity criterion)

Q12 :

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see the given figure). Show that $\triangle ABC \sim \triangle PQR$.

Answer :



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{BD}{QM} = \frac{AD}{PM} \end{aligned}$$

In $\triangle ABD$ and $\triangle PQM$,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (Proved above)}$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In $\triangle ABC$ and $\triangle PQR$,

$\angle ABD = \angle PQM$ (Proved above)

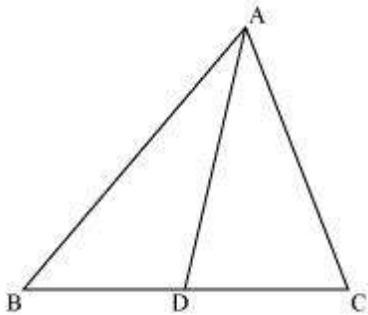
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Q13 :

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Answer :



In $\triangle ADC$ and $\triangle BAC$,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

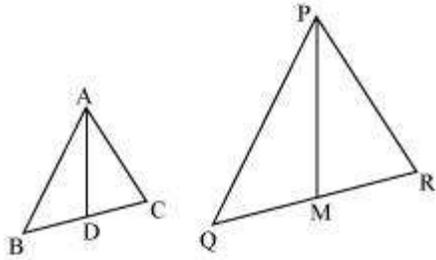
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Q14 :

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$

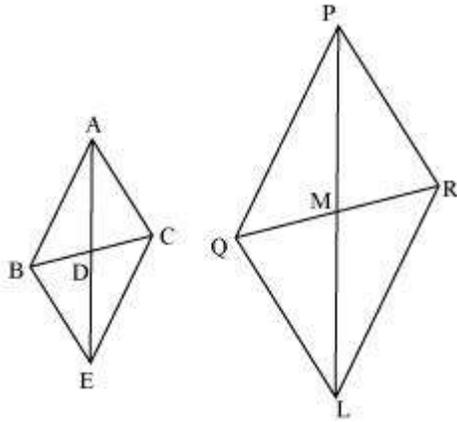
Answer :



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, $BD = DC$ and $QM = MR$

Also, $AD = DE$ (By construction)

And, $PM = ML$ (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore AC = BE$ and $AB = EC$ (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and $PR = QL$, $PQ = LR$

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$\therefore \angle BAE = \angle QPL \dots (1)$

Similarly, it can be proved that $\triangle AEC \sim \triangle PLR$ and

$\angle CAE = \angle RPL \dots (2)$

Adding equation (1) and (2), we obtain

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\Rightarrow \angle CAB = \angle RPQ \dots (3)$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{Given})$$

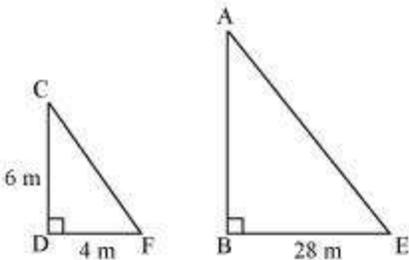
$\angle CAB = \angle RPQ$ [Using equation (3)]

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Q15 :

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer :



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

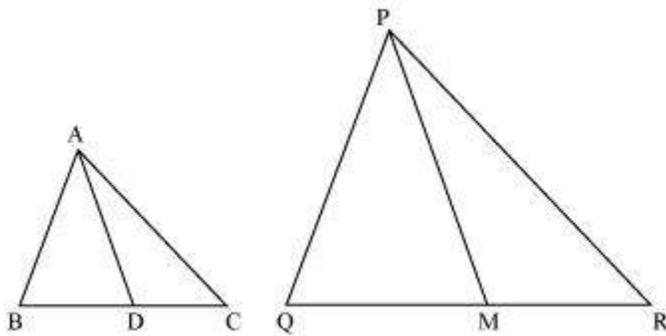
Therefore, the height of the tower will be 42 metres.

Q16 :

If AD and PM are medians of triangles ABC and PQR, respectively

$\Delta ABC \sim \Delta PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$
 where

Answer :



It is given that $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R \dots (2)$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In ΔABD and ΔPQM ,

$\angle B = \angle Q$ [Using equation (2)]

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad [\text{Using equation (4)}]$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Exercise 6.4 : Solutions of Questions on Page Number : 143

Q1 :

Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Answer :

It is given that $\triangle ABC \sim \triangle DEF$.

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$$EF = 15.4 \text{ cm},$$

$$\text{ar}(\triangle ABC) = 64 \text{ cm}^2,$$

$$\text{ar}(\triangle DEF) = 121 \text{ cm}^2$$

$$\therefore \frac{\text{ar}(ABC)}{\text{ar}(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

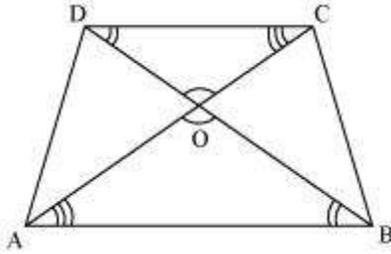
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) \text{ cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) \text{ cm} = (8 \times 1.4) \text{ cm} = 11.2 \text{ cm}$$

Q2 :

Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD .

Answer :



Since $AB \parallel CD$,

$\therefore \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (Alternate interior angles)

In $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$ (By AAA similarity criterion)

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

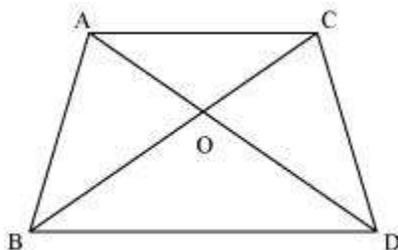
Since $AB = 2 CD$,

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2 CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

Q3 :

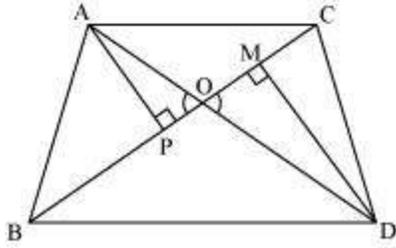
In the following figure, ABC and DBC are two triangles on the same base BC . If AD intersects BC at O , show

that
$$\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$$



Answer :

Let us draw two perpendiculars AP and DM on line BC .



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each = 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\therefore \triangle APO \sim \triangle DMO$ (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

Q4 :

If the areas of two similar triangles are equal, prove that they are congruent.

Answer :

Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad (1)$$

Given that, $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$\Rightarrow AB = PQ, BC = QR, \text{ and } AC = PR$

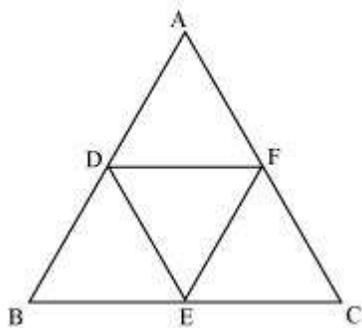
$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)



Q5 :

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC . Find the ratio of the area of ΔDEF and ΔABC .

Answer :



D and E are the mid-points of ΔABC .

$$\therefore DE \parallel AC \text{ and } DE = \frac{1}{2} AC$$

In $\triangle BED$ and $\triangle BCA$,

$$\angle BED = \angle BCA \quad (\text{Corresponding angles})$$

$$\angle BDE = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle EBD = \angle CBA \quad (\text{Common angles})$$

$$\therefore \triangle BED \sim \triangle BCA \quad (\text{AAA similarity criterion})$$

$$\frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \frac{1}{4}$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle BCA)$$

$$\text{Similarly, } \text{ar}(\triangle CFE) = \frac{1}{4} \text{ar}(\triangle CBA) \text{ and } \text{ar}(\triangle ADF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\text{Also, } \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - [\text{ar}(\triangle BED) + \text{ar}(\triangle CFE) + \text{ar}(\triangle ADF)]$$

$$\Rightarrow \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - \frac{3}{4} \text{ar}(\triangle ABC) = \frac{1}{4} \text{ar}(\triangle ABC)$$

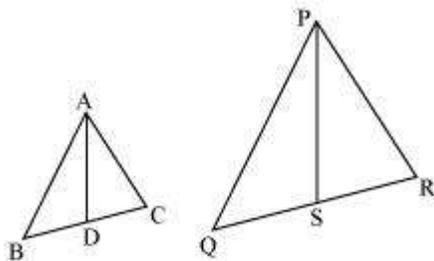
$$\Rightarrow \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$



Q6 :

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer :



Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$. Let AD and PS be the medians of these triangles.

$$\because \triangle ABC \sim \triangle PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \dots (1)$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PS are medians,

$$\therefore BD = DC = \frac{BC}{2}$$

$$\text{And, } QS = SR = \frac{QR}{2}$$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} \dots (3)$$

In $\triangle ABD$ and $\triangle PQS$,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\text{And, } \frac{AB}{PQ} = \frac{BD}{QS} \text{ [Using equation (3)]}$$

$\therefore \triangle ABD \sim \triangle PQS$ (SAS similarity criterion)

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \dots (4)$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

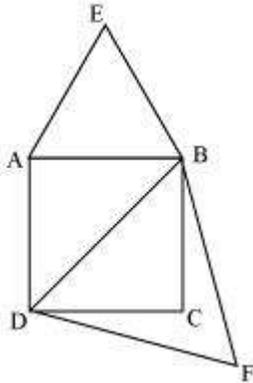
And hence,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AD}{PS}\right)^2$$

Q7 :

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer :



Let ABCD be a square of side a .

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as $\triangle ABE$ and $\triangle DBF$.

Side of an equilateral triangle, $\triangle ABE$, described on one of its sides $= a$

Side of an equilateral triangle, $\triangle DBF$, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle DBF} = \left(\frac{a}{\sqrt{2}a} \right)^2 = \frac{1}{2}$$

Q8 :

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

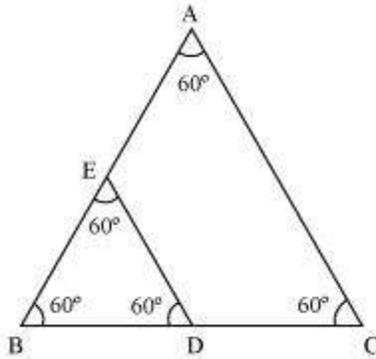
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer :



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of $\Delta ABC = x$

$$\Delta BDE = \frac{x}{2}$$

Therefore, side of

$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Q9 :

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

- (A) 2 : 3
- (B) 4 : 9
- (C) 81 : 16
- (D) 16 : 81

Answer :

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

$$\left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Therefore, ratio between areas of these triangles =

Hence, the correct answer is (D).

Exercise 6.5 : Solutions of Questions on Page Number : 150

Q1 :

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer :

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

$$49 + 576 = 625$$

$$\text{Or, } 7^2 + 24^2 = 25^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

$$\text{However, } 9 + 36 \neq 64$$

$$\text{Or, } 3^2 + 6^2 \neq 8^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii) Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

$$\text{However, } 2500 + 6400 \neq 10000$$

$$\text{Or, } 50^2 + 80^2 \neq 100^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, $144 + 25 = 169$

$$\text{Or, } 12^2 + 5^2 = 13^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

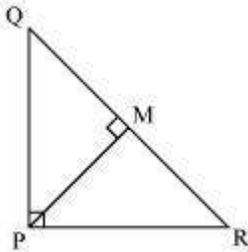
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Q2 :

PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

Answer :



Let $\angle MPR = x$

In $\triangle MPR$,

$$\angle MRP = 180^\circ - 90^\circ - x$$

$$\angle MRP = 90^\circ - x$$

Similarly, in $\triangle MPQ$,

$$\angle MPQ = 90^\circ - \angle MPR$$

$$= 90^\circ - x$$

$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$$

$$\angle MQP = x$$

In $\triangle QMP$ and $\triangle PMR$,

$$\angle MPQ = \angle MRP$$

$$\angle PMQ = \angle RMP$$

$$\angle MQP = \angle MPR$$

$\therefore \triangle QMP \sim \triangle PMR$ (By AAA similarity criterion)

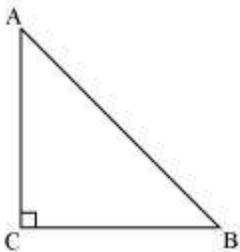
$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$

$$\Rightarrow PM^2 = QM \times MR$$

Q3 :

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2 AC^2$.

Answer :



Given that $\triangle ABC$ is an isosceles triangle.

$$\therefore AC = CB$$

Applying Pythagoras theorem in $\triangle ABC$ (i.e., right-angled at point C), we obtain

$$AC^2 + CB^2 = AB^2$$

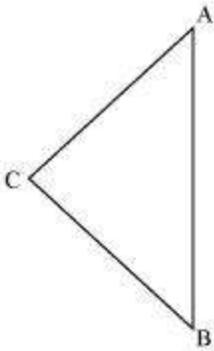
$$\Rightarrow AC^2 + AC^2 = AB^2 \quad (AC = CB)$$

$$\Rightarrow 2AC^2 = AB^2$$

Q4 :

ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2 AC^2$, prove that ABC is a right triangle.

Answer :



Given that,

$$AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$$

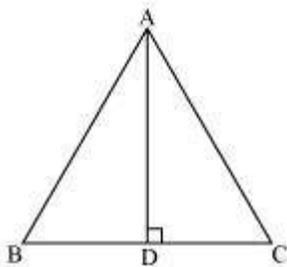
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Q5 :

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer :



Let AD be the altitude in the given equilateral triangle, ΔABC .

We know that altitude bisects the opposite side.

$$\therefore BD = DC = a$$

In $\triangle ADB$,

$$\angle ADB = 90^\circ$$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 + a^2 = (2a)^2$$

$$\Rightarrow AD^2 + a^2 = 4a^2$$

$$\Rightarrow AD^2 = 3a^2$$

$$\Rightarrow AD = a\sqrt{3}$$

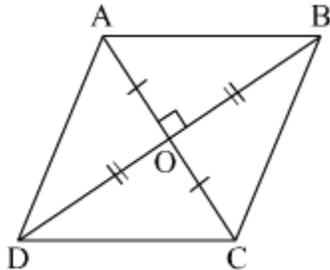
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Q6 :

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer :



In $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle AOD$,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

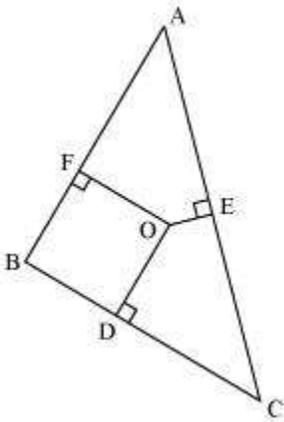
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Q7 :

In the following figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

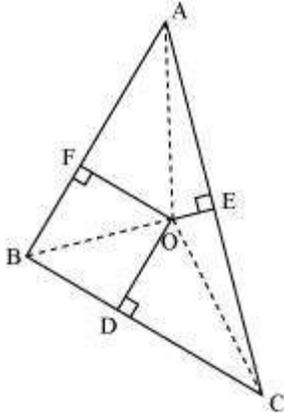


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer :

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in $\triangle AOF$, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in $\triangle BOD$,

$$OB^2 = OD^2 + BD^2$$

Similarly, in $\triangle COE$,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

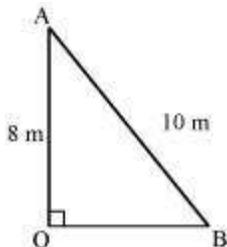
$$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Q8 :

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer :



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10 \text{ m})^2 = (8 \text{ m})^2 + OB^2$$

$$100 \text{ m}^2 = 64 \text{ m}^2 + OB^2$$

$$OB^2 = 36 \text{ m}^2$$

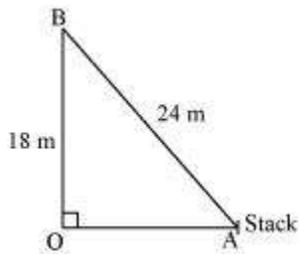
$$OB = 6 \text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Q9 :

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer :



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$(24 \text{ m})^2 = (18 \text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324) \text{ m}^2 = 252 \text{ m}^2$$

$$OA = \sqrt{252} \text{ m} = \sqrt{6 \times 6 \times 7} \text{ m} = 6\sqrt{7} \text{ m}$$

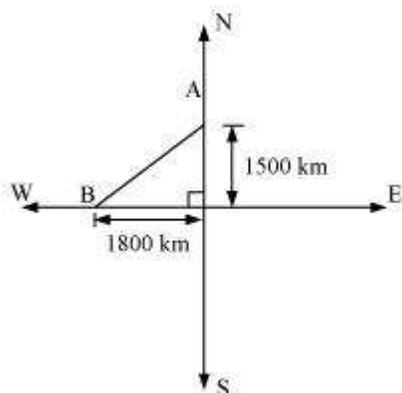
Therefore, the distance from the base is $6\sqrt{7}$ m.

Q10 :

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be

the two planes after $1\frac{1}{2}$ hours?

Answer :



Distance travelled by the plane flying towards north in $1\frac{1}{2}$ hrs = $1,000 \times 1\frac{1}{2} = 1,500$ km

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hrs = $1,200 \times 1\frac{1}{2} = 1,800$ km

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

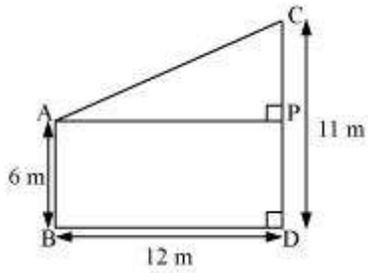
$$\begin{aligned} \text{Distance between these planes after } 1\frac{1}{2} \text{ hrs, } AB &= \sqrt{OA^2 + OB^2} \\ &= \left(\sqrt{(1,500)^2 + (1,800)^2} \right) \text{ km} = \left(\sqrt{2250000 + 3240000} \right) \text{ km} \\ &= \left(\sqrt{5490000} \right) \text{ km} = \left(\sqrt{9 \times 610000} \right) \text{ km} = 300\sqrt{61} \text{ km} \end{aligned}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Q11 :

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer :



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, $CP = 11 - 6 = 5$ m

From the figure, it can be observed that $AP = 12$ m

Applying Pythagoras theorem for $\triangle APC$, we obtain

$$AP^2 + PC^2 = AC^2$$

$$(12 \text{ m})^2 + (5 \text{ m})^2 = AC^2$$

$$AC^2 = (144 + 25) \text{ m}^2 = 169 \text{ m}^2$$

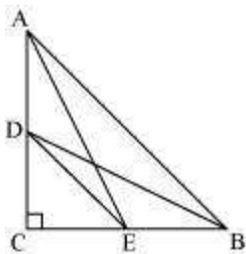
$$AC = 13 \text{ m}$$

Therefore, the distance between their tops is 13 m.

Q12 :

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer :



Applying Pythagoras theorem in $\triangle ACE$, we obtain

$$AC^2 + CE^2 = AE^2 \quad \dots (1)$$

Applying Pythagoras theorem in $\triangle BCD$, we obtain

$$BC^2 + CD^2 = BD^2 \quad \dots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \dots (3)$$

Applying Pythagoras theorem in $\triangle CDE$, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AB^2 = AC^2 + CB^2$$

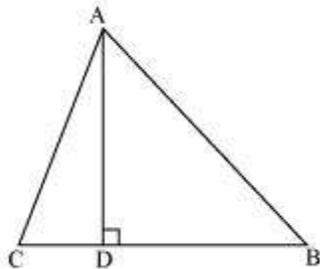
Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$



Q13 :

The perpendicular from A on side BC of a $\triangle ABC$ intersect BC at D such that $DB = 3 CD$. Prove that $2 AB^2 = 2 AC^2 + BC^2$



Answer :

Applying Pythagoras theorem for $\triangle ACD$, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \quad \dots (1)$$

Applying Pythagoras theorem in $\triangle ABD$, we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \quad \dots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \dots (3)$$

It is given that $3DC = DB$

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

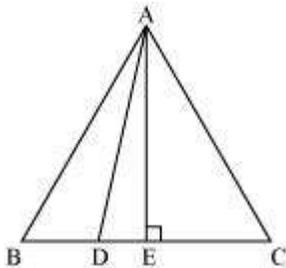
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Q14 :

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9 AD^2 = 7 AB^2$.

Answer :



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

$$\text{And, } AE = \frac{a\sqrt{3}}{2}$$

$$\text{Given that, } BD = \frac{1}{3} BC$$

$$\therefore BD = \frac{a}{3}$$

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

Applying Pythagoras theorem in $\triangle ADE$, we obtain

$$AD^2 = AE^2 + DE^2$$

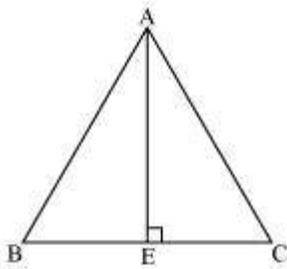
$$\begin{aligned} AD^2 &= \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2 \\ &= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) \\ &= \frac{28a^2}{36} \\ &= \frac{7}{9}AB^2 \end{aligned}$$

$$\Rightarrow 9 AD^2 = 7 AB^2$$

Q15 :

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer :



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

Applying Pythagoras theorem in $\triangle ABE$, we obtain

$$AB^2 = AE^2 + BE^2$$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$$4AE^2 = 3a^2$$

$\Rightarrow 4 \times (\text{Square of altitude}) = 3 \times (\text{Square of one side})$

Q16 :

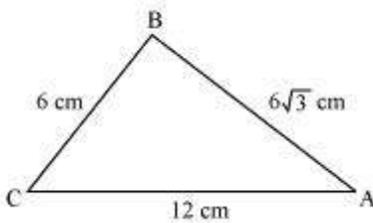
Tick the correct answer and justify: In ΔABC , $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer :



Given that, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, and $BC = 6$ cm

It can be observed that

$$AB^2 = 108$$

$$AC^2 = 144$$

$$\text{And, } BC^2 = 36$$

$$AB^2 + BC^2 = AC^2$$

The given triangle, ΔABC , is satisfying Pythagoras theorem.

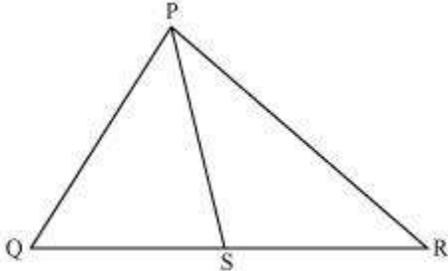
Therefore, the triangle is a right triangle, right-angled at B.

$$\therefore \angle B = 90^\circ$$

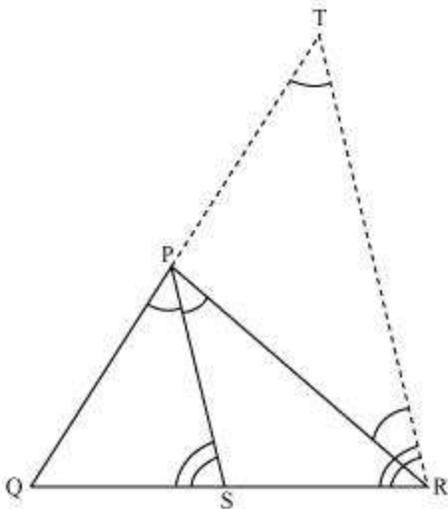
Hence, the correct answer is (C).

Q1 :

In the given figure, PS is the bisector of $\angle QPR$ of ΔPQR . Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer :



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of $\angle QPR$.

$$\angle QPS = \angle SPR \dots (1)$$

By construction,

$$\angle SPR = \angle PRT \text{ (As } PS \parallel TR \text{) } \dots (2)$$

$$\angle QPS = \angle QTR \text{ (As } PS \parallel TR \text{) } \dots (3)$$

Using these equations, we obtain

$$\angle PRT = \angle QTR$$

$$\therefore PT = PR$$

By construction,

$$PS \parallel TR$$

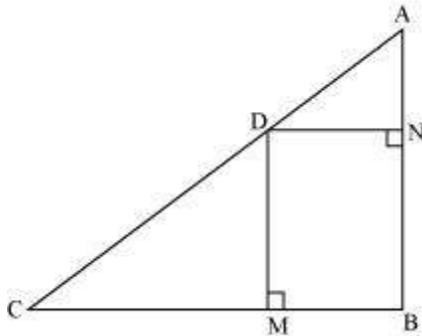
By using basic proportionality theorem for ΔQTR ,
 $QSSR=QPPT$
 $\Rightarrow QSSR$

Q2 :

In the given figure, D is a point on hypotenuse AC of ΔABC , $DM \perp BC$ and $DN \perp AB$, Prove that:

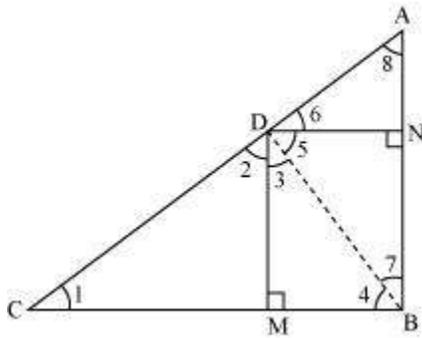
(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$



Answer :

(i) Let us join DB.



We have, $DN \parallel CB$, $DM \parallel AB$, and $\angle B = 90^\circ$

\therefore DMBN is a rectangle.

$\therefore DN = MB$ and $DM = NB$

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle CDB = 90^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \dots (1)$

In ΔCDM ,

$\angle 1 + \angle 2 + \angle DMC = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots (2)$

In $\triangle DMB$,

$$\angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots (3)$$

From equation (1) and (2), we obtain

$$\angle 1 = \angle 3$$

From equation (1) and (3), we obtain

$$\angle 2 = \angle 4$$

In $\triangle DCM$ and $\triangle BDM$,

$$\angle 1 = \angle 3 \text{ (Proved above)}$$

$$\angle 2 = \angle 4 \text{ (Proved above)}$$

$\therefore \triangle DCM \sim \triangle BDM$ (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$
$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In right triangle DBN ,

$$\angle 5 + \angle 7 = 90^\circ \dots (4)$$

In right triangle DAN ,

$$\angle 6 + \angle 8 = 90^\circ \dots (5)$$

D is the foot of the perpendicular drawn from B to AC .

$$\therefore \angle ADB = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = 90^\circ \dots (6)$$

From equation (4) and (6), we obtain

$$\angle 6 = \angle 7$$

From equation (5) and (6), we obtain

$$\angle 8 = \angle 5$$

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7 \text{ (Proved above)}$$

$$\angle 8 = \angle 5 \text{ (Proved above)}$$

$\therefore \triangle DNA \sim \triangle BND$ (AA similarity criterion)

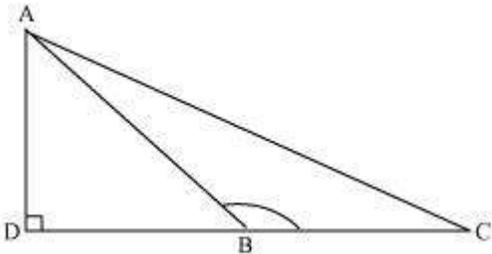
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \text{ (As } NB = DM)$$

Q3 :

In the given figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer :

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AB^2 = AD^2 + DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ADC$, we obtain

$$AC^2 = AD^2 + DC^2$$

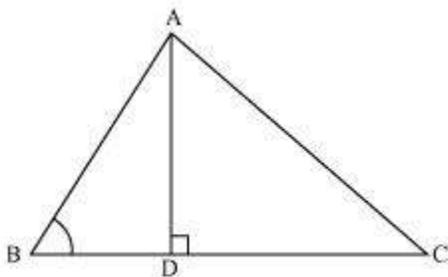
$$AC^2 = AD^2 + (DB + BC)^2$$

$$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

$$AC^2 = AB^2 + BC^2 + 2DB \times BC \text{ [Using equation (1)]}$$

Q4 :

In the given figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer :

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ADC$, we obtain

$$AD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2 \text{ [Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$= AB^2 + BC^2 - 2BC \times BD$$

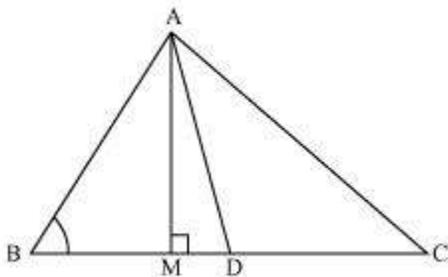
Q5 :

In the given figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

$$(i) \quad AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(ii) \quad AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(iii) \quad AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$



Answer :

(i) Applying Pythagoras theorem in $\triangle AMD$, we obtain

$$AM^2 + MD^2 = AD^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MC^2 = AC^2$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

$$AD^2 + DC^2 + 2MD \cdot DC = AC^2 \text{ [Using equation (1)]}$$

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in $\triangle ABM$, we obtain

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii) Applying Pythagoras theorem in $\triangle ABM$, we obtain

$$AM^2 + MB^2 = AB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MC^2 = AC^2 \dots (2)$$

Adding equations (1) and (2), we obtain

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD \cdot DM + MD^2 + DC^2 + 2MD \cdot DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER 6: TRIANGLES

MIND MAP

This chapter consists of seven different topics. The most probable questions from examination point of view are given below.

TYPE: 1 SIMILARITY OF TRIANGLES

Q.1. If $\Delta ABC \sim \Delta PQR$, $AB = 5\text{cm}$, $BC = 6\text{cm}$, $PQ = 8\text{cm}$, then find QR.

TYPE: 2 BASIC PROPORTIONALITY THEOREM

Q.1. State and prove BPT.

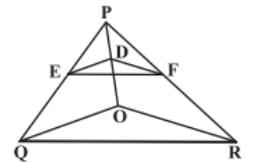
Q.2. In ΔABC , D and E are points on the sides AB and AC respectively, such that $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x .

Q.3. In ΔABC , $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$

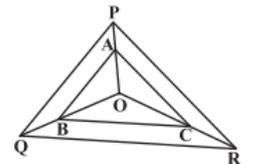
TYPE: 3 CONVERSE OF BASIC PROPORTIONALITY THEOREM

Q.1. E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$ for $PE = 3.9\text{ cm}$, $EQ = 3\text{ cm}$, $PF = 3.6\text{ cm}$ and $FR = 2.4\text{ cm}$

Q.2. In the adjoining figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



Q.3. In the adjoining figure, A , B and C are points on OP , OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

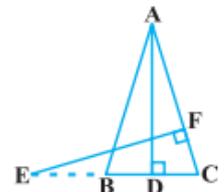


TYPE: 4 CRITERIA FOR SIMILARITY OF TRIANGLES

Q.1. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$.

Show that $\Delta RPQ \sim \Delta RTS$.

Q.2. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



Q.3. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

TYPE: 5 AREAS OF SIMILAR TRIANGLES

- Q.1. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
- Q.2. If $\Delta ABC \sim \Delta DEF$, $BC = 3\text{cm}$ $EF = 4\text{cm}$ and area of $\Delta ABC = 54\text{cm}^2$. Determine the area of ΔDEF .
- Q.3. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.
- Q.4. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

TYPE: 6 PYTHAGORAS THEOREM

- Q.1. State and prove Pythagoras theorem.
- Q.2. In an equilateral ΔABC , D is a point on the side BC such that $BD = \frac{1}{3}BC$.
Prove that $9AD^2 = 7AB^2$.
- Q.3. BL and CM are medians of a triangle ABC right angled at A .
Prove that $4(BL^2 + CM^2) = 5BC^2$.

TYPE: 7 CONVERSE OF PYTHAGORAS THEOREM

- Q.1. State and prove converse of Pythagoras theorem.
- Q.2. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

NCERT Solutions for Class 10 Maths Unit 7

Coordinate Geometry Class 10

Unit 7 Coordinate Geometry Exercise 7.1, 7.2, 7.3, 7.4 Solutions

Exercise 7.1 : Solutions of Questions on Page Number : 161

Q1 :

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3) (iii) (a, b), (-a, -b)

Answer :

(i) Distance between the two points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between (2, 3) and (4, 1) is given by

$$\begin{aligned} l &= \sqrt{(2-4)^2 + (3-1)^2} = \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \end{aligned}$$

(ii) Distance between (-5, 7) and (-1, 3) is given by

$$\begin{aligned} l &= \sqrt{(-5-(-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

(iii) Distance between (a, b) and (-a, -b) is given by

$$\begin{aligned} l &= \sqrt{(a-(-a))^2 + (b-(-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \end{aligned}$$

Q2 :

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer :

Distance between points (0,0) and (36,15)

$$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$$

$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be 39 km.

Q3 :

Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Answer :

Let the points (1, 5), (2, 3), and (-2, -11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A = (1, 5), B = (2, 3), C = (-2, -11)$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$$

$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196} = \sqrt{212}$$

$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$$

Since $AB + BC \neq CA$,

Therefore, the points (1, 5), (2, 3), and (-2, -11) are not collinear.

Q4 :

Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Answer :

Let the points (5, -2), (6, 4), and (7, -2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

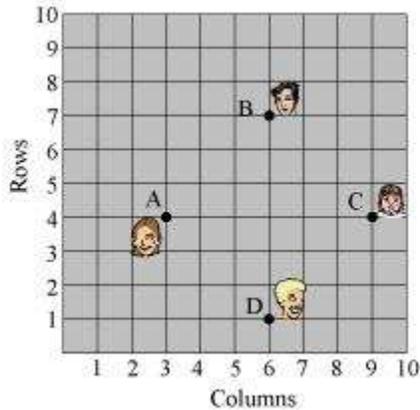
Therefore, $AB = BC$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

Q5 :

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

Using distance formula, find which of them is correct.



Answer :

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

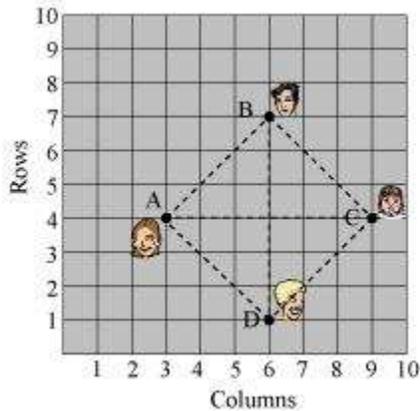
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CB = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Q6 :

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Answer :

(i) Let the points $(-1, -2), (1, 0), (-1, 2),$ and $(-3, 0)$ be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Let the points $(-3, 5), (3, 1), (0, 3),$ and $(-1, -4)$ be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal BD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Q7 :

Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Answer :

We have to find a point on x-axis. Therefore, its y-coordinate will be 0.

Let the point on x-axis be $(x, 0)$.

$$\text{Distance between } (x, 0) \text{ and } (2, -5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x, 0) \text{ and } (-2, 9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = 25 - 81$$

$$8x = -56$$

$$x = -7$$

Therefore, the point is (-7, 0).

Q8 :

Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.

Answer :

It is given that the distance between (2, -3) and (10, y) is 10.

$$\text{Therefore, } \sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\sqrt{(-8)^2 + (3+y)^2} = 10$$

$$64 + (y+3)^2 = 100$$

$$(y+3)^2 = 36$$

$$y+3 = \pm 6$$

$$y+3 = 6 \text{ or } y+3 = -6$$

$$\text{Therefore, } y = 3 \text{ or } -9$$

Q9 :

If Q (0, 1) is equidistant from P (5, -3) and R (x , 6), find the values of x . Also find the distance QR and PR.

Answer :

$$PQ = QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

$$41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (-4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Q10 :

If Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also find the distance QR and PR.

Answer :

$$PQ = QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

$$41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (-4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Q11 :

Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Answer :

Point (x, y) is equidistant from $(3, 6)$ and $(-3, 4)$.

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

Q12 :

Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Answer :

Point (x, y) is equidistant from $(3, 6)$ and $(-3, 4)$.

$$\begin{aligned}
\therefore \sqrt{(x-3)^2 + (y-6)^2} &= \sqrt{(x-(-3))^2 + (y-4)^2} \\
\sqrt{(x-3)^2 + (y-6)^2} &= \sqrt{(x+3)^2 + (y-4)^2} \\
(x-3)^2 + (y-6)^2 &= (x+3)^2 + (y-4)^2 \\
x^2 + 9 - 6x + y^2 + 36 - 12y &= x^2 + 9 + 6x + y^2 + 16 - 8y \\
36 - 16 &= 6x + 6x + 12y - 8y \\
20 &= 12x + 4y \\
3x + y &= 5 \\
3x + y - 5 &= 0
\end{aligned}$$

Exercise 7.2 : Solutions of Questions on Page Number : 167

Q1 :

Find the coordinates of the point which divides the join of (- 1, 7) and (4, - 3) in the ratio 2:3.

Answer :

Let P(x, y) be the required point. Using the section formula, we obtain

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the point is (1, 3).

Q2 :

Find the coordinates of the point which divides the join of (- 1, 7) and (4, - 3) in the ratio 2:3.

Answer :

Let P(x, y) be the required point. Using the section formula, we obtain

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

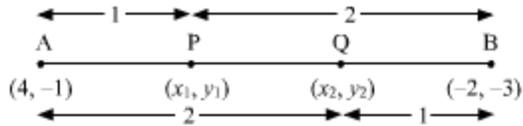
$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the point is (1, 3).

Q3 :

Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Answer :



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., $AP = PQ = QB$

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1 + 2}, \quad y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}$$

$$x_1 = \frac{-2 + 8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3 - 2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2 + 1}, \quad y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2 + 1}$$

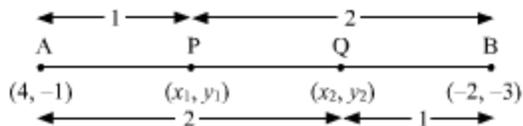
$$x_2 = \frac{-4 + 4}{3} = 0, \quad y_2 = \frac{-6 - 1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Q4 :

Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Answer :



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., $AP = PQ = QB$

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1+2}, \quad y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2+1}, \quad y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

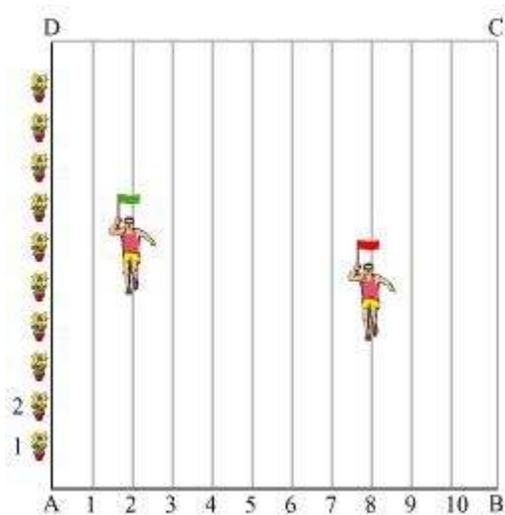
$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Q5 :

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each

other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and

posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer :

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4} \times 100\right) \text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5} \times 100\right) \text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x,y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

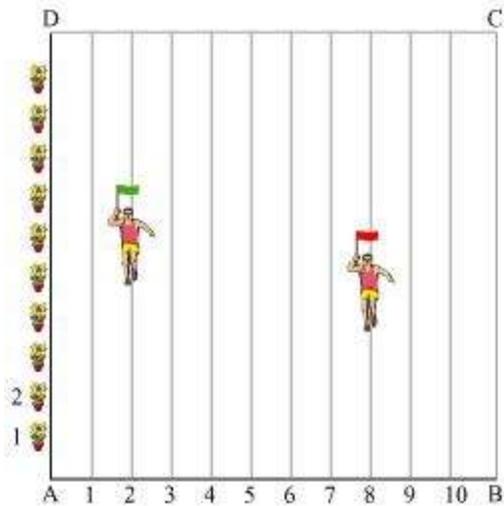
Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Q6 :

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each

other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and

posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer :

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4} \times 100\right) \text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5} \times 100\right) \text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x,y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Q7 :

Find the ratio in which the line segment joining the points (- 3, 10) and (6, - 8) is divided by (- 1, 6).

Answer :

Let the ratio in which the line segment joining $(-3, 10)$ and $(6, -8)$ is divided by point $(-1, 6)$ be $k : 1$.

$$\text{Therefore, } -1 = \frac{6k - 3}{k + 1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore, the required ratio is $2 : 7$.

Q8 :

Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1, 6)$.

Answer :

Let the ratio in which the line segment joining $(-3, 10)$ and $(6, -8)$ is divided by point $(-1, 6)$ be $k : 1$.

$$\text{Therefore, } -1 = \frac{6k - 3}{k + 1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore, the required ratio is $2 : 7$.

Q9 :

Find the ratio in which the line segment joining A $(1, -5)$ and B $(-4, 5)$ is divided by the x-axis. Also find the coordinates of the point of division.

Answer :

Let the ratio in which the line segment joining A $(1, -5)$ and B $(-4, 5)$ is divided by x-axis be $k : 1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k + 1}{k + 1}, \frac{5k - 5}{k + 1} \right)$.

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \frac{5k - 5}{k + 1} = 0$$

$$k = 1$$

Therefore, x-axis divides it in the ratio 1:1.

$$\text{Division point} = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1} \right) = \left(\frac{-4+1}{2}, \frac{5-5}{2} \right) = \left(\frac{-3}{2}, 0 \right)$$

Q10 :

Find the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer :

Let the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by x-axis be $k:1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1} \right)$.

We know that y-coordinate of any point on x-axis is 0.

$$\begin{aligned} \therefore \frac{5k-5}{k+1} &= 0 \\ k &= 1 \end{aligned}$$

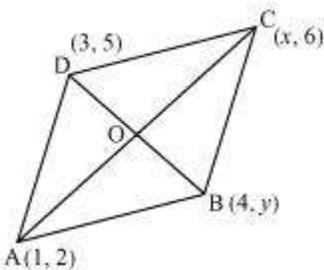
Therefore, x-axis divides it in the ratio 1:1.

$$\text{Division point} = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1} \right) = \left(\frac{-4+1}{2}, \frac{5-5}{2} \right) = \left(\frac{-3}{2}, 0 \right)$$

Q11 :

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer :



Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

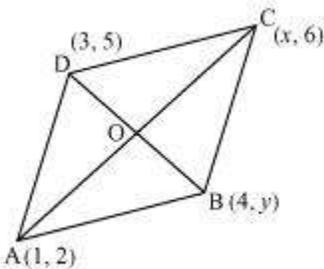
Since both the coordinates are of the same point O,

$$\begin{aligned} \therefore \frac{x+1}{2} &= \frac{7}{2} \text{ and } 4 = \frac{5+y}{2} \\ \Rightarrow x+1 &= 7 \text{ and } 5+y = 8 \\ \Rightarrow x &= 6 \text{ and } y = 3 \end{aligned}$$

Q12 :

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer :



Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\begin{aligned}\therefore \frac{x+1}{2} &= \frac{7}{2} \text{ and } 4 = \frac{5+y}{2} \\ \Rightarrow x+1 &= 7 \text{ and } 5+y = 8 \\ \Rightarrow x &= 6 \text{ and } y = 3\end{aligned}$$

Q13 :

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, -3) and B is (1, 4)

Answer :

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, -3), which is the center of the circle.

$$\begin{aligned}\therefore (2, -3) &= \left(\frac{x+1}{2}, \frac{y+4}{2} \right) \\ \Rightarrow \frac{x+1}{2} &= 2 \text{ and } \frac{y+4}{2} = -3 \\ \Rightarrow x+1 &= 4 \text{ and } y+4 = -6 \\ \Rightarrow x &= 3 \text{ and } y = -10\end{aligned}$$

Therefore, the coordinates of A are (3, -10).

Q14 :

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, -3) and B is (1, 4)

Answer :

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, -3), which is the center of the circle.

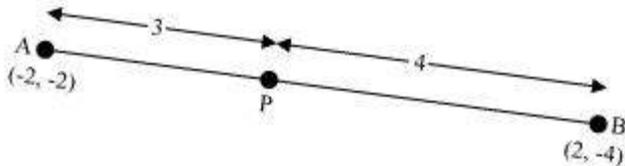
$$\begin{aligned}\therefore (2, -3) &= \left(\frac{x+1}{2}, \frac{y+4}{2} \right) \\ \Rightarrow \frac{x+1}{2} &= 2 \text{ and } \frac{y+4}{2} = -3 \\ \Rightarrow x+1 &= 4 \text{ and } y+4 = -6 \\ \Rightarrow x &= 3 \text{ and } y = -10\end{aligned}$$

Therefore, the coordinates of A are (3, -10).

Q15 :

If A and B are $(-2, -2)$ and $(2, -4)$, respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Answer :



The coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively.

Since $AP = \frac{3}{7} AB$,

Therefore, AP: PB = 3:4

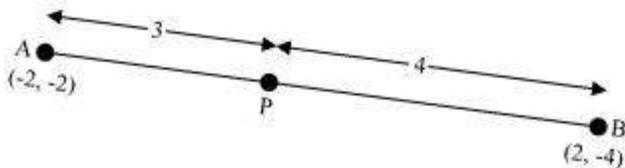
Point P divides the line segment AB in the ratio 3:4.

$$\begin{aligned} \text{Coordinates of P} &= \left(\frac{3 \times 2 + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \right) \\ &= \left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right) \\ &= \left(-\frac{2}{7}, -\frac{20}{7} \right) \end{aligned}$$

Q16 :

If A and B are $(-2, -2)$ and $(2, -4)$, respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Answer :



The coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively.

Since $AP = \frac{3}{7} AB$,

Therefore, AP: PB = 3:4

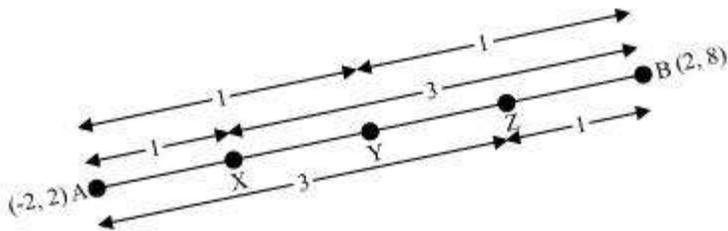
Point P divides the line segment AB in the ratio 3:4.

$$\begin{aligned}\text{Coordinates of P} &= \left(\frac{3 \times 2 + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \right) \\ &= \left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right) \\ &= \left(-\frac{2}{7}, -\frac{20}{7} \right)\end{aligned}$$

Q17 :

Find the coordinates of the points which divide the line segment joining A (-2, 2) and B (2, 8) into four equal parts.

Answer :



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\begin{aligned}\text{Coordinates of P} &= \left(\frac{1 \times 2 + 3 \times (-2)}{1 + 3}, \frac{1 \times 8 + 3 \times 2}{1 + 3} \right) \\ &= \left(-1, \frac{7}{2} \right)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of Q} &= \left(\frac{2 + (-2)}{2}, \frac{2 + 8}{2} \right) \\ &= (0, 5)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of R} &= \left(\frac{3 \times 2 + 1 \times (-2)}{3 + 1}, \frac{3 \times 8 + 1 \times 2}{3 + 1} \right) \\ &= \left(1, \frac{13}{2} \right)\end{aligned}$$

Q18 :

Find the coordinates of the points which divide the line segment joining A (- 2, 2) and B (2, 8) into four equal parts.

Answer :



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1 \times 2 + 3 \times (-2)}{1+3}, \frac{1 \times 8 + 3 \times 2}{1+3} \right)$$

$$= \left(-1, \frac{7}{2} \right)$$

$$\text{Coordinates of Q} = \left(\frac{2 + (-2)}{2}, \frac{2 + 8}{2} \right)$$

$$= (0, 5)$$

$$\text{Coordinates of R} = \left(\frac{3 \times 2 + 1 \times (-2)}{3+1}, \frac{3 \times 8 + 1 \times 2}{3+1} \right)$$

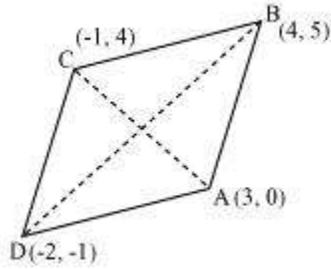
$$= \left(1, \frac{13}{2} \right)$$

Q19 :

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) taken in order. [Hint: Area of a

rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer :



Let (3, 0), (4, 5), (-1, 4) and (-2, -1) are the vertices A, B, C, D of a rhombus ABCD.

$$\begin{aligned} \text{Length of diagonal AC} &= \sqrt{[3 - (-1)]^2 + (0 - 4)^2} \\ &= \sqrt{16 + 16} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Length of diagonal BD} &= \sqrt{[4 - (-2)]^2 + [5 - (-1)]^2} \\ &= \sqrt{36 + 36} = 6\sqrt{2} \end{aligned}$$

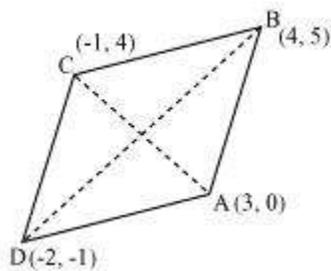
$$\begin{aligned} \text{Therefore, area of rhombus ABCD} &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square units} \end{aligned}$$

Q20 :

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order. [Hint: Area of a

rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer :



Let (3, 0), (4, 5), (-1, 4) and (-2, -1) are the vertices A, B, C, D of a rhombus ABCD.

$$\begin{aligned}\text{Length of diagonal AC} &= \sqrt{[3 - (-1)]^2 + (0 - 4)^2} \\ &= \sqrt{16 + 16} = 4\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Length of diagonal BD} &= \sqrt{[4 - (-2)]^2 + [5 - (-1)]^2} \\ &= \sqrt{36 + 36} = 6\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Therefore, area of rhombus ABCD} &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square units}\end{aligned}$$

Exercise 7.3 : Solutions of Questions on Page Number : 170

Q1 :

Find the area of the triangle whose vertices are:

(i) (2, 3), (-1, 0), (2, -4) (ii) (-5, -1), (3, -5), (5, 2)

Answer :

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned}\text{Area of the given triangle} &= \frac{1}{2} [2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)] \\ &= \frac{1}{2} \{8 + 7 + 6\} \\ &= \frac{21}{2} \text{ square units}\end{aligned}$$

$$\text{(ii) Area of the given triangle} = \frac{1}{2} [(-5)\{(-5) - (2)\} + 3(2 - (-1)) + 5\{-1 - (-5)\}]$$

$$\begin{aligned}&= \frac{1}{2} \{35 + 9 + 20\} \\ &= 32 \text{ square units}\end{aligned}$$

Q2 :

Find the area of the triangle whose vertices are:

(i) (2, 3), (-1, 0), (2, -4) (ii) (-5, -1), (3, -5), (5, 2)

Answer :

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of the given triangle} &= \frac{1}{2} [2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)] \\ &= \frac{1}{2} \{8 + 7 + 6\} \\ &= \frac{21}{2} \text{ square units} \end{aligned}$$

(ii)
$$\text{Area of the given triangle} = \frac{1}{2} [(-5)\{(-5) - (2)\} + 3(2 - (-1)) + 5\{-1 - (-5)\}]$$

$$\begin{aligned} &= \frac{1}{2} \{35 + 9 + 20\} \\ &= 32 \text{ square units} \end{aligned}$$

Q3 :

In each of the following find the value of 'k', for which the points are collinear.

(i) (7, - 2), (5, 1), (3, - k) (ii) (8, 1), (k, - 4), (2, - 5)

Answer :

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, - 2) (5, 1), and (3, k), area = 0

$$\frac{1}{2} [7\{1 - k\} + 5\{k - (-2)\} + 3\{(-2) - 1\}] = 0$$

$$7 - 7k + 5k + 10 - 9 = 0$$

$$-2k + 8 = 0$$

$$k = 4$$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, - 4), and (2, - 5), area = 0

$$\frac{1}{2} [8\{-4 - (-5)\} + k\{(-5) - (1)\} + 2\{1 - (-4)\}] = 0$$

$$8 - 6k + 10 = 0$$

$$6k = 18$$

$$k = 3$$

Q4 :

In each of the following find the value of 'k', for which the points are collinear.

(i) (7, - 2), (5, 1), (3, - k) (ii) (8, 1), (k, - 4), (2, - 5)

Answer :

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, - 2) (5, 1), and (3, k), area = 0

$$\frac{1}{2} [7\{1-k\} + 5\{k-(-2)\} + 3\{(-2)-1\}] = 0$$

$$7 - 7k + 5k + 10 - 9 = 0$$

$$-2k + 8 = 0$$

$$k = 4$$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, - 4), and (2, - 5), area = 0

$$\frac{1}{2} [8\{-4-(-5)\} + k\{(-5)-(1)\} + 2\{1-(-4)\}] = 0$$

$$8 - 6k + 10 = 0$$

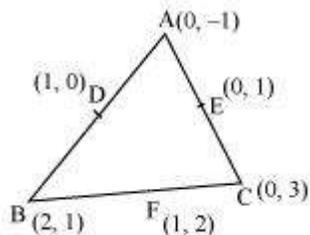
$$6k = 18$$

$$k = 3$$

Q5 :

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, - 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer :



Let the vertices of the triangle be A (0, - 1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2} \right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2} \right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle DEF &= \frac{1}{2} \{1(2-1) + 1(1-0) + 0(0-2)\} \\ &= \frac{1}{2} (1+1) = 1 \text{ square units} \end{aligned}$$

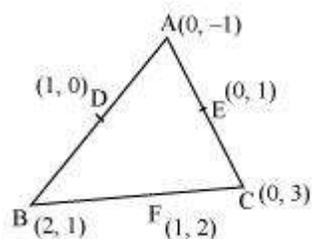
$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [0(1-3) + 2\{3-(-1)\} + 0(-1-1)] \\ &= \frac{1}{2} \{8\} = 4 \text{ square units} \end{aligned}$$

Therefore, required ratio = 1 : 4

Q6 :

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer :



Let the vertices of the triangle be A (0, -1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2} \right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2} \right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle DEF &= \frac{1}{2} \{1(2-1) + 1(1-0) + 0(0-2)\} \\ &= \frac{1}{2} (1+1) = 1 \text{ square units} \end{aligned}$$

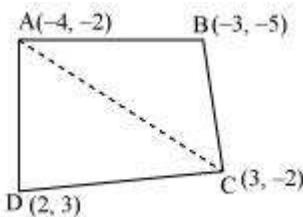
$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [0(1-3) + 2\{3-(-1)\} + 0(-1-1)] \\ &= \frac{1}{2} \{8\} = 4 \text{ square units} \end{aligned}$$

Therefore, required ratio = 1 : 4

Q7 :

Find the area of the quadrilateral whose vertices, taken in order, are (- 4, - 2), (- 3, - 5), (3, - 2) and (2, 3)

Answer :



Let the vertices of the quadrilateral be A (- 4, - 2), B (- 3, - 5), C (3, - 2), and D (2, 3). Join AC to form two triangles $\triangle ABC$ and $\triangle ACD$.

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [(-4)\{(-5) - (-2)\} + (-3)\{(-2) - (-2)\} + 3\{(-2) - (-5)\}] \\ &= \frac{1}{2} (12 + 0 + 9) = \frac{21}{2} \text{ square units} \end{aligned}$$

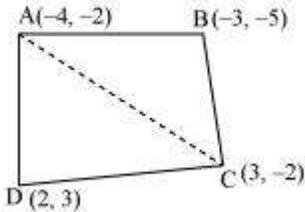
$$\begin{aligned} \text{Area of } \triangle ACD &= \frac{1}{2} [(-4)\{(-2) - (3)\} + 3\{(3) - (-2)\} + 2\{(-2) - (-2)\}] \\ &= \frac{1}{2} \{20 + 15 + 0\} = \frac{35}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area of } \square ABCD &= \text{Area of } \triangle ABC + \text{Area of } \triangle ACD \\ &= \left(\frac{21}{2} + \frac{35}{2} \right) \text{ square units} = 28 \text{ square units} \end{aligned}$$

Q8 :

Find the area of the quadrilateral whose vertices, taken in order, are (- 4, - 2), (- 3, - 5), (3, - 2) and (2, 3)

Answer :



Let the vertices of the quadrilateral be A (- 4, - 2), B (- 3, - 5), C (3, - 2), and D (2, 3). Join AC to form two triangles $\triangle ABC$ and $\triangle ACD$.

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [(-4)\{(-5) - (-2)\} + (-3)\{(-2) - (-2)\} + 3\{(-2) - (-5)\}] \\ &= \frac{1}{2} (12 + 0 + 9) = \frac{21}{2} \text{ square units} \end{aligned}$$

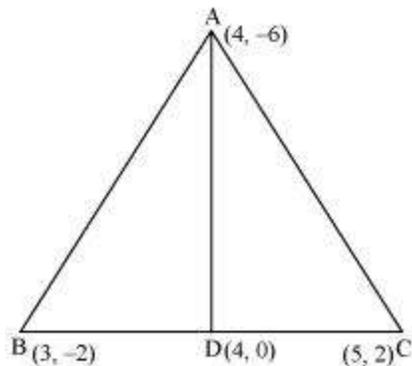
$$\begin{aligned} \text{Area of } \triangle ACD &= \frac{1}{2} [(-4)\{(-2) - (3)\} + 3\{(3) - (-2)\} + 2\{(-2) - (-2)\}] \\ &= \frac{1}{2} \{20 + 15 + 0\} = \frac{35}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area of } \square ABCD &= \text{Area of } \triangle ABC + \text{Area of } \triangle ACD \\ &= \left(\frac{21}{2} + \frac{35}{2}\right) \text{ square units} = 28 \text{ square units} \end{aligned}$$

Q9 :

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for $\triangle ABC$ whose vertices are A (4, - 6), B (3, - 2) and C (5, 2)

Answer :



Let the vertices of the triangle be A (4, - 6), B (3, - 2), and C (5, 2).

Let D be the mid-point of side BC of $\triangle ABC$. Therefore, AD is the median in $\triangle ABC$.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABD &= \frac{1}{2} [(4)\{(-2) - (0)\} + (3)\{(0) - (-6)\} + (4)\{(-6) - (-2)\}] \\ &= \frac{1}{2} (-8 + 18 - 16) = -3 \text{ square units} \end{aligned}$$

However, area cannot be negative. Therefore, area of $\triangle ABD$ is 3 square units.

$$\begin{aligned} \text{Area of } \triangle ADC &= \frac{1}{2} [(4)\{0 - (2)\} + (4)\{(2) - (-6)\} + (5)\{(-6) - (0)\}] \\ &= \frac{1}{2} (-8 + 32 - 30) = -3 \text{ square units} \end{aligned}$$

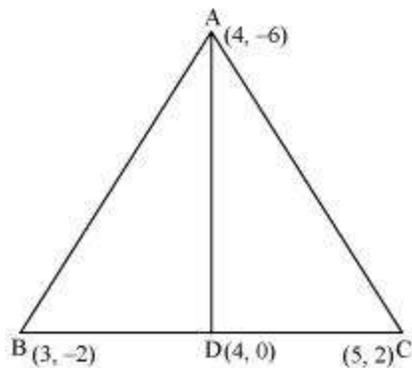
However, area cannot be negative. Therefore, area of $\triangle ADC$ is 3 square units.

Clearly, median AD has divided $\triangle ABC$ in two triangles of equal areas.

Q10 :

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for $\triangle ABC$ whose vertices are A (4, - 6), B (3, - 2) and C (5, 2)

Answer :



Let the vertices of the triangle be A (4, - 6), B (3, - 2), and C (5, 2).

Let D be the mid-point of side BC of $\triangle ABC$. Therefore, AD is the median in $\triangle ABC$.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2} \right) = (4, 0)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABD &= \frac{1}{2} [(4)\{(-2) - (0)\} + (3)\{(0) - (-6)\} + (4)\{(-6) - (-2)\}] \\ &= \frac{1}{2} (-8 + 18 - 16) = -3 \text{ square units} \end{aligned}$$

However, area cannot be negative. Therefore, area of $\triangle ABD$ is 3 square units.

$$\begin{aligned} \text{Area of } \triangle ADC &= \frac{1}{2} [(4)\{0 - (2)\} + (4)\{(2) - (-6)\} + (5)\{(-6) - (0)\}] \\ &= \frac{1}{2} (-8 + 32 - 30) = -3 \text{ square units} \end{aligned}$$

However, area cannot be negative. Therefore, area of $\triangle ADC$ is 3 square units.

Clearly, median AD has divided $\triangle ABC$ in two triangles of equal areas.

Exercise 7.4 : Solutions of Questions on Page Number : 171

Q1 :

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7)

Answer :

Let the given line divide the line segment joining the points A(2, - 2) and B(3, 7) in a ratio $k : 1$.

$$\text{Coordinates of the point of division} = \left(\frac{3k + 2}{k + 1}, \frac{7k - 2}{k + 1} \right)$$

This point also lies on $2x + y - 4 = 0$

$$\begin{aligned} \therefore 2 \left(\frac{3k + 2}{k + 1} \right) + \left(\frac{7k - 2}{k + 1} \right) - 4 &= 0 \\ \Rightarrow \frac{6k + 4 + 7k - 2 - 4k - 4}{k + 1} &= 0 \\ \Rightarrow 9k - 2 &= 0 \\ \Rightarrow k &= \frac{2}{9} \end{aligned}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7) is 2:9.

Q2 :

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7)

Answer :

Let the given line divide the line segment joining the points A(2, - 2) and B(3, 7) in a ratio $k : 1$.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1} \right)$

This point also lies on $2x + y - 4 = 0$

$$\begin{aligned} \therefore 2 \left(\frac{3k+2}{k+1} \right) + \left(\frac{7k-2}{k+1} \right) - 4 &= 0 \\ \Rightarrow \frac{6k+4+7k-2-4k-4}{k+1} &= 0 \\ \Rightarrow 9k-2 &= 0 \\ \Rightarrow k &= \frac{2}{9} \end{aligned}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7) is 2:9.

Q3 :

Find a relation between x and y if the points (x, y) , $(1, 2)$ and $(7, 0)$ are collinear.

Answer :

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area} = \frac{1}{2} [x(2-0) + 1(0-y) + 7(y-2)]$$

$$0 = \frac{1}{2} [2x - y + 7y - 14]$$

$$0 = \frac{1}{2} [2x + 6y - 14]$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between x and y .

Q4 :

Find a relation between x and y if the points (x, y) , $(1, 2)$ and $(7, 0)$ are collinear.

Answer :

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area} = \frac{1}{2} [x(2 - 0) + 1(0 - y) + 7(y - 2)]$$

$$0 = \frac{1}{2} [2x - y + 7y - 14]$$

$$0 = \frac{1}{2} [2x + 6y - 14]$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between x and y .

Q5 :

Find the centre of a circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.

Answer :

Let $O(x, y)$ be the centre of the circle. And let the points $(6, -6)$, $(3, -7)$, and $(3, 3)$ be representing the points A, B, and C on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However, $OA = OB$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$\Rightarrow -6x - 2y + 14 = 0$$

$$\Rightarrow 3x + y = 7 \quad \dots (1)$$

Similarly, $OA = OC$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$\Rightarrow -6x + 18y + 54 = 0$$

$$\Rightarrow -3x + 9y = -27 \quad \dots (2)$$

On adding equation (1) and (2), we obtain

$$10y = -20$$

$$y = -2$$

From equation (1), we obtain

$$3x - 2 = 7$$

$$3x = 9$$

$$x = 3$$

Therefore, the centre of the circle is (3, -2).

Q6 :

Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).

Answer :

Let O (x, y) be the centre of the circle. And let the points (6, -6), (3, -7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However, $OA = OB$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$\Rightarrow -6x - 2y + 14 = 0$$

$$\Rightarrow 3x + y = 7 \quad \dots (1)$$

Similarly, $OA = OC$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$\Rightarrow -6x + 18y + 54 = 0$$

$$\Rightarrow -3x + 9y = -27 \quad \dots (2)$$

On adding equation (1) and (2), we obtain

$$10y = -20$$

$$y = -2$$

From equation (1), we obtain

$$3x - 2 = 7$$

$$3x = 9$$

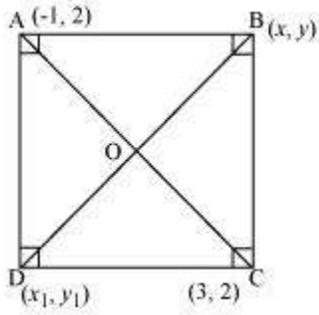
$$x = 3$$

Therefore, the centre of the circle is (3, -2).

Q7 :

The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices.

Answer :



Let ABCD be a square having $(-1, 2)$ and $(3, 2)$ as vertices A and C respectively. Let (x, y) , (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

$$\therefore AB = BC$$

$$\begin{aligned} \Rightarrow \sqrt{(x+1)^2 + (y-2)^2} &= \sqrt{(x-3)^2 + (y-2)^2} \\ \Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 &= x^2 + 9 - 6x + y^2 + 4 - 4y \\ \Rightarrow 8x &= 8 \\ \Rightarrow x &= 1 \end{aligned}$$

We know that in a square, all interior angles are of 90° .

In $\triangle ABC$,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \left(\sqrt{(1+1)^2 + (y-2)^2} \right)^2 + \left(\sqrt{(1-3)^2 + (y-2)^2} \right)^2 = \left(\sqrt{(3+1)^2 + (2-2)^2} \right)^2$$

$$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$$

$$\Rightarrow 2y^2 + 16 - 8y = 16$$

$$\Rightarrow 2y^2 - 8y = 0$$

$$\Rightarrow y(y - 4) = 0$$

$$\Rightarrow y = 0 \text{ or } 4$$

We know that in a square, the diagonals are of equal length and bisect each other at 90° . Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2} \right)$$

$$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2} \right) = (1, 2)$$

$$\frac{1+x_1}{2} = 1$$

$$1+x_1 = 2$$

$$x_1 = 1$$

$$\text{and } \frac{y+y_1}{2} = 2$$

$$\Rightarrow y + y_1 = 4$$

$$\text{If } y = 0,$$

$$y_1 = 4$$

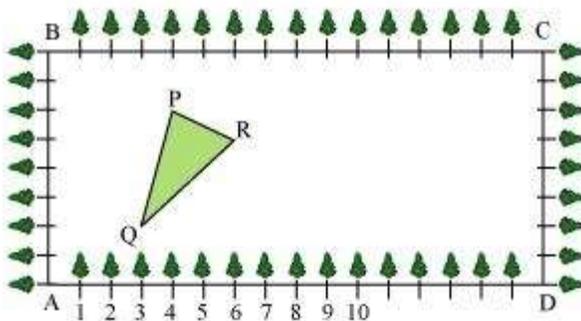
$$\text{If } y = 4,$$

$$y_1 = 0$$

Therefore, the required coordinates are (1, 0) and (1, 4).

Q8 :

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of ΔPQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer :

(i) Taking A as origin, we will take AD as x-axis and AB as y-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\begin{aligned} \text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [4(2 - 5) + 3(5 - 6) + 6(6 - 2)] \\ &= \frac{1}{2} [-12 - 3 + 24] \\ &= \frac{9}{2} \text{ square units} \end{aligned}$$

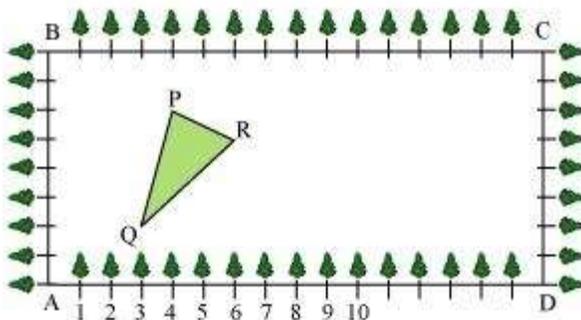
(ii) Taking C as origin, CB as x-axis, and CD as y-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

$$\begin{aligned} \text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [12(6 - 3) + 13(3 - 2) + 10(2 - 6)] \\ &= \frac{1}{2} [36 + 13 - 40] \\ &= \frac{9}{2} \text{ square units} \end{aligned}$$

It can be observed that the area of the triangle is same in both the cases.

Q9 :

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer :

(i) Taking A as origin, we will take AD as x-axis and AB as y-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\begin{aligned}\text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [4(2 - 5) + 3(5 - 6) + 6(6 - 2)] \\ &= \frac{1}{2} [-12 - 3 + 24] \\ &= \frac{9}{2} \text{ square units}\end{aligned}$$

(ii) Taking C as origin, CB as x-axis, and CD as y-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

$$\begin{aligned}\text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [12(6 - 3) + 13(3 - 2) + 10(2 - 6)] \\ &= \frac{1}{2} [36 + 13 - 40] \\ &= \frac{9}{2} \text{ square units}\end{aligned}$$

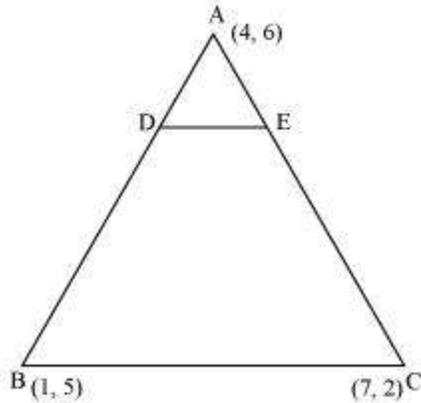
It can be observed that the area of the triangle is same in both the cases.

Q10 :

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and

E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC . (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer :



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\begin{aligned} \text{Coordinates of Point D} &= \left(\frac{1 \times 1 + 3 \times 4}{1+3}, \frac{1 \times 5 + 3 \times 6}{1+3} \right) \\ &= \left(\frac{13}{4}, \frac{23}{4} \right) \end{aligned}$$

$$\begin{aligned} \text{Coordinates of point E} &= \left(\frac{1 \times 7 + 3 \times 4}{1+3}, \frac{1 \times 2 + 3 \times 6}{1+3} \right) \\ &= \left(\frac{19}{4}, \frac{20}{4} \right) \end{aligned}$$

$$\text{Area of a triangle} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\begin{aligned} \text{Area of } \triangle ADE &= \frac{1}{2} \left[4 \left(\frac{23}{4} - \frac{20}{4} \right) + \frac{13}{4} \left(\frac{20}{4} - 6 \right) + \frac{19}{4} \left(6 - \frac{23}{4} \right) \right] \\ &= \frac{1}{2} \left[3 - \frac{13}{4} + \frac{19}{16} \right] = \frac{1}{2} \left[\frac{48 - 52 + 19}{16} \right] = \frac{15}{32} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [4(5-2) + 1(2-6) + 7(6-5)] \\ &= \frac{1}{2} [12 - 4 + 7] = \frac{15}{2} \text{ square units} \end{aligned}$$

Clearly, the ratio between the areas of $\triangle ADE$ and $\triangle ABC$ is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here $\triangle ADE$ and $\triangle ABC$) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

$$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

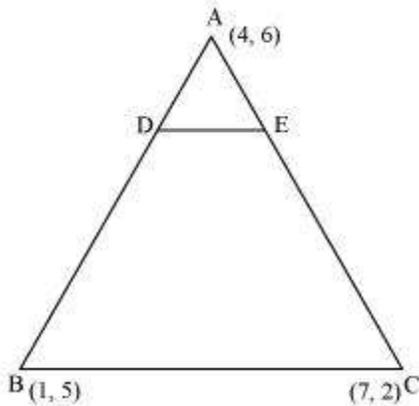
Therefore, ratio between the areas of $\triangle ADE$ and $\triangle ABC =$

Q11 :

The vertices of a $\triangle ABC$ are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and

E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the $\triangle ADE$ and compare it with the area of $\triangle ABC$. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer :



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

Given that,

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\begin{aligned}\text{Coordinates of Point D} &= \left(\frac{1 \times 1 + 3 \times 4}{1+3}, \frac{1 \times 5 + 3 \times 6}{1+3} \right) \\ &= \left(\frac{13}{4}, \frac{23}{4} \right)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of point E} &= \left(\frac{1 \times 7 + 3 \times 4}{1+3}, \frac{1 \times 2 + 3 \times 6}{1+3} \right) \\ &= \left(\frac{19}{4}, \frac{20}{4} \right)\end{aligned}$$

$$\text{Area of a triangle} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\begin{aligned}\text{Area of } \triangle ADE &= \frac{1}{2} \left[4 \left(\frac{23}{4} - \frac{20}{4} \right) + \frac{13}{4} \left(\frac{20}{4} - 6 \right) + \frac{19}{4} \left(6 - \frac{23}{4} \right) \right] \\ &= \frac{1}{2} \left[3 - \frac{13}{4} + \frac{19}{16} \right] = \frac{1}{2} \left[\frac{48 - 52 + 19}{16} \right] = \frac{15}{32} \text{ square units}\end{aligned}$$

$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{1}{2} [4(5-2) + 1(2-6) + 7(6-5)] \\ &= \frac{1}{2} [12 - 4 + 7] = \frac{15}{2} \text{ square units}\end{aligned}$$

Clearly, the ratio between the areas of $\triangle ADE$ and $\triangle ABC$ is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here $\triangle ADE$ and $\triangle ABC$) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

$$\left(\frac{1}{4} \right)^2 = \frac{1}{16}$$

Therefore, ratio between the areas of $\triangle ADE$ and $\triangle ABC$ =

Q12 :

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of $\triangle ABC$.

(i) The median from A meets BC at D. Find the coordinates of point D.

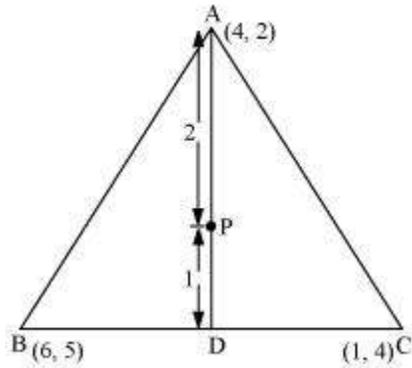
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are the vertices of ΔABC , find the coordinates of the centroid of the triangle.

Answer :



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2} \right) = \left(\frac{7}{2}, \frac{9}{2} \right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2} \right) = \left(\frac{5}{2}, 3 \right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2} \right) = \left(5, \frac{7}{2} \right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2 \times 5 + 1 \times 1}{2 + 1}, \frac{2 \times \frac{7}{2} + 1 \times 4}{2 + 1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC , having its vertices as $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

$$\begin{aligned} \text{Coordinates of O} &= \left(\frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2 + 1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2 + 1} \right) \\ &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \end{aligned}$$

Q13 :

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC .

(i) The median from A meets BC at D. Find the coordinates of point D.

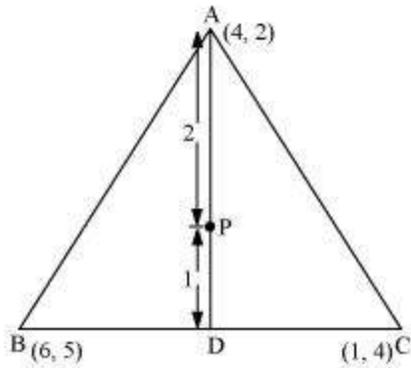
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are the vertices of ΔABC , find the coordinates of the centroid of the triangle.

Answer :



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2} \right) = \left(\frac{7}{2}, \frac{9}{2} \right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2} \right) = \left(\frac{5}{2}, 3 \right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2} \right) = \left(5, \frac{7}{2} \right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2 \times 5 + 1 \times 1}{2+1}, \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1} \right) = \left(\frac{11}{3}, \frac{11}{3} \right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC , having its vertices as $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC .

$$\text{Coordinates of } D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

Let the centroid of this triangle be O .

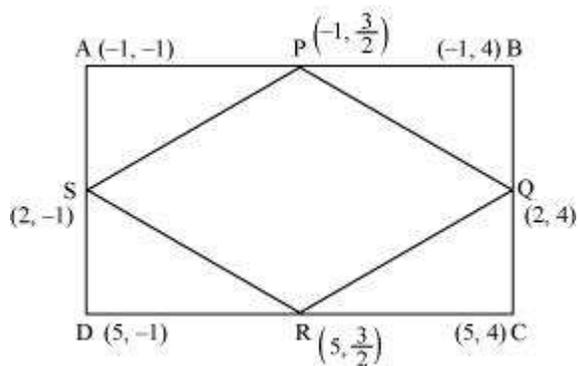
Point O divides the side AD in a ratio 2:1.

$$\begin{aligned} \text{Coordinates of } O &= \left(\frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2 + 1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2 + 1} \right) \\ &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \end{aligned}$$

Q14 :

$ABCD$ is a rectangle formed by the points $A(-1, -1)$, $B(-1, 4)$, $C(5, 4)$ and $D(5, -1)$. P , Q , R and S are the mid-points of AB , BC , CD , and DA respectively. Is the quadrilateral $PQRS$ a square? a rectangle? or a rhombus? Justify your answer.

Answer :



P is the mid-point of side AB .

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2} \right) = \left(-1, \frac{3}{2} \right)$

Similarly, the coordinates of Q , R , and S are $(2, 4)$, $\left(5, \frac{3}{2} \right)$, and $(2, -1)$ respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

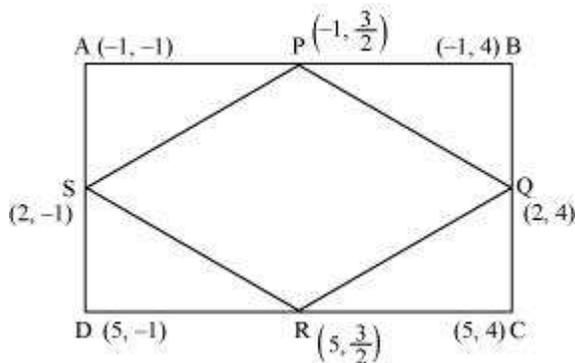
$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.

Q15 :

ABCD is a rectangle formed by the points A (-1, -1), B (-1, 4), C (5, 4) and D (5, -1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.

Answer :



P is the mid-point of side AB.

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R, and S are (2, 4), $\left(5, \frac{3}{2}\right)$, and (2, -1) respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.

DELHI PUBLIC SCHOOL – GANDHINAGAR

CHAPTER 7 COORDINATE GEOMETRY

MIND MAP

This chapter consists of three different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 DISTANCE FORMULA

- Q.1. Find the distance between the points (a, b) and $(-a, -b)$.
- Q.2. Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.
- Q.3. Find the values of y for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10 units.

TYPE: 2 SECTION FORMULA

- Q.1. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4,-3)$ in the ratio 2:3.
- Q.2. Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the X-axis.
- Q.3. Find the coordinates of the points of trisection of the line segment joining $(4,-1)$ and $(2,-3)$.
- Q.4. If $(1, 2)$, $(4, y)$, $(x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y .
- Q.5. Find the coordinates of the points which divide the line segment joining $(-2, 2)$ and $(2, 8)$ into four equal parts.

TYPE: 3 AREA OF A TRIANGLE

- Q.1. Find the area of a triangle whose vertices are $(-5,-1)$, $(3,-5)$ and $(5, 2)$.
- Q.2. Find k if the points $(8, 1)$, $(k,-4)$ and $(2,-5)$ are collinear.
- Q.3. Find the area of the quadrilateral whose vertices, taken in order, are $(-4,-2)$, $(-3,-5)$, $(3,-2)$ and $(2, 3)$.

NCERT Solutions for Class 10 Maths Unit 8

Introduction to Trigonometry Class 10

Unit 8 Introduction to Trigonometry Exercise 8.1, 8.2, 8.3, 8.4 Solutions

Exercise 8.1 : Solutions of Questions on Page Number : 181

Q1 :

In $\triangle ABC$ right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Answer :

Applying Pythagoras theorem for $\triangle ABC$, we obtain

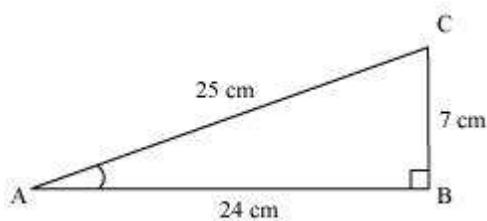
$$AC^2 = AB^2 + BC^2$$

$$= (24 \text{ cm})^2 + (7 \text{ cm})^2$$

$$= (576 + 49) \text{ cm}^2$$

$$= 625 \text{ cm}^2$$

$$\therefore AC = \sqrt{625} \text{ cm} = 25 \text{ cm}$$

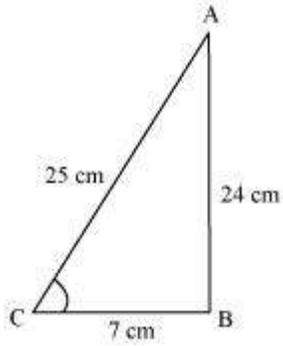


$$(i) \sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

(ii)



$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24}{25}$$

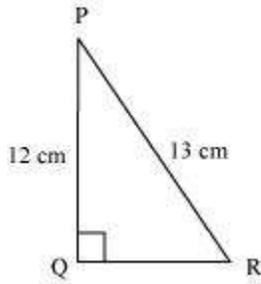
$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$



Q2 :

In the given figure find $\tan P - \cot R$



Answer :

Applying Pythagoras theorem for ΔPQR , we obtain

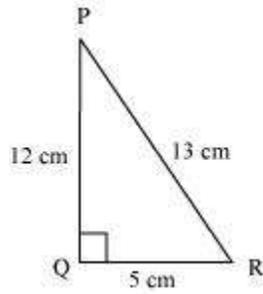
$$PR^2 = PQ^2 + QR^2$$

$$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$$

$$169 \text{ cm}^2 = 144 \text{ cm}^2 + QR^2$$

$$25 \text{ cm}^2 = QR^2$$

$$QR = 5 \text{ cm}$$



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

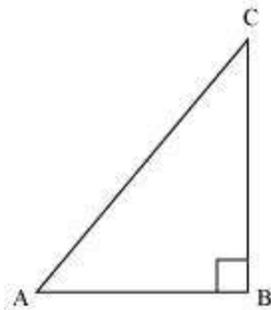


Q3 :

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer :

Let $\triangle ABC$ be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be $3k$. Therefore, AC will be $4k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = AB^2 + (3k)^2$$

$$16k^2 - 9k^2 = AB^2$$

$$7k^2 = AB^2$$

$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

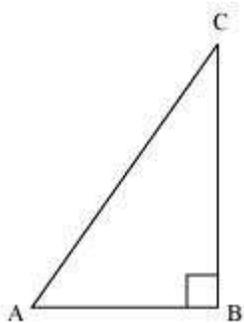
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Q4 :

Given $15 \cot A = 8$. Find $\sin A$ and $\sec A$

Answer :

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

$$\cot A = \frac{8}{15}$$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$. Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$$AC = 17k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

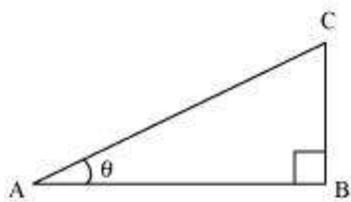
$$= \frac{AC}{AB} = \frac{17}{8}$$

Q5 :

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer :

Consider a right-angle triangle $\triangle ABC$, right-angled at point B.



$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is $13k$, AB will be $12k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + BC^2$$

$$25k^2 = BC^2$$

$$BC = 5k$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

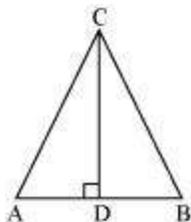
Q6 :

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that

$\angle A = \angle B$.

Answer :

Let us consider a triangle ABC in which $CD \perp AB$.

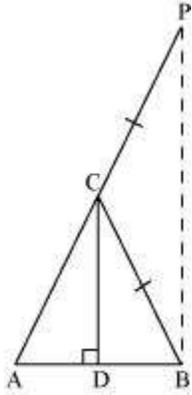


It is given that

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \dots (1)$$

We have to prove $\angle A = \angle B$. To prove this, let us extend AC to P such that $BC = CP$.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP} \quad (\text{By construction, we have } BC = CP) \quad \dots (2)$$

By using the converse of B.P.T,

$CD \parallel BP$

$\Rightarrow \angle ACD = \angle CPB$ (Corresponding angles) ... (3)

And, $\angle BCD = \angle CBP$ (Alternate interior angles) ... (4)

By construction, we have $BC = CP$.

$\therefore \angle CBP = \angle CPB$ (Angle opposite to equal sides of a triangle) ... (5)

From equations (3), (4), and (5), we obtain

$\angle ACD = \angle BCD$... (6)

In $\triangle CAD$ and $\triangle CBD$,

$\angle ACD = \angle BCD$ [Using equation (6)]

$\angle CDA = \angle CDB$ [Both 90°]

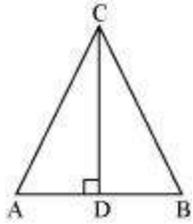
Therefore, the remaining angles should be equal.

$\therefore \angle CAD = \angle CBD$

$\Rightarrow \angle A = \angle B$

Alternatively,

Let us consider a triangle ABC in which $CD \perp AB$.



It is given that,

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

$$\text{Let } \frac{AD}{BD} = \frac{AC}{BC} = k$$

$$\Rightarrow AD = k BD \dots (1)$$

$$\text{And, } AC = k BC \dots (2)$$

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$$CD^2 = AC^2 - AD^2 \dots (3)$$

$$\text{And, } CD^2 = BC^2 - BD^2 \dots (4)$$

From equations (3) and (4), we obtain

$$AC^2 - AD^2 = BC^2 - BD^2$$

$$\Rightarrow (k BC)^2 - (k BD)^2 = BC^2 - BD^2$$

$$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$$

$$\Rightarrow k^2 = 1$$

$$\Rightarrow k = 1$$

Putting this value in equation (2), we obtain

$$AC = BC$$

$$\Rightarrow \angle A = \angle B (\text{Angles opposite to equal sides of a triangle})$$

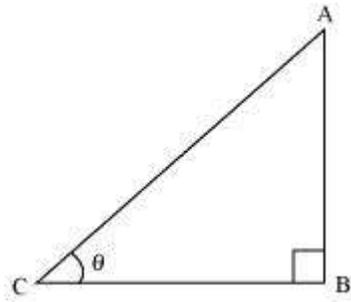
Q7 :

If $\cot \theta = \frac{7}{8}$, evaluate

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \quad \text{(i) } \cot^2 \theta$$

Answer :

Let us consider a right triangle ABC, right-angled at point B.



$$\begin{aligned}\cot \theta &= \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{BC}{AB} \\ &= \frac{7}{8}\end{aligned}$$

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (7k)^2$$

$$= 64k^2 + 49k^2$$

$$= 113k^2$$

$$AC = \sqrt{113}k$$

$$\begin{aligned}\sin \theta &= \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} \\ &= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}\end{aligned}$$

$$\begin{aligned}\cos \theta &= \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} \\ &= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}\end{aligned}$$

$$(i) \quad \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1 - \sin^2 \theta)}{(1 - \cos^2 \theta)}$$

$$\begin{aligned}
 &= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} \\
 &= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}
 \end{aligned}$$

$$(ii) \cot^2 \theta = (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Q8 :

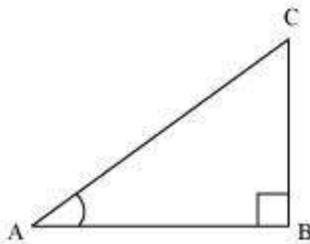
If $3 \cot A = 4$, Check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer :

It is given that $3 \cot A = 4$

$$\text{Or, } \cot A = \frac{4}{3}$$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is $4k$, then BC will be $3k$, where k is a positive integer.

In $\triangle ABC$,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (4k)^2 + (3k)^2$$

$$= 16k^2 + 9k^2$$

$$= 25k^2$$

$$AC = 5k$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

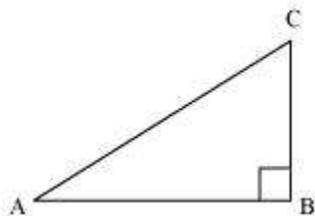
Q9 :

In $\triangle ABC$, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Answer :



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is k , then AB will be $\sqrt{3}k$, where k is a positive integer.

In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$= (\sqrt{3}k)^2 + (k)^2$$

$$= 3k^2 + k^2 = 4k^2$$

$$\therefore AC = 2k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) $\cos A \cos C - \sin A \sin C$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q10 :

In ΔPQR , right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

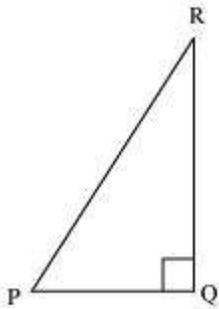
Answer :

Given that, $PR + QR = 25$

$PQ = 5$

Let PR be x .

Therefore, $QR = 25 - x$



Applying Pythagoras theorem in ΔPQR , we obtain

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = (5)^2 + (25 - x)^2$$

$$x^2 = 25 + 625 + x^2 - 50x$$

$$50x = 650$$

$$x = 13$$

Therefore, $PR = 13$ cm

$QR = (25 - 13)$ cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q11 :

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A .

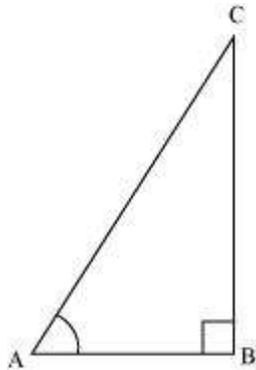
(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

(iv) $\cot A$ is the product of \cot and A

(v) $\sin \theta = \frac{4}{3}$, for some angle θ

Answer :

(i) Consider a $\triangle ABC$, right-angled at B .



$$\begin{aligned}\tan A &= \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} \\ &= \frac{12}{5}\end{aligned}$$

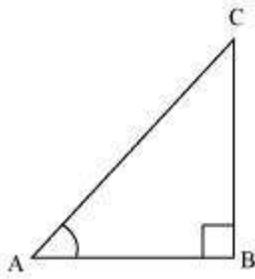
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, $\tan A < 1$ is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$(12k)^2 = (5k)^2 + BC^2$$

$$144k^2 = 25k^2 + BC^2$$

$$BC^2 = 119k^2$$

$$BC = 10.9k$$

It can be observed that for given two sides $AC = 12k$ and $AB = 5k$,

BC should be such that,

$$AC - AB < BC < AC + AB$$

$$12k - 5k < BC < 12k + 5k$$

$$7k < BC < 17k$$

However, $BC = 10.9k$. Clearly, such a triangle is possible and hence, such value of $\sec A$ is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) $\cot A$ is not the product of cot and A. It is the cotangent of $\angle A$.

Hence, the given statement is false.

$$(v) \sin \theta = \frac{4}{3}$$

We know that in a right-angled triangle,

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of $\sin \theta$ is not possible.

Hence, the given statement is false

Exercise 8.2 : Solutions of Questions on Page Number : 187

Q1 :

Evaluate the following

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer :

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$\begin{aligned} &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$\begin{aligned} &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 2 + \frac{3}{4} - \frac{3}{4} = 2 \end{aligned}$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$\begin{aligned}
&= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} \\
&= \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \\
&= \frac{\sqrt{3}(2\sqrt{6}-2\sqrt{2})}{(2\sqrt{6}+2\sqrt{2})(2\sqrt{6}-2\sqrt{2})} \\
&= \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{24-8} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{16} \\
&= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}
\end{aligned}$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\begin{aligned}
&= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}
\end{aligned}$$

$$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

$$(v) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\begin{aligned}
&= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
&= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}} \\
&= \frac{15 + 64 - 12}{\frac{12}{4}} = \frac{67}{12}
\end{aligned}$$

Q2 :

Choose the correct option and justify your choice.

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

- (A). $\sin 60^\circ$
- (B). $\cos 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

- (A). $\tan 90^\circ$
- (B). 1
- (C). $\sin 45^\circ$
- (D). 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

- (A). 0°
- (B). 30°
- (C). 45°
- (D). 60°

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A). $\cos 60^\circ$

(B). $\sin 60^\circ$

(C). $\tan 60^\circ$

(D). $\sin 30^\circ$

Answer :

$$\begin{aligned} & \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \\ \text{(i)} & \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} \\ & = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only

Hence, (A) is correct.

$$\begin{aligned} & \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} \\ \text{(ii)} & \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0 \end{aligned}$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only $A = 0^\circ$ is correct.

$$\text{As } \sin 2A = \sin 0^\circ = 0$$

$$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$$

Hence, (A) is correct.

$$\begin{aligned} & \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} \\ \text{(iv)} & \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 - \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} \\ & = \sqrt{3} \end{aligned}$$

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct.

Q3 :

$$\text{If } \tan(A + B) = \sqrt{3} \text{ and } \tan(A - B) = \frac{1}{\sqrt{3}};$$

$0^\circ < A + B \leq 90^\circ$, $A > B$ find A and B.

Answer :

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60$$

$$\Rightarrow A + B = 60 \dots (1)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30$$

$$\Rightarrow A - B = 30 \dots (2)$$

On adding both equations, we obtain

$$2A = 90$$

$$\Rightarrow A = 45$$

From equation (1), we obtain

$$45 + B = 60$$

$$B = 15$$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

Q4 :

State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin \hat{A}$ increases as \hat{A} increases

(iii) The value of $\cos \hat{A}$ increases as \hat{A} increases

(iv) $\sin \hat{A} = \cos \hat{A}$ for all values of \hat{A} ,

(v) $\cot A$ is not defined for $A = 0^\circ$

Answer :

$$(i) \sin(A + B) = \sin A + \sin B$$

$$\text{Let } A = 30^\circ \text{ and } B = 60^\circ$$

$$\sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Clearly, $\sin(A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$ as

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

$$(iii) \cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

It can be observed that the value of $\cos \theta$ does not increase in the interval of $0^\circ < \theta < 90^\circ$.

Hence, the given statement is false.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

This is true when $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ .

$$\text{As } \sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2},$$

Hence, the given statement is false.

(v) $\cot A$ is not defined for $A = 0^\circ$

$$\text{As } \cot A = \frac{\cos A}{\sin A},$$

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined}$$

Hence, the given statement is true.

Exercise 8.3 : Solutions of Questions on Page Number : 189

Q1 :

Evaluate

$$(I) \frac{\sin 18^\circ}{\cos 72^\circ}$$

$$(II) \frac{\tan 26^\circ}{\cot 64^\circ}$$

$$(III) \cos 48^\circ - \sin 42^\circ$$

$$(IV) \operatorname{cosec} 31^\circ - \sec 59^\circ$$

Answer :

$$(I) \frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$$

$$(II) \frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$$

$$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$$

$$(III) \cos 48^\circ - \sin 42^\circ = \cos (90^\circ - 42^\circ) - \sin 42^\circ$$

$$= \sin 42^\circ - \sin 42^\circ$$

$$= 0$$

$$(IV) \operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ$$

$$= \sec 59^\circ - \sec 59^\circ$$

$$= 0$$

Q2 :

Show that

$$(I) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

$$(II) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

Answer :

$$(I) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

$$= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ)$$

$$= (1) (1)$$

$$= 1$$

$$(II) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$$

$$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$$

$$= 0$$

Q3 :

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Answer :

Given that,

$$\tan 2A = \cot (A - 18^\circ)$$

$$\cot (90^\circ - 2A) = \cot (A - 18^\circ)$$

$$90^\circ - 2A = A - 18^\circ$$

$$108^\circ = 3A$$

$$A = 36^\circ$$

Q4 :

If $\tan A = \cot B$, prove that $A + B = 90^\circ$

Answer :

Given that,

$$\tan A = \cot B$$

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

Q5 :

If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Answer :

Given that,

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

$$\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

$$90^\circ - 4A = A - 20^\circ$$

$$110^\circ = 5A$$

$$A = 22^\circ$$

Q6 :

If A , B and C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer :

We know that for a triangle ABC ,

$$\angle A + \angle B + \angle C = 180^\circ$$

- $\angle B + \angle C = 180^\circ - \angle A$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$= \cos\left(\frac{A}{2}\right)$$

Q7 :

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Answer :

$$\begin{aligned} & \sin 67^\circ + \cos 75^\circ \\ &= \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ) \\ &= \cos 23^\circ + \sin 15^\circ \end{aligned}$$

Exercise 8.4 : Solutions of Questions on Page Number : 193

Q1 :

Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Answer :

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

$\sqrt{1 + \cot^2 A}$ will always be positive as we are adding two positive quantities.

$$\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

Therefore,

$$\tan A = \frac{\sin A}{\cos A}$$

We know that,

$$\cot A = \frac{\cos A}{\sin A}$$

However,

$$\tan A = \frac{1}{\cot A}$$

Therefore,

$$\text{Also, } \sec^2 A = 1 + \tan^2 A$$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2 :

Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Answer :

We know that,

$$\cos A = \frac{1}{\sec A}$$

$$\text{Also, } \sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\begin{aligned} \sin A &= \sqrt{1 - \left(\frac{1}{\sec A}\right)^2} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \end{aligned}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{1}{\sec A} \sqrt{\sec^2 A - 1}}$$
$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Q3 :

Evaluate

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

Answer :

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$
$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$
$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$
$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$
$$= \frac{1}{1} \quad (\text{As } \sin^2 A + \cos^2 A = 1)$$

= 1

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= (\sin 25^\circ) \{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ \{\sin(90^\circ - 25^\circ)\}$$

$$= (\sin 25^\circ)(\sin 25^\circ) + (\cos 25^\circ)(\cos 25^\circ)$$

$$= \sin^2 25^\circ + \cos^2 25^\circ$$

$$= 1 \text{ (As } \sin^2 A + \cos^2 A = 1)$$

Q4 :

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

(A) 0

(B) 1

(C) 2

(D) - 1

(iii) $(\sec A + \tan A) (1 - \sin A) =$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

(A) $\sec^2 A$

(B) - 1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer :

(i) $9 \sec^2 A - 9 \tan^2 A$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 (1) \text{ [As } \sec^2 A - \tan^2 A = 1]$$

$$= 9$$

Hence, alternative (B) is correct.

(ii)

$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$$\begin{aligned} &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\ &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\ &= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2 \end{aligned}$$

Hence, alternative (C) is correct.

(iii) $(\sec A + \tan A) (1 - \sin A)$

$$\begin{aligned} &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) (1 - \sin A) \\ &= \left(\frac{1 + \sin A}{\cos A}\right) (1 - \sin A) \\ &= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} \end{aligned}$$

$= \cos A$

Hence, alternative (D) is correct.

$$\begin{aligned} \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} \\ \text{(iv)} \quad &= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \end{aligned}$$

Hence, alternative (D) is correct.

Q5 :

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer :

$$(i) \quad (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\begin{aligned} \text{L.H.S.} &= (\operatorname{cosec} \theta - \cot \theta)^2 \\ &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\ &= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\ &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \text{R.H.S.} \end{aligned}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} \\ &= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)(\cos A)} \\ &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{\sin^2 A + \cos^2 A + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{1 + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} = \frac{2 + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{2(1 + \sin A)}{(1 + \sin A)(\cos A)} = \frac{2}{\cos A} = 2 \sec A \\ &= \text{R.H.S.} \end{aligned}$$

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\begin{aligned}
\text{L.H.S.} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
&= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}
\end{aligned}$$

DELHI PUBLIC SCHOOL – GANDHINAGAR

CHAPTER 8 INTRODUCTION TO TRIGONOMETRY

MIND MAP

This chapter consists of four different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 TRIGONOMETRIC RATIOS

Q.1. If $\tan\theta = \frac{4}{3}$, what is the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.

Q.2. If $\sec\theta = \frac{5}{4}$, verify that $\frac{\tan\theta}{1 + \tan^2\theta} = \frac{\sin\theta}{\sec\theta}$

TYPE: 2 TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

Q.1. Determine the value of x such that
$$2\operatorname{cosec}^2 30^\circ + x\sin^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 10.$$

Q.2. If $\theta = 30^\circ$, verify that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

TYPE: 3 TRIGONOMETRIC RATIOS OF COMPLEMENTRY ANGLES

Q.1. If $\cos(40 + x) = \sin 30^\circ$, find the value of x .

Q.2. Evaluate :

(a) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \cdot \operatorname{cosec} 20^\circ$

(b) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ$

TYPE: 4 TRIGONOMETRIC IDENTITIES

Q.1. Prove the following identities:

(a) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{2\sec^2\theta}{\tan^2\theta - 1}$

(b) $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$

(c) $\frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$

Q.2. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

NCERT Solutions for Class 10 Maths Unit 14

Statistics Class 10

Unit 14 Statistics Exercise 14.1, 14.2, 14.3, 14.4 Solutions

Exercise 14.1 : Solutions of Questions on Page Number : 270

Q1 :

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Answer :

To find the class mark (x_i) for each interval, the following relation is used.

$$\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class mark (x_i) =

2

x_i and $f_i x_i$ can be calculated as follows.

Number of plants	Number of houses (f_i)	x_i	$f_i x_i$
0 - 2	1	1	$1 \times 1 = 1$
2 - 4	2	3	$2 \times 3 = 6$
4 - 6	1	5	$1 \times 5 = 5$
6 - 8	5	7	$5 \times 7 = 35$
8 - 10	6	9	$6 \times 9 = 54$
10 - 12	2	11	$2 \times 11 = 22$
12 - 14	3	13	$3 \times 13 = 39$
Total	20		162

From the table, it can be observed that

$$\sum f_i = 20$$

$$\sum f_i x_i = 162$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Mean,

$$= \frac{162}{20} = 8.1$$

Therefore, mean number of plants per house is 8.1.

Here, direct method has been used as the values of class marks (x_i) and f_i are small.

Q2 :

Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer :

To find the class mark for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 20

Taking 150 as assured mean (a), d_i , u_i , and $f_i u_i$ can be calculated as follows.

Daily wages (in Rs)	Number of workers (f_i)	x_i	$d_i = x_i - 150$	$u_i = \frac{d_i}{20}$	$f_i u_i$
100 - 120	12	110	- 40	- 2	- 24
120 - 140	14	130	- 20	- 1	- 14
140 - 160	8	150	0	0	0
160 - 180	6	170	20	1	6
180 - 200	10	190	40	2	20
Total	50				- 12

From the table, it can be observed that

$$\sum f_i = 50$$

$$\sum f_i u_i = -12$$

$$\begin{aligned} \text{Mean } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h \\ &= 150 + \left(\frac{-12}{50} \right) 20 \\ &= 150 - \frac{24}{5} \\ &= 150 - 4.8 \\ &= 145.2 \end{aligned}$$

Therefore, the mean daily wage of the workers of the factory is Rs 145.20.

Q3 :

The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency f .

Daily pocket allowance (in Rs)	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of workers	7	6	9	13	f	5	4

Answer :

To find the class mark (x_i) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Given that, mean pocket allowance, $\bar{x} = \text{Rs } 18$

Taking 18 as assured mean (a), d_i and $f d_i$ are calculated as follows.

Daily pocket allowance (in Rs)	Number of children f_i	Class mark x_i	$d_i = x_i - 18$	$f d_i$
11 - 13	7	12	- 6	- 42
13 - 15	6	14	- 4	- 24
15 - 17	9	16	- 2	- 18
17 - 19	13	18	0	0

19 - 21	f	20	2	$2f$
21 - 23	5	22	4	20
23 - 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$2f - 40$

From the table, we obtain

$$\sum f_i = 44 + f$$

$$\sum f_i u_i = 2f - 40$$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f} \right)$$

$$0 = \left(\frac{2f - 40}{44 + f} \right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence, the missing frequency, f , is 20.

Q4 :

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	65 - 68	68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women	2	4	3	8	7	4	2

Answer :

To find the class mark of each interval (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size, h , of this data = 3

Taking 75.5 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Number of heart beats per minute	Number of women f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
65 - 68	2	66.5	- 9	- 3	- 6
68 - 71	4	69.5	- 6	- 2	- 8
71 - 74	3	72.5	- 3	- 1	- 3
74 - 77	8	75.5	0	0	0
77 - 80	7	78.5	3	1	7
80 - 83	4	81.5	6	2	8
83 - 86	2	84.5	9	3	6
Total	30				4

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = 4$$

$$\begin{aligned} \text{Mean } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 75.5 + \left(\frac{4}{30} \right) \times 3 \\ &= 75.5 + 0.4 = 75.9 \end{aligned}$$

Therefore, mean hear beats per minute for these women are 75.9 beats per minute.

Q5 :

In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Answer :

Number of mangoes	Number of boxes f_i
--------------------------	-----------------------------------------

50 - 52	15
53 - 55	110
56 - 58	135
59 - 61	115
62 - 64	25

It can be observed that class intervals are not continuous. There is a gap of 1 between two class intervals. Therefore,

$\frac{1}{2}$ has to be added to the upper class limit and $\frac{1}{2}$ has to be subtracted from the lower class limit of each interval.

Class mark (x_i) can be obtained by using the following relation.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 3

Taking 57 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Class interval	f_i	x_i	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	$f_i u_i$
49.5 - 52.5	15	51	- 6	- 2	- 30
52.5 - 55.5	110	54	- 3	- 1	- 110
55.5 - 58.5	135	57	0	0	0
58.5 - 61.5	115	60	3	1	115
61.5 - 64.5	25	63	6	2	50
Total	400				25

It can be observed that

$$\sum f_i = 400$$

$$\sum f_i u_i = 25$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 57 + \left(\frac{25}{400} \right) \times 3 \\ &= 57 + \frac{3}{16} = 57 + 0.1875 \\ &= 57.1875 \\ &= 57.19 \end{aligned}$$

Mean number of mangoes kept in a packing box is 57.19.

Step deviation method is used here as the values of f_i , d_i are big and also, there is a common

Q6 :

The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs)	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Answer :

To find the class mark (x_i) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size = 50

Taking 225 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Daily expenditure (in Rs)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	$f_i u_i$
100 - 150	4	125	- 100	- 2	- 8
150 - 200	5	175	- 50	- 1	- 5
200 - 250	12	225	0	0	0
250 - 300	2	275	50	1	2
300 - 350	2	325	100	2	4
Total	25				- 7

From the table, we obtain

$$\sum f_i = 25$$

$$\sum f_i u_i = -7$$

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\
 &= 225 + \left(\frac{-7}{25} \right) \times (50) \\
 &= 225 - 14 \\
 &= 211
 \end{aligned}$$

Therefore, mean daily expenditure on food is Rs 211.

Q7 :

To find out the concentration of SO₂ in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

concentration of SO ₂ (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2

Find the mean concentration of SO₂ in the air.

Answer :

To find the class marks for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size of this data = 0.04

Taking 0.14 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Concentration of SO ₂ (in ppm)	Frequency f_i	Class mark x_i	$d_i = x_i - 0.14$	$u_i = \frac{d_i}{0.04}$	$f_i u_i$
0.00 - 0.04	4	0.02	- 0.12	- 3	- 12
0.04 - 0.08	9	0.06	- 0.08	- 2	- 18
0.08 - 0.12	9	0.10	- 0.04	- 1	- 9

0.12 - 0.16	2	0.14	0	0	0
0.16 - 0.20	4	0.18	0.04	1	4
0.20 - 0.24	2	0.22	0.08	2	4
Total	30				- 31

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = -31$$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 0.14 + \left(\frac{-31}{30} \right) (0.04)$$

$$= 0.14 - 0.04133$$

$$= 0.09867$$

$$\approx 0.099 \text{ ppm}$$

Therefore, mean concentration of SO₂ in the air is 0.099 ppm.

Q8 :

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 - 6	6 - 10	10 - 14	14 - 20	20 - 28	28 - 38	38 - 40
Number of students	11	10	7	4	4	3	1

Answer :

To find the class mark of each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 17 as assumed mean (*a*), *d_i* and *f_id_i* are calculated as follows.

Number of days	Number of students <i>f_i</i>	<i>x_i</i>	<i>d_i = x_i - 17</i>	<i>f_id_i</i>
0 - 6	11	3	- 14	- 154
6 - 10	10	8	- 9	- 90

10 - 14	7	12	- 5	- 35
14 - 20	4	17	0	0
20 - 28	4	24	7	28
28 - 38	3	33	16	48
38 - 40	1	39	22	22
Total	40			- 181

From the table, we obtain

$$\sum f_i = 40$$

$$\sum f_i d_i = -181$$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i d_i}{\sum f_i} \right)$$

$$= 17 + \left(\frac{-181}{40} \right)$$

$$= 17 - 4.525$$

$$= 12.475$$

$$\approx 12.48$$

Therefore, the mean number of days is 12.48 days for which a student was absent.

Q9 :

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 - 55	55 - 65	65 - 75	75 - 85	85 - 95
Number of cities	3	10	11	8	3

Answer :

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) for this data = 10

Taking 70 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated as follows.

Literacy rate (in %)	Number of cities	x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	$f_i u_i$
	f_i				

45 - 55	3	50	- 20	- 2	- 6
55 - 65	10	60	- 10	- 1	- 10
65 - 75	11	70	0	0	0
75 - 85	8	80	10	1	8
85 - 95	3	90	20	2	6
Total	35				- 2

From the table, we obtain

$$\sum f_i = 35$$

$$\sum f_i u_i = -2$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 70 + \left(\frac{-2}{35} \right) \times (10) \\ &= 70 - \frac{20}{35} \\ &= 70 - \frac{4}{7} \\ &= 70 - 0.57 \\ &= 69.43 \end{aligned}$$

Therefore, mean literacy rate is 69.43%.

Exercise 14.2 : Solutions of Questions on Page Number : 275

Q1 :

The following table shows the ages of the patients admitted in a hospital during a year:

age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Answer :

To find the class marks (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 30 as assumed mean (a), d_i and $f_i d_i$ are calculated as follows.

Age (in years)	Number of patients f_i	Class mark x_i	$d_i = x_i - 30$	$f_i d_i$
5 - 15	6	10	- 20	- 120
15 - 25	11	20	- 10	- 110
25 - 35	21	30	0	0
35 - 45	23	40	10	230
45 - 55	14	50	20	280
55 - 65	5	60	30	150
Total	80			430

From the table, we obtain

$$\sum f_i = 80$$

$$\sum f_i d_i = 430$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \frac{\sum f_i d_i}{\sum f_i} \\ &= 30 + \left(\frac{430}{80} \right) \\ &= 30 + 5.375 \\ &= 35.375 \\ &\approx 35.38 \end{aligned}$$

Mean of this data is 35.38. It represents that on an average, the age of a patient admitted to hospital was 35.38 years.

It can be observed that the maximum class frequency is 23 belonging to class interval 35 - 45.

Modal class = 35 - 45

Lower limit (l) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal class = 21

Frequency (f_2) of class succeeding the modal class = 14

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$\begin{aligned}
&= 35 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10 \\
&= 35 + \left[\frac{2}{46 - 35} \right] \times 10 \\
&= 35 + \frac{20}{11} \\
&= 35 + 1.81 \\
&= 36.8
\end{aligned}$$

Mode is 36.8. It represents that the age of maximum number of patients admitted in hospital was 36.8 years.

Q2 :

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Answer :

From the data given above, it can be observed that the maximum class frequency is 61, belonging to class interval 60 - 80.

Therefore, modal class = 60 - 80

Lower class limit (l) of modal class = 60

Frequency (f_1) of modal class = 61

Frequency (f_0) of class preceding the modal class = 52

Frequency (f_2) of class succeeding the modal class = 38

Class size (h) = 20

$$\begin{aligned}
\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
&= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38} \right) (20)
\end{aligned}$$

$$\begin{aligned}
&= 60 + \left(\frac{9}{122 - 90} \right) (20) \\
&= 60 + \left(\frac{9 \times 20}{32} \right) \\
&= 60 + \frac{90}{16} = 60 + 5.625 \\
&= 65.625
\end{aligned}$$

Therefore, modal lifetime of electrical components is 65.625 hours.

Q3 :

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	Number of families
1000 - 1500	24
1500 - 2000	40
2000 - 2500	33
2500 - 3000	28
3000 - 3500	30
3500 - 4000	22
4000 - 4500	16
4500 - 5000	7

Answer :

It can be observed from the given data that the maximum class frequency is 40, belonging to 1500 - 2000 intervals.

Therefore, modal class = 1500 - 2000

Lower limit (l) of modal class = 1500

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

Class size (h) = 500

$$\begin{aligned}
\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
&= 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33} \right) \times 500 \\
&= 1500 + \left(\frac{16}{80 - 57} \right) \times 500 \\
&= 1500 + \frac{8000}{23} \\
&= 1500 + 347.826 \\
&= 1847.826 = 1847.83
\end{aligned}$$

Therefore, modal monthly expenditure was Rs 1847.83.

To find the class mark, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of the given data = 500

Taking 2750 as assumed mean (a), d_i , u_i , and fu_i are calculated as follows.

Expenditure (in Rs)	Number of families f_i	x_i	$d_i = x_i - 2750$	$u_i = \frac{d_i}{500}$	fu_i
1000 - 1500	24	1250	- 1500	- 3	- 72
1500 - 2000	40	1750	- 1000	- 2	- 80
2000 - 2500	33	2250	- 500	- 1	- 33
2500 - 3000	28	2750	0	0	0
3000 - 3500	30	3250	500	1	30
3500 - 4000	22	3750	1000	2	44
4000 - 4500	16	4250	1500	3	48
4500 - 5000	7	4750	2000	4	28
Total	200				- 35

From the table, we obta

Q4 :

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states/U.T
15 - 20	3
20 - 25	8
25 - 30	9
30 - 35	10
35 - 40	3
40 - 45	0
45 - 50	0
50 - 55	2

Answer :

It can be observed from the given data that the maximum class frequency is 10 belonging to class interval 30 - 35.

Therefore, modal class = 30 - 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 30 + \left(\frac{10 - 9}{2(10) - 9 - 3} \right) \times (5) \\
 &= 30 + \left(\frac{1}{20 - 12} \right) 5 \\
 &= 30 + \frac{5}{8} = 30.625
 \end{aligned}$$

Mode = 30.6

It represents that most of the states/U.T have a teacher-student ratio as 30.6.

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 32.5 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated as follows.

Number of students per teacher	Number of states/U.T (f_i)	x_i	$d_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	$f_i u_i$
15 - 20	3	17.5	- 15	- 3	- 9
20 - 25	8	22.5	- 10	- 2	- 16
25 - 30	9	27.5	- 5	- 1	- 9
30 - 35	10	32.5	0	0	0
35 - 40	3	37.5	5	1	3
40 - 45	0	42.5	10	2	0
45 - 50	0	47.5	15	3	0
50 - 55	2	52.5	20	4	8
Total	35				- 23

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h \\
 &= 32.5 + \left(\frac{-23}{35} \right) \times 5 \\
 &= 32.5 - \frac{23}{7} = 32.5 - 3.28 \\
 &= 29.22
 \end{aligned}$$

Therefore, mean of the data is 29.2.

It repre

Q5 :

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7

7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Find the mode of the data.

Answer :

From the given data, it can be observed that the maximum class frequency is 18, belonging to class interval 4000 - 5000.

Therefore, modal class = 4000 - 5000

Lower limit (l) of modal class = 4000

Frequency (f_1) of modal class = 18

Frequency (f_0) of class preceding modal class = 4

Frequency (f_2) of class succeeding modal class = 9

Class size (h) = 1000

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9} \right) \times 1000 \\
 &= 4000 + \left(\frac{14000}{23} \right) \\
 &= 4000 + 608.695 \\
 &= 4608.695
 \end{aligned}$$

Therefore, mode of the given data is 4608.7 runs.

Q6 :

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

Answer :

From the given data, it can be observed that the maximum class frequency is 20, belonging to 40 - 50 class intervals.

Therefore, modal class = 40 - 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left[\frac{20 - 12}{2(20) - 12 - 11} \right] \times 10 \\ &= 40 + \left(\frac{80}{40 - 23} \right) \\ &= 40 + \frac{80}{17} \\ &= 40 + 4.7 \\ &= 44.7\end{aligned}$$

Therefore, mode of this data is 44.7 cars.

Exercise 14.3 : Solutions of Questions on Page Number : 287

Q1 :

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

Answer :

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 135 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated according to step deviation method as follows.

Monthly consumption (in units)	Number of consumers (f_i)	x_i class mark	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 - 85	4	75	- 60	- 3	- 12
85 - 105	5	95	- 40	- 2	- 10
105 - 125	13	115	- 20	- 1	- 13
125 - 145	20	135	0	0	0
145 - 165	14	155	20	1	14
165 - 185	8	175	40	2	16
185 - 205	4	195	60	3	12
Total	68				7

From the table, we obtain

$$\sum f_i u_i = 7$$

$$\sum f_i = 68$$

$$\text{Class size } (h) = 20$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 135 + \frac{7}{68} \times 20 \\ &= 135 + \frac{140}{68} \\ &= 137.058 \end{aligned}$$

From the table, it can be observed that the maximum class frequency is 20, belonging to class interval 125 - 145.

Modal class = 125 - 145

Lower limit (l) of modal class = 125

Class size (h) = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

Q2 :

If the median of the distribution is given below is 28.5, find the values of x and y .

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

Answer :

The cumulative frequency for the given data is calculated as follows.

Class interval	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	$5 + x$
20 - 30	20	$25 + x$
30 - 40	15	$40 + x$
40 - 50	y	$40 + x + y$
50 - 60	5	$45 + x + y$
Total (n)	60	

From the table, it can be observed that $n = 60$

$$45 + x + y = 60$$

$$x + y = 15 \quad (1)$$

Median of the data is given as 28.5 which lies in interval 20 - 30.

Therefore, median class = 20 - 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$
$$28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20} \right] \times 10$$

$$8.5 = \left(\frac{25 - x}{2} \right)$$

$$17 = 25 - x$$

$$x = 8$$

From equation (1),

$$8 + y = 15$$

$$y = 7$$

Hence, the values of x and y are 8 and 7 respectively.

Q3 :

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Answer :

Here, class width is not the same. There is no requirement of adjusting the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	Number of policy holders (f_i)	Cumulative frequency (cf)
18 - 20	2	2
20 - 25	$6 - 2 = 4$	6
25 - 30	$24 - 6 = 18$	24
30 - 35	$45 - 24 = 21$	45
35 - 40	$78 - 45 = 33$	78
40 - 45	$89 - 78 = 11$	89
45 - 50	$92 - 89 = 3$	92
50 - 55	$98 - 92 = 6$	98
55 - 60	$100 - 98 = 2$	100
Total (n)		

From the table, it can be observed that $n = 100$.

Cumulative frequency (cf) just greater than $\frac{n}{2}$ (i.e., $\frac{100}{2} = 50$) is 78, belonging to interval 35 - 40.

Therefore, median class = 35 - 40

Lower limit (l) of median class = 35

Class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 35 + \left(\frac{50 - 45}{33} \right) \times 5 \\
 &= 35 + \frac{25}{33} \\
 &= 35.76
 \end{aligned}$$

Therefore, median age is 35.76 years.

Q4 :

The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number or leaves f_i
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

Find the median length of the leaves.

(Hint: The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5... 171.5 - 180.5)

Answer :

The given data does not have continuous class intervals. It can be observed that the difference between two class

intervals is 1. Therefore, $\frac{1}{2} = 0.5$ has to be added and subtracted to upper class limits and lower class limits respectively.

Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	Number or leaves f_i	Cumulative frequency
----------------	------------------------	----------------------

117.5 - 126.5	3	3
126.5 - 135.5	5	3 + 5 = 8
135.5 - 144.5	9	8 + 9 = 17
144.5 - 153.5	12	17 + 12 = 29
153.5 - 162.5	5	29 + 5 = 34
162.5 - 171.5	4	34 + 4 = 38
171.5 - 180.5	2	38 + 2 = 40

$$\frac{n}{2} \left(\text{i.e., } \frac{40}{2} = 20 \right) \text{ is } 29,$$

From the table, it can be observed that the cumulative frequency just greater than belonging to class interval 144.5 - 153.5.

Median class = 144.5 - 153.5

Lower limit (l) of median class = 144.5

Class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 144.5 + \left(\frac{20 - 17}{12} \right) \times 9$$

$$= 144.5 + \frac{9}{4} = 146.75$$

Therefore, median length of leaves is 146.75 mm.

Q5 :

Find the following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60

3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp.

Answer :

The cumulative frequencies with their respective class intervals are as follows.

Life time	Number of lamps (f)	Cumulative frequency
1500 - 2000	14	14
2000 - 2500	56	$14 + 56 = 70$
2500 - 3000	60	$70 + 60 = 130$
3000 - 3500	86	$130 + 86 = 216$
3500 - 4000	74	$216 + 74 = 290$
4000 - 4500	62	$290 + 62 = 352$
4500 - 5000	48	$352 + 48 = 400$
Total (n)	400	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{400}{2} = 200$) is 216, belonging to class interval 3000 - 3500.

Median class = 3000 - 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 3000 + \left(\frac{200 - 130}{86} \right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3406.976$$

Therefore, median life time of lamps is 3406.98 hours.

Q6 :

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	6	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Answer :

The cumulative frequencies with their respective class intervals are as follows.

Number of letters	Frequency (f_i)	Cumulative frequency
1 - 4	6	6
4 - 7	30	$30 + 6 = 36$
7 - 10	40	$36 + 40 = 76$
10 - 13	16	$76 + 16 = 92$
13 - 16	4	$92 + 4 = 96$
16 - 19	4	$96 + 4 = 100$
Total (n)	100	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{100}{2} = 50$) is 76, belonging to class interval 7 - 10.

Median class = 7 - 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 7 + \left(\frac{50 - 36}{40} \right) \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

= 8.05

To find the class marks of the given class intervals, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 11.5 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated according to step deviation method as follows.

Number of letters	Number of surnames f_i	x_i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
1 - 4	6	2.5	- 9	- 3	- 18
4 - 7	30	5.5	- 6	- 2	- 60
7 - 10	40	8.5	- 3	- 1	- 40
10 - 13	16				

Q7 :

The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
Number of students	2	3	8	6	6	3	2

Answer :

The cumulative frequencies with their respective class intervals are as follows.

Weight (in kg)	Frequency (f_i)	Cumulative frequency
40 - 45	2	2
45 - 50	3	2 + 3 = 5

50 - 55	8	$5 + 8 = 13$
55 - 60	6	$13 + 6 = 19$
60 - 65	6	$19 + 6 = 25$
65 - 70	3	$25 + 3 = 28$
70 - 75	2	$28 + 2 = 30$
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{30}{2} = 15$) is 19, belonging to class interval 55 - 60.

Median class = 55 - 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) of median class = 13

Class size (h) = 5

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 55 + \left(\frac{15 - 13}{6} \right) \times 5$$

$$= 55 + \frac{10}{6}$$

= 56.67

Therefore, median weight is 56.67 kg.

Exercise 14.4 : Solutions of Questions on Page Number : 293

Q1 :

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

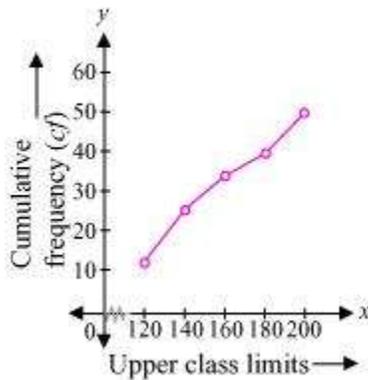
Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Answer :

The frequency distribution table of less than type is as follows.

Daily income (in Rs) (upper class limits)	Cumulative frequency
Less than 120	12
Less than 140	$12 + 14 = 26$
Less than 160	$26 + 8 = 34$
Less than 180	$34 + 6 = 40$
Less than 200	$40 + 10 = 50$

Taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis, its ogive can be drawn as follows.



Q2 :

During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32

Less than 52	35
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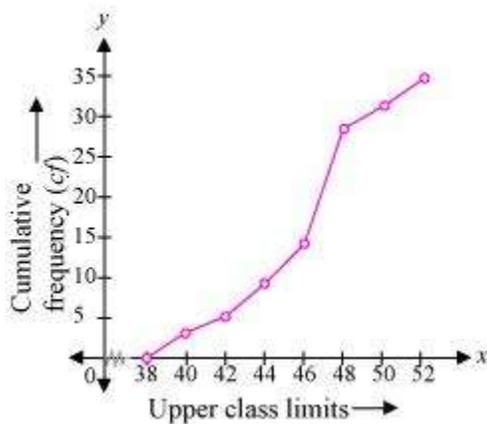
Draw a less than type ogive for the given data. Hence obtain the median weight from the graph verify the result by using the formula.

Answer :

The given cumulative frequency distributions of less than type are

Weight (in kg) upper class limits	Number of students (cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

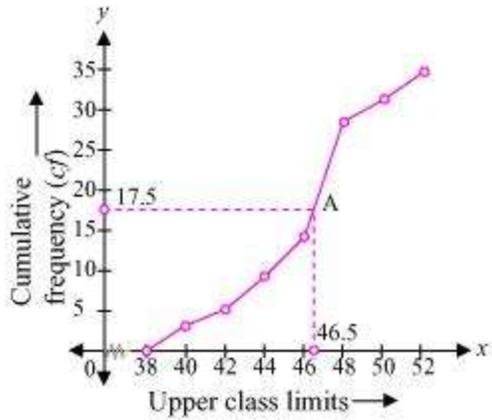
Taking upper class limits on x-axis and their respective cumulative frequencies on y-axis, its ogive can be drawn as follows.



Here, $n = 35$

$$\text{So, } \frac{n}{2} = 17.5$$

Mark the point A whose ordinate is 17.5 and its x-coordinate is 46.5. Therefore, median of this data is 46.5.



It can be observed that the difference between two consecutive upper class limits is 2. The class marks with their respective frequencies are obtained as below.

Weight (in kg)	Frequency (f)	Cumulative frequency
Less than 38	0	0
38 - 40	$3 - 0 = 3$	3
40 - 42	$5 - 3 = 2$	5
42 - 44	$9 - 5 = 4$	9
44 - 46	$14 - 9 = 5$	14
46 - 48	$28 - 14 = 14$	28
48 - 50	$32 - 28 = 4$	32
50 - 52	$35 - 32 = 3$	35
Total (n)	35	

$$\frac{n}{2} \left(\text{i.e., } \frac{35}{2} = 17.5 \right)$$

The cumulative frequency just greater than $\frac{n}{2}$ is 28, belonging to class interval 46 - 48.

Median class = 46 - 48

Lower class limit (l) of median class = 46

Frequency (f) of median class = 14

Cumulative frequency (cf) of class preceding median class = 14

Class size (h) = 2

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2 \\
 &= 46 + \frac{3.5}{7} \\
 &= 46.5
 \end{aligned}$$

Q3 :

The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

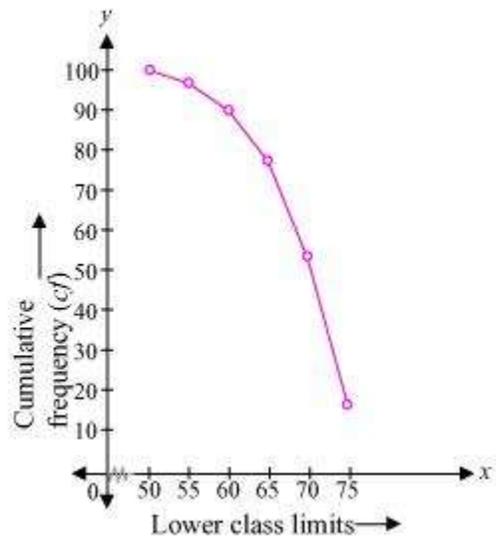
Change the distribution to a more than type distribution and draw ogive.

Answer :

The cumulative frequency distribution of more than type can be obtained as follows.

Production yield (lower class limits)	Cumulative frequency
more than or equal to 50	100
more than or equal to 55	100 - 2 = 98
more than or equal to 60	98 - 8 = 90
more than or equal to 65	90 - 12 = 78
more than or equal to 70	78 - 24 = 54
more than or equal to 75	54 - 38 = 16

Taking the lower class limits on x-axis and their respective cumulative frequencies on y-axis, its ogive can be obtained as follows.



DELHI PUBLIC SCHOOL – GANDHINAGAR

CHAPTER 14 STATISTICS

MIND MAP

This chapter consists of four different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 MEAN

Q.1. Find the mean of the following data by direct method.

Class-interval	100-150	150-200	200-250	250-300	300-350
frequency	4	5	12	2	2

Q.2. Find the mean in question 1 by assumed mean method.

Q.3. Find the mean in question 1 by step deviation method.

Q.4. Find the missing frequency of the following data if the mean pocket allowance is rupees 18.

Daily allowance	11-13	13-15	15-17	17-19	19-21	21-23	23-25
No. of children	7	6	9	13	f	5	4

TYPE: 2 MEDIAN

Q.1. If the median of the following data is 28.5, find the values of x and y.

Class-interval	0 - 10	10-20	20-30	30- 40	40- 50	50 -60	Total
frequency	5	x	20	15	y	5	60

Q.2. Find the median of the following data:

Class-interval	100-150	150-200	200-250	250-300	300-350
frequency	4	5	12	2	2

Q.3. Find the median of the following data:

Age (years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No. of policy holders	2	6	24	45	78	89	92	98	100

Q.4. Find the median of the following data:

Length(mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves	3	5	9	12	5	4	2

TYPE: 3 MODE

Q.1. Find the mode of the following data:

Class-interval	100-150	150-200	200-250	250-300	300-350
frequency	4	5	12	2	2

TYPE: 4 OGIVE

Q.1. Convert the following distribution to a less than type cumulative frequency distribution and draw its ogive. Also, find out median graphically.

Class-interval	100-120	120-140	140-150	150-160	160-170
frequency	4	5	12	2	2

Q.2. Convert the following distribution to a more than type cumulative frequency distribution and draw its ogive. Find out median graphically and also verify it using formula.

Class-interval	100-120	120-140	140-150	150-160	160-170
frequency	4	5	12	2	2

Ch.1 Real Numbers

Exercise 1.1 :

Q1 :

Use Euclid's division algorithm to find the HCF of:

- (i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer :

(i) 135 and 225

Since $225 > 135$, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since $38220 > 196$, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since $867 > 255$, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$$102 = 51 \times 2 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,
Therefore, HCF of 867 and 255 is 51.

Q2 :

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Answer :

Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,
 $a = 6q + r$ for some integer q is not 0, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1, 6q + 3, 6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1, 6q + 3, 6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,
or $6q + 5$

Q3 :

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer :

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Q4 :

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m .

[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer :

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$\begin{aligned} a^2 &= (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2 \\ a^2 &= (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4 \\ &= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

Where k_1 , k_2 , and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Q5 :

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer :

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$$\therefore a = 3q \text{ or } 3q + 1 \text{ or } 3q + 2$$

Therefore, every number can be represented as these three forms. There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$$a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9(3q^3 + 3q^2 + q) + 1$$

$$a^3 = 9m + 1$$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8$$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m, 9m + 1$,
or $9m + 8$.

Exercise 1.2 :

Q1 :

Express each number as product of its prime factors:

- (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer :

(i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) $5005 = 5 \times 7 \times 11 \times 13$

(v) $7429 = 17 \times 19 \times 23$

Q2 :

Find the LCM and HCF of the following pairs of integers and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$.

- (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer :

(i) 26 and 91
 $26 = 2 \times 13$
 $91 = 7 \times 13$
HCF = 13
 $LCM = 2 \times 7 \times 13 = 182$
Product of the two numbers = $26 \times 91 = 2366$
 $HCF \times LCM = 13 \times 182 = 2366$

Hence, product of two numbers = HCF \times LCM

(ii) 510 and 92
 $510 = 2 \times 3 \times 5 \times 17$
 $92 = 2 \times 2 \times 23$
HCF = 2
 $LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
Product of the two numbers = $510 \times 92 = 46920$
 $HCF \times LCM = 2 \times 23460$
 $= 46920$

Hence, product of two numbers = HCF \times LCM

(iii) 336 and 54
 $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
 $336 = 2^4 \times 3 \times 7$
 $54 = 2 \times 3 \times 3 \times 3$
 $54 = 2 \times 3^3$
HCF = $2 \times 3 = 6$
 $LCM = 2^4 \times 3^3 \times 7 = 3024$
Product of the two numbers = $336 \times 54 = 18144$
 $HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF \times LCM

Q3 :

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer :

(i) 12, 15 and 21
 $12 = 2^2 \times 3$
 $15 = 3 \times 5$
 $21 = 3 \times 7$
HCF = 3
 $LCM = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29
 $17 = 1 \times 17$
 $23 = 1 \times 23$
 $29 = 1 \times 29$
HCF = 1
 $LCM = 17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25
 $8 = 2 \times 2 \times 2$
 $9 = 3 \times 3$
 $25 = 5 \times 5$
HCF = 1
 $LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Q4 :

Given that $HCF(306, 657) = 9$, find $LCM(306, 657)$.

Answer :

$$HCF(306, 657) = 9$$

We know that, $LCM \times HCF = \text{Product of two numbers}$

$$\therefore LCM \times HCF = 306 \times 657$$

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

$$LCM = 22338$$

Q5 :

Check whether 6^n can end with the digit 0 for any natural number n .

Answer :

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n .

Hence, for any value of n , 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n .

Q6 :

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer :

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$$

$$= 13 \times 78$$

$$= 13 \times 13 \times 6$$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

$$= 5 \times (1008 + 1)$$

$$= 5 \times 1009$$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Q7 :

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer :

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the

LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$$18 = 2 \times 3 \times 3$$

And, $12 = 2 \times 2 \times 3$

$$\text{LCM of 12 and 18} = 2 \times 2 \times 3 \times 3 = 36$$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

[]

Exercise 1.3 :

Q1 :

Prove that $\sqrt{5}$ is irrational.

Answer :

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$$(5k)^2 = 5b^2 \quad \text{This means that } b^2 \text{ is divisible by 5 and hence, } b \text{ is divisible by 5.}$$

$$b^2 = 5k^2 \quad \text{This implies that } a \text{ and } b \text{ have 5 as a common factor.}$$

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Q2 :

Prove that $3 + 2\sqrt{5}$ is irrational.

Answer :

Let $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2} \left(\frac{a}{b} - 3 \right)$$

Since a and b are integers, $\frac{1}{2} \left(\frac{a}{b} - 3 \right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false.

Therefore, $3 + 2\sqrt{5}$ is irrational.

Q3 :

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Answer :

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b} \text{ for some integers } a \text{ and } b$$

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that

$\sqrt{2}$ is irrational. Therefore, our assumption is false and hence,

[]

Exercise 1.4 :

Q1 :

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$	(ii) $\frac{17}{8}$	(iii) $\frac{64}{455}$	(iv) $\frac{15}{1600}$
(v) $\frac{29}{343}$	(vi) $\frac{23}{2^3 5^2}$	(vii) $\frac{129}{2^2 5^7 7^5}$	(viii) $\frac{6}{15}$
(ix) $\frac{35}{50}$	(x) $\frac{77}{210}$		

Answer :

(i) $\frac{13}{3125}$

$$3125 = 5^5$$

The denominator is of the form 5^n .

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$$8 = 2^3$$

The denominator is of the form 2^m .

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$$455 = 5 \times 7 \times 13$$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$$1600 = 2^6 \times 5^2$$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$$343 = 7^3$$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

$$(vi) \frac{23}{2^3 \times 5^2}$$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

$$(vii) \frac{129}{2^2 \times 5^7 \times 7^5}$$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion

of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

$$(viii) \frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$$

The denominator is of the form 5^n .

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

$$(ix) \frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$$

$$10 = 2 \times 5$$

The denominator is of the form $2^m \times 5^n$.

Q2 :

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer :

(i) $\frac{13}{3125} = 0.00416$

$$\begin{array}{r}
 3125 \overline{)13.00000} \\
 \underline{0} \\
 130 \\
 \underline{0} \\
 1300 \\
 \underline{0} \\
 13000 \\
 \underline{12500} \\
 5000 \\
 \underline{3125} \\
 18750 \\
 \underline{18750} \\
 \times
 \end{array}$$

(ii) $\frac{17}{8} = 2.125$

$$\begin{array}{r}
 8 \overline{)17} \\
 \underline{16} \\
 10 \\
 \underline{8} \\
 20 \\
 \underline{16} \\
 40 \\
 \underline{40} \\
 \times
 \end{array}$$

$$(iv) \quad \frac{15}{1600} = 0.009375$$

$$\begin{array}{r} 0.009375 \\ 1600 \overline{)15.000000} \\ \underline{0} \\ 150 \\ \underline{0} \\ 1500 \\ \underline{0} \\ 15000 \\ \underline{14400} \\ 6000 \\ \underline{4800} \\ 12000 \\ \underline{11200} \\ 8000 \\ \underline{8000} \\ \times \end{array}$$

$$(vi) \quad \frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$$

$$\begin{array}{r} 0.115 \\ 200 \overline{)23.000} \\ \underline{0} \\ 230 \\ \underline{200} \\ 300 \\ \underline{200} \\ 1000 \\ \underline{1000} \\ \times \end{array}$$

$$(viii) \quad \frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$$

$$\begin{array}{r} 0.4 \\ 5 \overline{)2.0} \\ \underline{0} \\ 20 \\ \underline{20} \\ \times \end{array}$$

$$(ix) \quad \frac{35}{50} = 0.7$$

$$\begin{array}{r} 0.7 \\ 50 \overline{)35.0} \\ \underline{0} \\ 350 \\ \underline{350} \\ \times \end{array}$$

Q3 :

The following real numbers have decimal expansions as given below. In each case, decide whether they are

rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q ?

(i) 43.123456789 (ii) 0.120120012000120000... (iii) $\overline{43.123456789}$

Answer :

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 ...

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $\overline{43.123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.

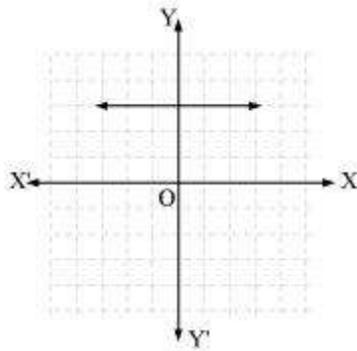
Ch.2 POLYNOMIALS

Exercise 2.1

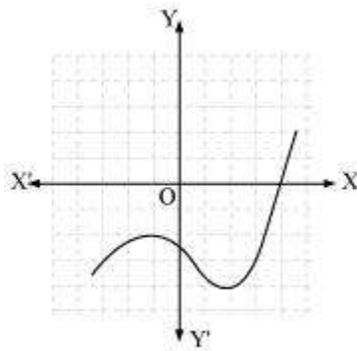
Q1 :

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

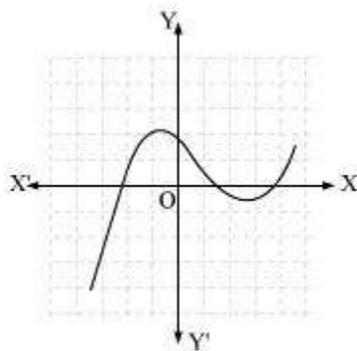
(i)



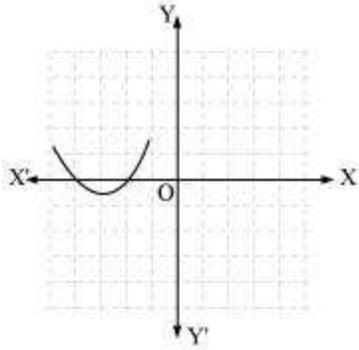
(ii)



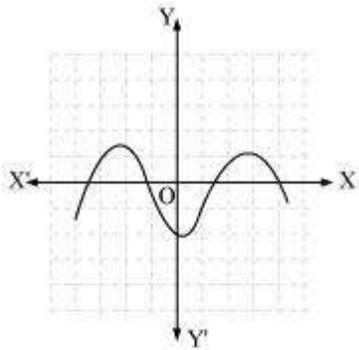
(iii)



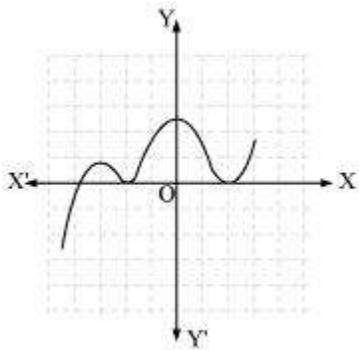
(iv)



(v)



(v)



Answer :

- (i) The number of zeroes is 0 as the graph does not cut the x -axis at any point.
- (ii) The number of zeroes is 1 as the graph intersects the x -axis at only 1 point.
- (iii) The number of zeroes is 3 as the graph intersects the x -axis at 3 points.
- (iv) The number of zeroes is 2 as the graph intersects the x -axis at 2 points.
- (v) The number of zeroes is 4 as the graph intersects the x -axis at 4 points.
- (vi) The number of zeroes is 3 as the graph intersects the x -axis at 3 points.

Exercise 2.2

Q1 :

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Answer :

(i) $x^2 - 2x - 8 = (x - 4)(x + 2)$

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

$$\text{Sum of zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\begin{aligned} \text{(iv)} \quad 4u^2 + 8u &= 4u^2 + 8u + 0 \\ &= 4u(u + 2) \end{aligned}$$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

$$\begin{aligned} \text{(v)} \quad t^2 - 15 \\ &= t^2 - 0t - 15 \\ &= (t - \sqrt{15})(t + \sqrt{15}) \end{aligned}$$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$$\text{(i)} \quad \frac{1}{4}, -1 \quad \text{(ii)} \quad \sqrt{2}, \frac{1}{3} \quad \text{(iii)} \quad 0, \sqrt{5}$$

$$\text{(iv)} \quad 1, 1 \quad \text{(v)} \quad -\frac{1}{4}, \frac{1}{4} \quad \text{(vi)} \quad 4, 1$$

Answer :

$$\text{(i)} \quad \frac{1}{4}, -1$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

$$(ii) \quad \sqrt{2}, \frac{1}{3}$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

$$(iii) \quad 0, \sqrt{5}$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

$$(iv) \quad 1, 1$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -1$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

$$(v) \quad -\frac{1}{4}, \frac{1}{4}$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and

Exercise 2.3

Q1 :

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$
 $q(x) = x^2 - 2$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3 \quad -2x} \\ -3x^2+7x-3 \\ \underline{-3x^2 \quad +6} \\ + \quad - \\ \hline 7x-9 \end{array}$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$
 $q(x) = x^2 + 1 - x = x^2 - x + 1$

$$\begin{array}{r}
 x^2 + x - 3 \\
 x^2 - x + 1 \overline{) x^4 + 0x^3 - 3x^2 + 4x + 5} \\
 \underline{x^4 - x^3 + x^2} \\
 x^3 - 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 -3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x - 3} \\
 8
 \end{array}$$

Quotient = $x^2 + x - 3$

Remainder = 8

$$\begin{aligned}
 \text{(iii)} \quad p(x) &= x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6 \\
 q(x) &= 2 - x^2 = -x^2 + 2
 \end{aligned}$$

$$\begin{array}{r}
 -x^2 - 2 \\
 -x^2 + 2 \overline{) x^4 + 0x^2 - 5x + 6} \\
 \underline{x^4 - 2x^2} \\
 2x^2 - 5x + 6 \\
 \underline{2x^2 - 4} \\
 -5x + 10
 \end{array}$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

$$\text{(i)} \quad 2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$$

$$\text{(ii)} \quad x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$$

Answer :

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 \\ &= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 5 \times 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2 \\ &= -16 + 4 + 10 + 2 = 0 \end{aligned}$$

Therefore, $\frac{1}{2}$, 1, and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 2, b = 1, c = -5, d = 2$

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(-2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$\begin{aligned} p(2) &= 2^3 - 4(2^2) + 5(2) - 2 \\ &= 8 - 16 + 10 - 2 = 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 1^3 - 4(1)^2 + 5(1) - 2 \\ &= 1 - 4 + 5 - 2 = 0 \end{aligned}$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r}
 \overline{3x^2 - 4x + 2} \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 \overline{x^2 - 1} \\
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \\
 -x^3 + 3x + 1 \\
 \underline{-x^3 + 3x - 1} \\
 2
 \end{array}$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q4 :

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β , and γ .

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q5 :

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) \text{ is a factor of } 3x^4 + 6x^3 - 2x^2 - 10x - 5.$$

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\text{Remainder} = (-2x + 4)$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$\begin{array}{r} x^2 - x + 1 \\ x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\ \underline{x^3 - 2x^2} \\ -x^2 + 3x - 2 \\ \underline{-x^2 + 2x} \\ +x - 2 \\ \underline{+x - 2} \\ 0 \end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

Q7 :

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Answer :

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x) \times q(x) + r(x),$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

$$(i) \deg p(x) = \deg q(x)$$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

$$\text{Here, } p(x) = 6x^2 + 2x + 2$$

$$g(x) = 2$$

$$q(x) = 3x^2 + x + 1 \text{ and } r(x) = 0$$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$\begin{aligned} 6x^2 + 2x + 2 &= 2(3x^2 + x + 1) \\ &= 6x^2 + 2x + 2 \end{aligned}$$

Thus, the division algorithm is satisfied.

$$(ii) \deg q(x) = \deg r(x)$$

Let us assume the division of $x^3 + x$ by x^2 ,

$$\text{Here, } p(x) = x^3 + x$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = x$$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$\begin{aligned} x^3 + x &= (x^2) \times x + x \\ x^3 + x &= x^3 + x \end{aligned}$$

Thus, the division algorithm is satisfied.

$$(iii) \deg r(x) = 0$$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2 .

$$\text{Here, } p(x) = x^3 + 1$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = 1$$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + 1 = (x^2) \times x + 1$$

$$x^3 + 1 = x^3 + 1$$

Thus, the division algorithm is satisfied.

Exercise 2.4

Q1 :

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b .

Answer :

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are $a - b, a, a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$$p = 1, q = -3, r = 1, t = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1 - b, 1, 1 + b$.

$$\text{Multiplication of zeroes} = 1(1 - b)(1 + b)$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Q2 :

It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer :

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r} x^2 - 2x - 35 \\ x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\ \underline{x^4 - 4x^3 + \quad x^2} \\ -2x^3 - 27x^2 + 138x - 35 \\ \underline{-2x^3 + 8x^2 - 2x} \\ +35x^2 + 140x - 35 \\ \underline{-35x^2 + 140x - 35} \\ 0 \end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

$$-5 - a = 0$$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

NCERT Solutions for Class 10 Maths Unit 3

Pair of Linear Equations in Two Variables Class 10

Unit 3 Pair of Linear Equations in Two Variables Exercise 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7 Solutions

If two linear equations have the two same variables, they are called a pair of linear equations in two variables. Following is the most general form of linear equations:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Here, a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers such that;

$$a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$$

A pair of linear equations can be represented and solved by the following methods:

- a. Graphical method
- b. Algebraic method

Exercise 3.1 : Solutions of Questions on Page Number : 44

Q1 :

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Answer :

Let the present age of Aftab be x .

And, present age of his daughter = y

Seven years ago,

Age of Aftab = $x - 7$

Age of his daughter = $y - 7$

According to the question,

$$(x - 7) = 7(y - 7)$$

$$x - 7 = 7y - 49$$

$$x - 7y = -42 \quad (1)$$

Three years hence,

Age of Aftab = $x + 3$

Age of his daughter = $y + 3$

According to the question,

$$(x+3) = 3(y+3)$$

$$x+3 = 3y+9$$

$$x-3y = 6 \quad (2)$$

Therefore, the algebraic representation is

$$x-7y = -42$$

$$x-3y = 6$$

For $x-7y = -42$,

$$x = -42 + 7y$$

The solution table is

x	-7	0	7
y	5	6	7

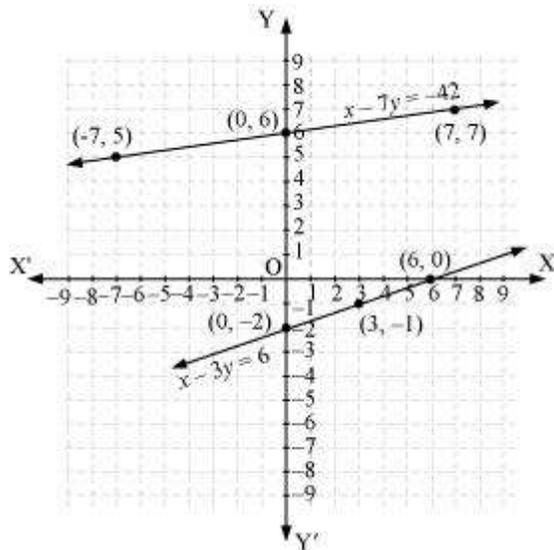
For $x-3y = 6$,

$$x = 6 + 3y$$

The solution table is

x	6	3	0
y	0	-1	-2

The graphical representation is as follows.



Q2 :

The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer :

Let the cost of a bat be Rs x .

And, cost of a ball = Rs y

According to the question, the algebraic representation is

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For $3x + 6y = 3900$,

$$x = \frac{3900 - 6y}{3}$$

The solution table is

x	300	100	-100
y	500	600	700

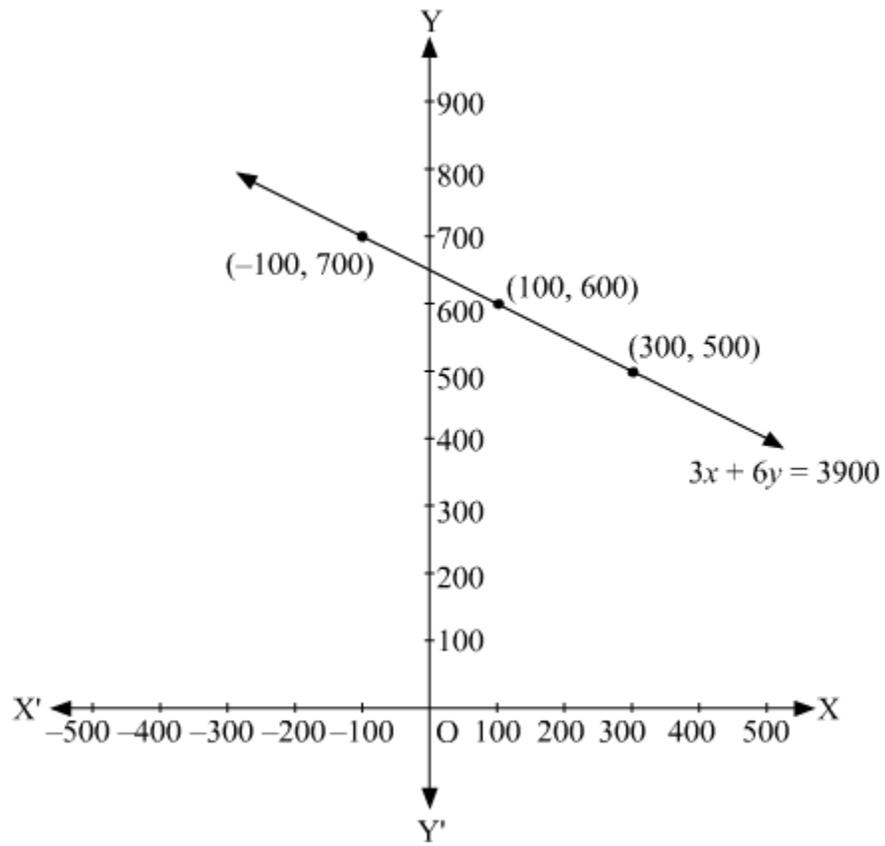
For $x + 3y = 1300$,

$$x = 1300 - 3y$$

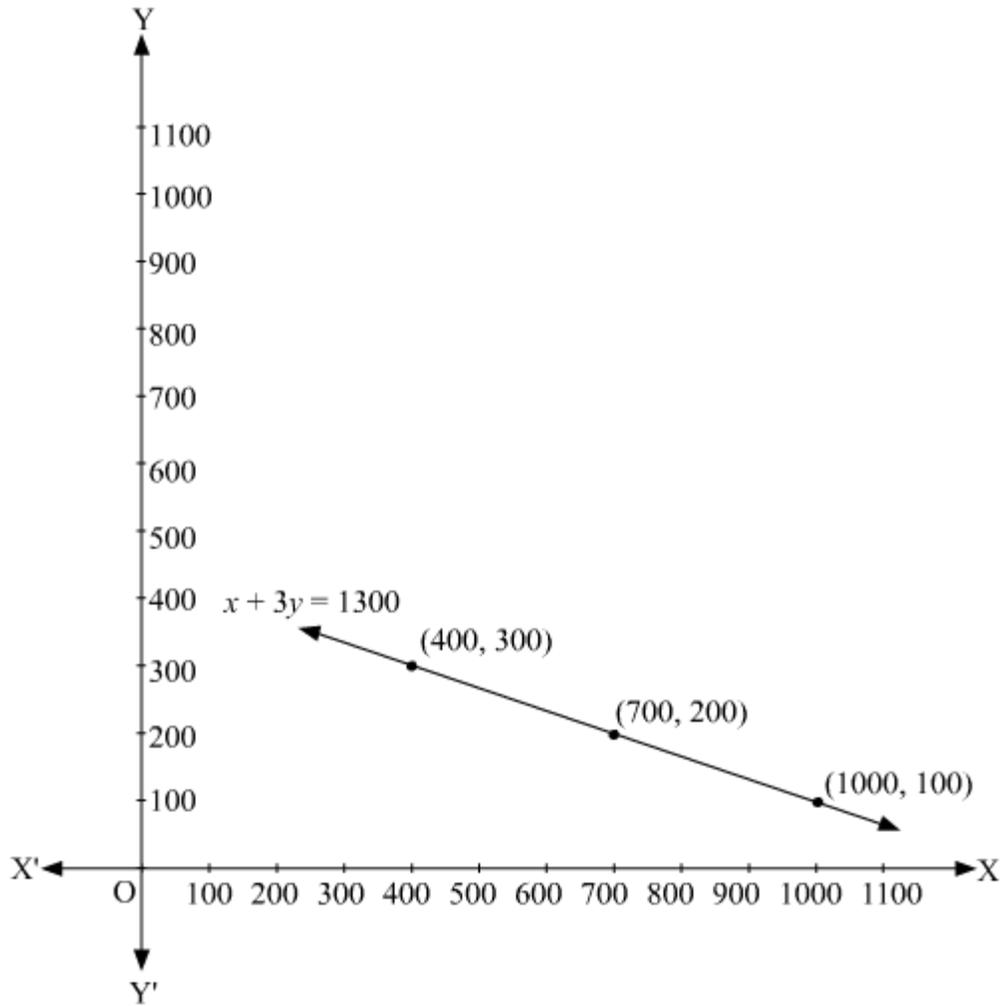
The solution table is

x	400	700	1000
y	300	200	100

The graphical representation for first line is as follows.



And graph for second line will be,



Q3 :

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer :

Let the cost of 1 kg of apples be Rs x .

And, cost of 1 kg of grapes = Rs y

According to the question, the algebraic representation is

$$2x + y = 160$$

$$4x + 2y = 300$$

For $2x + y = 160$,

$$y = 160 - 2x$$

The solution table is

x	50	60	70
y	60	40	20

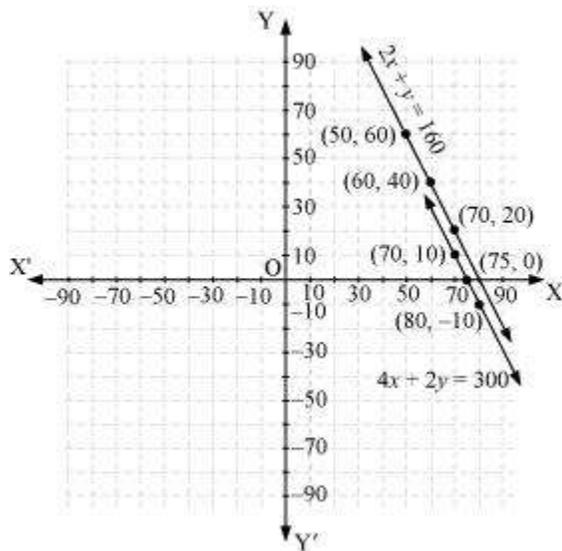
For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	- 10	0

The graphical representation is as follows.



Exercise 3.2 : Solutions of Questions on Page Number : 49

Q1 :

Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer :

(i) Let the number of girls be x and the number of boys be y .

According to the question, the algebraic representation is

$$x + y = 10$$

$$x - y = 4$$

For $x + y = 10$,

$$x = 10 - y$$

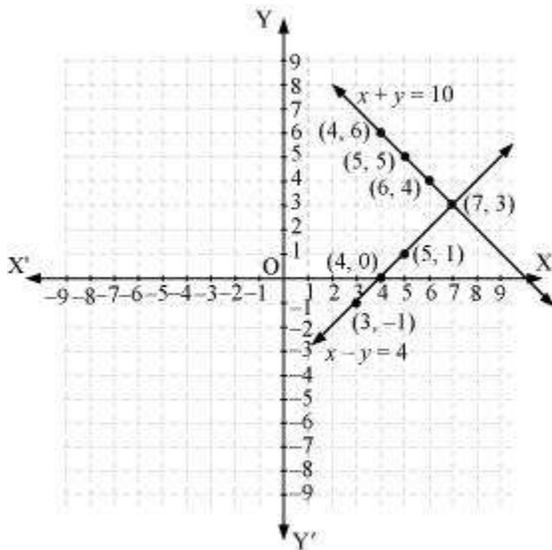
x	5	4	6
y	5	6	4

For $x - y = 4$,

$$x = 4 + y$$

x	5	4	3
y	1	0	-1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y .

According to the question, the algebraic representation is

$$5x + 7y = 50$$

$$7x + 5y = 46$$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

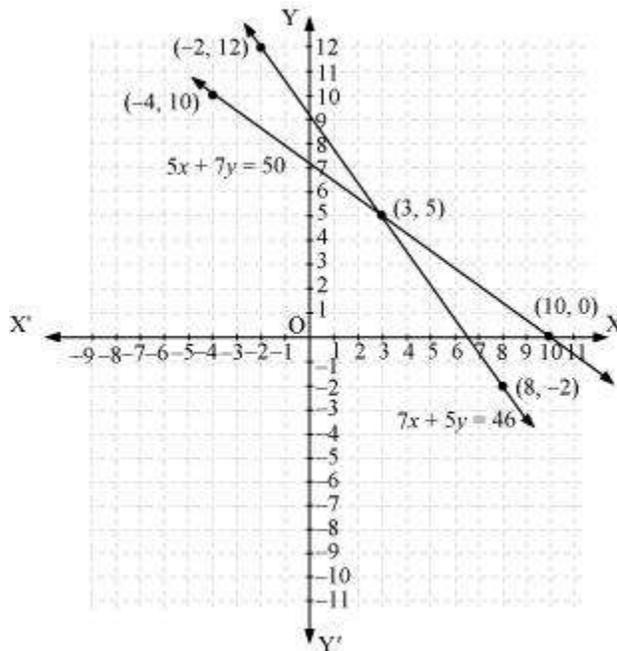
x	3	10	-4
y	5	0	10

$$7x + 5y = 46$$

$$x = \frac{46 - 5y}{7}$$

x	8	3	-2
y	-2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Q2 :

$$\frac{a_1}{a_2}, \frac{b_1}{b_2} \text{ and } \frac{c_1}{c_2}$$

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

- (i) $5x - 4y + 8 = 0$ (ii) $9x + 3y + 12 = 0$ (iii) $6x - 3y + 10 = 0$
 $7x + 6y - 9 = 0$ $18x + 6y + 24 = 0$ $2x - y + 9 = 0$

Answer :

$$(i) 5x - 4y + 8 = 0$$

$$7x + 6y - 9 = 0$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$$a_1 = 5, \quad b_1 = -4, \quad c_1 = 8$$

$$a_2 = 7, \quad b_2 = 6, \quad c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

$$(ii) 9x + 3y + 12 = 0$$

$$18x + 6y + 24 = 0$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$$

$$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

$$(iii) 6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$$

$$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$$

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Q3 :

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

$$(i) \quad 3x + 2y = 5; \quad 2x - 3y = 7 \quad (ii) \quad 2x - 3y = 8; \quad 4x - 6y = 9$$

$$(iii) \quad \frac{3}{2}x + \frac{5}{3}y = 7; \quad 9x - 10y = 14 \quad (iv) \quad 5x - 3y = 11; \quad -10x + 6y = -22$$

$$(v) \quad \frac{4}{3}x + 2y = 8; \quad 2x + 3y = 12$$

Answer :

$$(i) \quad 3x + 2y = 5$$

$$2x - 3y = 7$$

$$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$$(ii) \quad 2x - 3y = 8$$

$$4x - 6y = 9$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{8}{9}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2},$$

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

$$(iii) \quad \frac{3}{2}x + \frac{5}{3}y = 7$$

$$9x - 10y = 14$$

$$\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

$$\text{Since } \frac{a_1}{a_2} \neq \frac{b_1}{b_2},$$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$$(iv) 5x - 3y = 11$$

$$-10x + 6y = -22$$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$$(v) \quad \frac{4}{3}x + 2y = 8$$

$$2x + 3y = 12$$

$$\frac{a_1}{a_2} = \frac{4}{2} = \frac{2}{1}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

Q4 :

Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

- (i) $x + y = 5, \quad 2x + 2y = 10$
- (ii) $x - y = 8, \quad 3x - 3y = 16$
- (iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$
- (iv) $2x - 2y - 2 = 0, \quad 4x - 4y - 5 = 0$

Answer :

(i) $x + y = 5$

$2x + 2y = 10$

$$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$

$x = 5 - y$

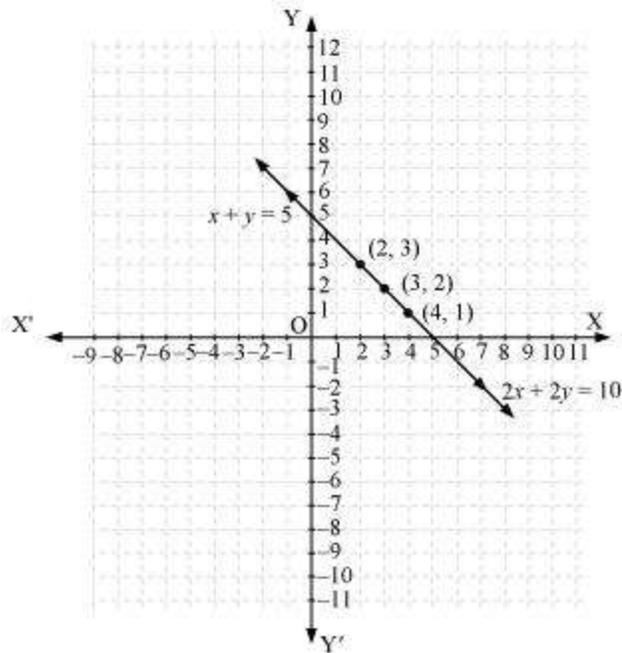
x	4	3	2
y	1	2	3

And, $2x + 2y = 10$

$$x = \frac{10 - 2y}{2}$$

x	4	3	2
y	1	2	3

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.

(ii) $x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

x	0	1	2
y	6	4	2

And $4x - 2y - 4 = 0$

$$y = \frac{4x - 4}{2}$$

x	1
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Q5 :

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer :

Let the width of the garden be x and length be y .

According to the question,

$$y - x = 4 \quad (1)$$

$$y + x = 36 \quad (2)$$

$$y - x = 4$$

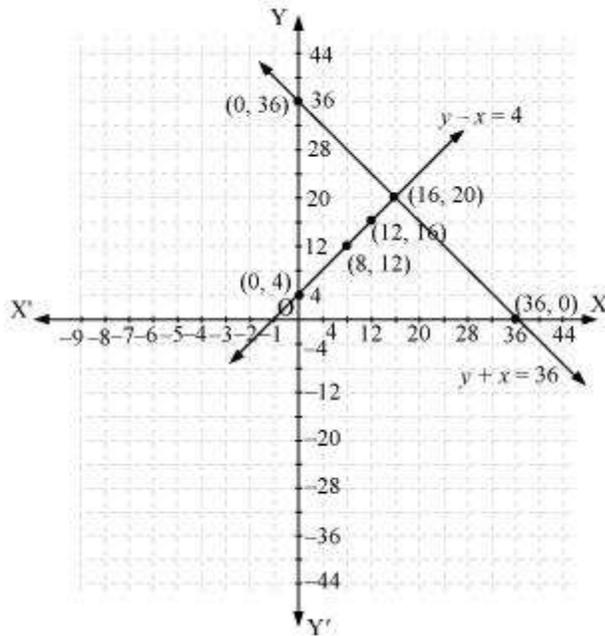
$$y = x + 4$$

x	0	8	12
y	4	12	16

$$y + x = 36$$

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Q6 :

Given the linear equation $2x + 3y - 8 = 0$, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

- (i) intersecting lines (ii) parallel lines**
- (iii) coincident lines**

Answer :

(i) Intersecting lines:

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line

is $2x + 4y - 6 = 0$ as $\frac{a_1}{a_2} = \frac{2}{2} = 1$, $\frac{b_1}{b_2} = \frac{3}{4}$ and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) Parallel lines:

For this condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

$$4x + 6y - 8 = 0$$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-8}{-8} = 1$$

$$\text{And clearly, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(iii) Coincident lines:

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the second line can be

$$6x + 9y - 24 = 0$$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$$

$$\text{And clearly, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Q7 :

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x -axis, and shade the triangular region.

Answer :

$$x - y + 1 = 0$$

$$x = y - 1$$

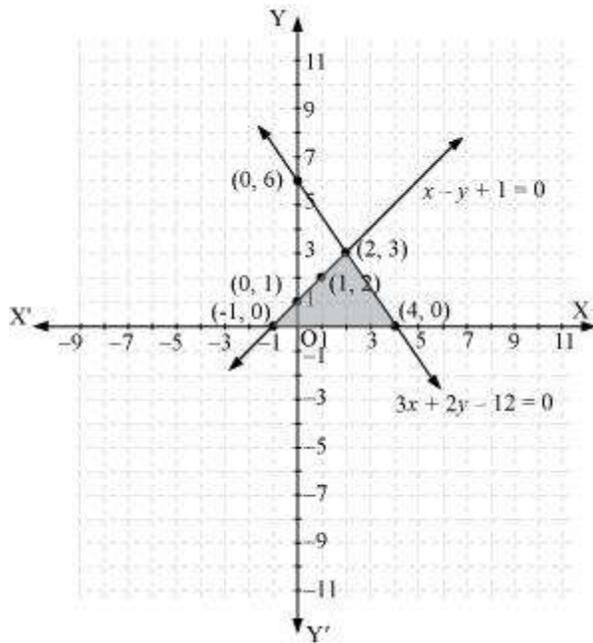
x	0	1	2
y	1	2	3

$$3x + 2y - 12 = 0$$

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and x-axis at (-1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (-1, 0), and (4, 0).

Exercise 3.3 : Solutions of Questions on Page Number : 53

Q1 :

Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(iii) $3x - y = 3$

$9x - 3y = 9$

(v) $\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

(ii) $s - t = 3$

$\frac{s}{3} + \frac{t}{2} = 6$

(iv) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Answer :

(i) $x + y = 14$ (1)

$x - y = 4$ (2)

From (1), we obtain

$x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$$\begin{aligned}(14 - y) - y &= 4 \\ 14 - 2y &= 4 \\ 10 &= 2y \\ y &= 5 \quad (4)\end{aligned}$$

Substituting this in equation (3), we obtain

$$\begin{aligned}x &= 9 \\ \therefore x = 9, y = 5\end{aligned}$$

$$(ii) \quad s - t = 3 \quad (1)$$

$$\frac{s}{3} + \frac{t}{2} = 6 \quad (2)$$

From (1), we obtain

$$s = t + 3 \quad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned}\frac{t+3}{3} + \frac{t}{2} &= 6 \\ 2t + 6 + 3t &= 36 \\ 5t &= 30 \\ t &= 6 \quad (4)\end{aligned}$$

Substituting in equation (3), we obtain

$$\begin{aligned}s &= 9 \\ \therefore s = 9, t = 6\end{aligned}$$

$$(iii) \quad 3x - y = 3 \quad (1)$$

$$9x - 3y = 9 \quad (2)$$

From (1), we obtain

$$y = 3x - 3 \quad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned}9x - 3(3x - 3) &= 9 \\ 9x - 9x + 9 &= 9\end{aligned}$$

$$9 = 9$$

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by

$$y = 3x - 3$$

Therefore, one of its possible solutions is $x = 1, y = 0$.

$$(iv) \quad 0.2x + 0.3y = 1.3 \quad (1)$$

$$0.4x + 0.5y = 2.3 \quad (2)$$

From equation (1), we obtain

$$x = \frac{1.3 - 0.3y}{0.2} \quad (3)$$

Substituting this value in equation (2), we obtain

$$0.4 \left(\frac{1.3 - 0.3y}{0.2} \right) + 0.5y = 2.3$$

$$2.6 - 0.6y + 0.5y = 2.3$$

$$2.6 - 2.3 = 0.1y$$

$$0.3 = 0.1y$$

$$y = 3 \quad (4)$$

Substituting this value in equation (3), we obtain

$$\begin{aligned} x &= \frac{1.3 - 0.3 \times 3}{0.2} \\ &= \frac{1.3 - 0.9}{0.2} = \frac{0.4}{0.2} = 2 \\ \therefore x &= 2, y = 3 \end{aligned}$$

$$(v) \quad \sqrt{2}x + \sqrt{3}y = 0 \quad (1)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \quad (2)$$

From equation (1), we obtain

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \quad (3)$$

Substituting this value in equation (2), we obtain

$$\sqrt{3} \left(-\frac{\sqrt{3}y}{\sqrt{2}} \right) - \sqrt{8}y = 0$$

$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y \left(-\frac{3}{\sqrt{2}} - 2\sqrt{2} \right) = 0$$

$$y = 0 \quad (4)$$

Substituting this value in equation (3), we obtain

x

Q2 :

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ' m ' for which $y = mx + 3$.

Answer :

$$2x + 3y = 11 \quad (1)$$

$$2x - 4y = -24 \quad (2)$$

From equation (1), we obtain

$$x = \frac{11 - 3y}{2} \quad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11 - 3y}{2}\right) - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5 \quad (4)$$

Putting this value in equation (3), we obtain

$$x = \frac{11 - 3 \times 5}{2} = -\frac{4}{2} = -2$$

Hence, $x = -2$, $y = 5$

Also,

$$y = mx + 3$$

$$5 = -2m + 3$$

$$-2m = 2$$

$$m = -1$$

Q3 :

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ' m ' for which $y = mx + 3$.

Answer :

$$2x + 3y = 11 \quad (1)$$

$$2x - 4y = -24 \quad (2)$$

From equation (1), we obtain

$$x = \frac{11 - 3y}{2} \quad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11 - 3y}{2}\right) - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5 \quad (4)$$

Putting this value in equation (3), we obtain

$$x = \frac{11 - 3 \times 5}{2} = -\frac{4}{2} = -2$$

Hence, $x = -2$, $y = 5$

Also,

$$y = mx + 3$$

$$5 = -2m + 3$$

$$-2m = 2$$

$$m = -1$$

Q4 :

Answer :

(i) Let the first number be x and the other number be y such that $y > x$.

According to the given information,

$$y = 3x \quad (1)$$

$$y - x = 26 \quad (2)$$

On substituting the value of y from equation (1) into equation (2), we obtain

$$3x - x = 26$$

$$x = 13 \quad (3)$$

Substituting this in equation (1), we obtain

$$y = 39$$

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be x and smaller angle be y .

We know that the sum of the measures of angles of a supplementary pair is always 180° .

According to the given information,

$$x + y = 180^\circ \quad (1)$$

$$x - y = 18^\circ \quad (2)$$

From (1), we obtain

$$x = 180^\circ - y \quad (3)$$

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$

$$162^\circ = 2y$$

$$81^\circ = y \quad (4)$$

Putting this in equation (3), we obtain

$$x = 180^\circ - 81^\circ$$

$$= 99^\circ$$

Hence, the angles are 99° and 81° .

(iii) Let the cost of a bat and a ball be x and y respectively.

According to the given information,

$$7x + 6y = 3800 \quad (1)$$

$$3x + 5y = 1750 \quad (2)$$

From (1), we obtain

$$y = \frac{3800 - 7x}{6} \quad (3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$

$$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$

$$3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\frac{18x - 35x}{6} = \frac{5250 - 9500}{3}$$

$$-\frac{17x}{6} = \frac{-4250}{3}$$

$$-17x = -8500$$

$$x = 500 \quad (4)$$

Substituting this in equation (3), we obtain

$$y = \frac{3800 - 7 \times 500}{6}$$

$$= \frac{300}{6} = 50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv) Let the fixed charge be Rs x and per km charge be Rs y .

According to the given information,

$$x + 10y = 105 \quad (1)$$

$$x + 15y = 155 \quad (2)$$

From (1), we obtain

$$x = 105 - 10y \quad (3)$$

Substituting this in equation (2), we obtain

$$105 - 10y + 15y = 155$$

$$5y = 50$$

$$y = 10 \quad (4)$$

Putting this in equation (3), we obtain

$$x = 105 - 10 \times 10$$

$$x = 5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = $x + 25y$

$$= 5 + 250 = \text{Rs } 255$$

(v) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = -4 \quad (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \quad (2)$$

From equation (1), we obtain

Exercise 3.4 : Solutions of Questions on Page Number : 56

Q1 :

Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x + y = 5$ and $2x - 3y = 4$ (ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ (iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Answer :

(i) **By elimination method**

$$x + y = 5 \quad (1)$$

$$2x - 3y = 4 \quad (2)$$

Multiplying equation (1) by 2, we obtain

$$2x + 2y = 10 \quad (3)$$

Subtracting equation (2) from equation (3), we obtain

$$5y = 6$$

$$y = \frac{6}{5} \quad (4)$$

Substituting the value in equation (1), we obtain

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

$$\therefore x = \frac{19}{5}, y = \frac{6}{5}$$

By substitution method

From equation (1), we obtain

$$x = 5 - y \quad (5)$$

Putting this value in equation (2), we obtain

$$2(5 - y) - 3y = 4$$

$$-5y = -6$$

$$y = \frac{6}{5}$$

Substituting the value in equation (5), we obtain

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

$$\therefore x = \frac{19}{5}, y = \frac{6}{5}$$

(ii) **By elimination method**

$$3x + 4y = 10 \quad (1)$$

$$2x - 2y = 2 \quad (2)$$

Multiplying equation (2) by 2, we obtain

$$4x - 4y = 4 \quad (3)$$

Adding equation (1) and (3), we obtain

$$7x = 14$$

$$x = 2 \quad (4)$$

Substituting in equation (1), we obtain

$$6 + 4y = 10$$

$$4y = 4$$

$$y = 1$$

Hence, $x = 2, y = 1$

By substitution method

From equation (2), we obtain

$$x = 1 + y \quad (5)$$

Putting this value in equation (1), we obtain

$$3(1 + y) + 4y = 10$$

$$7y = 7$$

$$y = 1$$

Substituting the value in equation (5), we obtain

$$x = 1 + 1 = 2$$

$$\therefore x = 2, y = 1$$

(iii) **By elimination method**

$$3x - 5y - 4 = 0 \quad (1)$$

Q2 :

Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer :

(i) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+1}{y-1} = 1 \quad \Rightarrow x - y = -2 \quad (1)$$

$$\frac{x}{y+1} = \frac{1}{2} \quad \Rightarrow 2x - y = 1 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x = 3 \quad (3)$$

Substituting this value in equation (1), we obtain

$$3 - y = -2$$

$$-y = -5$$

$$y = 5$$

Hence, the fraction is $\frac{3}{5}$.

(ii) Let present age of Nuri = x

and present age of Sonu = y

According to the given information,

$$(x - 5) = 3(y - 5)$$

$$x - 3y = -10 \quad (1)$$

$$(x + 10) = 2(y + 10)$$

$$x - 2y = 10 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$y = 20 \quad (3)$$

Substituting it in equation (1), we obtain

$$x - 60 = -10$$

$$x = 50$$

Hence, age of Nuri = 50 years

And, age of Sonu = 20 years

(iii) Let the unit digit and tens digits of the number be x and y respectively. Then, number = $10y + x$

Number after reversing the digits = $10x + y$

According to the given information,

$$x + y = 9 \quad (1)$$

$$9(10y + x) = 2(10x + y)$$

$$88y - 11x = 0$$

$$-x + 8y = 0 \quad (2)$$

Adding equation (1) and (2), we obtain

$$9y = 9$$

$$y = 1 \quad (3)$$

Substituting the value in equation (1), we obtain

$$x = 8$$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$

(iv) Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the given information,

$$x + y = 25 \quad (1)$$

$$50x + 100y = 2000 \quad (2)$$

Multiplying equation (1) by 50, we obtain

$$50x + 50y = 1250 \quad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$50y = 750$$

$$y = 15$$

Substituting in equation (1), we have $x = 10$

Hence, Meena has 10 notes of Rs 50 and 15 notes of Rs 100.

(v) Let the fixed charge for first three days and each day charge thereafter be Rs x and Rs y respectively.

According to the given information,

$$x + 4y = 27 \quad (1)$$

$$x + 2y = 21 \quad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$2y = 6$$

$$y = 3 \quad (3)$$

Substituting in equation (1), we obtain

$$x + 12 = 27$$

$$x = 15$$

Hence, fixed charge = Rs 15

And Charge per day = Rs 3

Exercise 3.5 : Solutions of Questions on Page Number : 62

Q1 :

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

$$(i) \quad x - 3y - 3 = 0$$

$$3x - 9y - 2 = 0$$

$$(iii) \quad 3x - 5y = 20$$

$$6x - 10y = 40$$

$$(ii) \quad 2x + y = 5$$

$$3x + 2y = 8$$

$$(iv) \quad x - 3y - 7 = 0$$

$$3x - 3y - 15 = 0$$

Answer :

$$\begin{aligned}
 \text{(i)} \quad & x - 3y - 3 = 0 \\
 & 3x - 9y - 2 = 0 \\
 & \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2} \\
 & \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}
 \end{aligned}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

$$\begin{aligned}
 \text{(ii)} \quad & 2x + y = 5 \\
 & 3x + 2y = 8 \\
 & \frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-5}{-8} \\
 & \frac{a_1}{a_2} \neq \frac{b_1}{b_2}
 \end{aligned}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\begin{aligned}
 \frac{x}{b_1c_2 - b_2c_1} &= \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \\
 \frac{x}{-8 - (-10)} &= \frac{y}{-15 + 16} = \frac{1}{4 - 3} \\
 \frac{x}{2} &= \frac{y}{1} = 1 \\
 \frac{x}{2} &= 1, \quad \frac{y}{1} = 1 \\
 x &= 2, \quad y = 1
 \end{aligned}$$

$\therefore x = 2, y = 1$

$$\begin{aligned}
 \text{(iii)} \quad & 3x - 5y = 20 \\
 & 6x - 10y = 40 \\
 & \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2} \\
 & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}
 \end{aligned}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

$$\begin{aligned}
 \text{(iv)} \quad & x - 3y - 7 = 0 \\
 & 3x - 3y - 15 = 0 \\
 & \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \quad \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15} \\
 & \frac{a_1}{a_2} \neq \frac{b_1}{b_2}
 \end{aligned}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\begin{aligned}
 \frac{x}{45 - (21)} &= \frac{y}{-21 - (-15)} = \frac{1}{-3 - (-9)} \\
 \frac{x}{24} &= \frac{y}{-6} = \frac{1}{6} \\
 \frac{x}{24} &= \frac{1}{6} \quad \text{and} \quad \frac{y}{-6} = \frac{1}{6} \\
 x &= 4 \quad \text{and} \quad y = -1 \\
 \therefore x &= 4, y = -1
 \end{aligned}$$

Q2 :

(i) For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$$\begin{aligned}
 2x + 3y &= 7 \\
 (a - b)x + (a + b)y &= 3a + b - 2
 \end{aligned}$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$\begin{aligned}
 3x + y &= 1 \\
 (2k - 1)x + (k - 1)y &= 2k + 1
 \end{aligned}$$

Answer :

$$\begin{aligned}
 \text{(i)} \quad & 2x + 3y - 7 = 0 \\
 & (a - b)x + (a + b)y - (3a + b - 2) = 0 \\
 & \frac{a_1}{a_2} = \frac{2}{a - b}, \quad \frac{b_1}{b_2} = \frac{3}{a + b}, \quad \frac{c_1}{c_2} = \frac{-7}{-(3a + b - 2)} = \frac{7}{(3a + b - 2)}
 \end{aligned}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \quad (1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \quad (2)$$

Subtracting (1) from (2), we obtain

$$4b=4$$

$$b=1$$

Substituting this in equation (2), we obtain

$$a-5 \times 1=0$$

$$a=5$$

Hence, $a=5$ and $b=1$ are the values for which the given equations give infinitely many solutions.

$$(ii) \quad 3x+y-1=0$$

$$(2k-1)x+(k-1)y-2k-1=0$$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3k-3=2k-1$$

$$k=2$$

Hence, for $k=2$, the given equation has no solution.

Q3 :

Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$\begin{aligned}8x + 5y &= 9 \\ 3x + 2y &= 4\end{aligned}$$

Answer :

$$\begin{aligned}8x + 5y &= 9 & (i) \\ 3x + 2y &= 4 & (ii)\end{aligned}$$

From equation (ii), we obtain

$$x = \frac{4-2y}{3} \quad (iii)$$

Substituting this value in equation (i), we obtain

$$\begin{aligned}8\left(\frac{4-2y}{3}\right) + 5y &= 9 \\ 32 - 16y + 15y &= 27 \\ -y &= -5 \\ y &= 5 & (iv)\end{aligned}$$

Substituting this value in equation (ii), we obtain

$$\begin{aligned}3x + 10 &= 4 \\ x &= -2\end{aligned}$$

Hence, $x = -2, y = 5$

Again, by cross-multiplication method, we obtain

$$\begin{aligned}8x + 5y - 9 &= 0 \\ 3x + 2y - 4 &= 0 \\ \frac{x}{-20 - (-18)} &= \frac{y}{-27 - (-32)} = \frac{1}{16 - 15} \\ \frac{x}{-2} &= \frac{y}{5} = \frac{1}{1} \\ \frac{x}{-2} &= 1 \text{ and } \frac{y}{5} = 1 \\ x &= -2 \text{ and } y = 5\end{aligned}$$

Q4 :

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer :

(i) Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

$$x + 20y = 1000 \quad (1)$$

$$x + 26y = 1180 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y = 180$$

$$y = 30$$

Substituting this value in equation (1), we obtain

$$x + 20 \times 30 = 1000$$

$$x = 1000 - 600$$

$$x = 400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x - y = 3 \quad (1)$$

$$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x - y = 8 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x = 5 \quad (3)$$

Putting this value in equation (1), we obtain

$$15 - y = 3$$

$$y = 12$$

Hence, the fraction is $\frac{5}{12}$.

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the given information,

$$3x - y = 40 \quad (1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \quad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$x = 15 \quad (3)$$

Substituting this in equation (2), we obtain

$$30 - y = 25$$

$$y = 5$$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of 1st car and 2nd car be u km/h and v km/h.

Respective speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = $(u + v)$ km/h

According to the given information,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \quad \dots(1)$$

$$1(u + v) = 100 \quad \dots(2)$$

Adding both the equations, we obtain

$$2u = 120$$

$$u = 60 \text{ km/h} \quad (3)$$

Substituting this value in equation (2), we obtain

$$v = 40 \text{ km/h}$$

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be x unit and y unit respectively.

$$\text{Area} = xy$$

According to the question,

$$(x-5)(y+3) = xy-9$$

$$\Rightarrow 3x-5y-6=0 \quad (1)$$

$$(x+3)(y+2) = xy+67$$

$$\Rightarrow 2x+3y-61=0 \quad (2)$$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x=17, y=9$$

Exercise 3.6 : Solutions of Questions on Page Number : 67

Q1 :

Solve the following pairs of equations by reducing them to a pair of linear equations:

$$(i) \quad \frac{1}{2x} + \frac{1}{3y} = 2 \quad (ii) \quad \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$
$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6} \quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

$$(iii) \quad \frac{4}{x} + 3y = 14 \quad (iv) \quad \frac{5}{x-1} + \frac{1}{y-2} = 2$$
$$\frac{3}{x} - 4y = 23 \quad \frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$(v) \quad \frac{7x-2y}{xy} = 5$$

$$\frac{8x+7y}{xy} = 15 \quad (vi) \quad \begin{aligned} 6x+3y &= 6xy \\ 2x+4y &= 5xy \end{aligned}$$

$$(vii) \quad \frac{10}{x+y} + \frac{2}{x-y} = 4 \quad (viii) \quad \frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \quad \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Answer :

$$(i) \quad \frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$, then the equations change as follows.

$$\frac{p}{2} + \frac{q}{3} = 2 \quad \Rightarrow \quad 3p + 2q - 12 = 0 \quad (1)$$

$$\frac{p}{3} + \frac{q}{2} = \frac{13}{6} \quad \Rightarrow \quad 2p + 3q - 13 = 0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26 - (-36)} = \frac{q}{-24 - (-39)} = \frac{1}{9 - 4}$$

$$\frac{p}{10} = \frac{q}{15} = \frac{1}{5}$$

$$\frac{p}{10} = \frac{1}{5} \quad \text{and} \quad \frac{q}{15} = \frac{1}{5}$$

$$p = 2 \quad \text{and} \quad q = 3$$

$$\frac{1}{x} = 2 \quad \text{and} \quad \frac{1}{y} = 3$$

$$x = \frac{1}{2} \quad \text{and} \quad y = \frac{1}{3}$$

$$(ii) \quad \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Putting $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$ in the given equations, we obtain

$$2p + 3q = 2 \quad (1)$$

$$4p - 9q = -1 \quad (2)$$

Multiplying equation (1) by 3, we obtain

$$6p + 9q = 6 \quad (3)$$

Adding equation (2) and (3), we obtain

$$10p = 5$$

$$p = \frac{1}{2} \quad (4)$$

Putting in equation (1), we obtain

$$2 \times \frac{1}{2} + 3q = 2$$

$$3q = 1$$

$$q = \frac{1}{3}$$

$$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$$

$$\sqrt{x} = 2$$

$$x = 4$$

$$\text{and } q = \frac{1}{\sqrt{y}} = \frac{1}{3}$$

$$\sqrt{y} = 3$$

$$y = 9$$

Hence, $x = 4, y = 9$

$$(iii) \quad \frac{4}{x} + 3y = 14$$

$$\frac{3}{x} - 4y = 23$$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$$4p + 3y = 14 \quad \Rightarrow \quad 4p + 3y - 14 = 0 \quad (1)$$

$$3p - 4y = 23 \quad \Rightarrow \quad 3p - 4y - 23 = 0 \quad (2)$$

By cross-multiplication, we obtain

$$\frac{p}{-69 - 56} = \frac{y}{-42 - (-92)} = \frac{1}{-16 - 9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$$\frac{p}{-125} = \frac{-1}{25} \quad \text{and} \quad \frac{y}{50} = \frac{-1}{25}$$

$$p = 5 \quad \text{and} \quad y = -2$$

$$p = \frac{1}{x} = 5$$

$$x = \frac{1}{5}$$

$$y = -2$$

$$(iv) \quad \frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Putting $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$ in the given equation, we obtain

$$5p + q = 2 \quad (1)$$

$$6p - 3q = 1 \quad (2)$$

Multiplying equation (1) by 3, we obtain

Q2 :

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer :

(i) Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

Speed of Ritu while rowing

$$\text{Upstream} = (x - y) \text{ km/h}$$

$$\text{Downstream} = (x + y) \text{ km/h}$$

According to question,

$$2(x + y) = 20$$

$$\Rightarrow x + y = 10 \quad (1)$$

$$2(x - y) = 4$$

$$\Rightarrow x - y = 2 \quad (2)$$

Adding equation (1) and (2), we obtain

$$2x = 12 \Rightarrow x = 6$$

Putting this in equation (1), we obtain

$$y = 4$$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii) Let the number of days taken by a woman and a man be x and y respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$$

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$$

$$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$$

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$$

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$ in these equations, we obtain

$$2p + 5q = \frac{1}{4}$$

$$\Rightarrow 8p + 20q = 1$$

$$3p + 6q = \frac{1}{3}$$

$$\Rightarrow 9p + 18q = 1$$

By cross-multiplication, we obtain

$$\frac{p}{-20 - (-18)} = \frac{q}{-9 - (-8)} = \frac{1}{144 - 180}$$

$$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$\frac{p}{-2} = \frac{-1}{36} \text{ and } \frac{q}{-1} = \frac{1}{-36}$$

$$p = \frac{1}{18} \text{ and } q = \frac{1}{36}$$

$$p = \frac{1}{x} = \frac{1}{18} \text{ and } q = \frac{1}{y} = \frac{1}{36}$$

$$x = 18 \quad y = 36$$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be u km/h and v km/h respectively.

According to the given information,

$$\frac{60}{u} + \frac{240}{v} = 4 \quad (1)$$

$$\frac{100}{u} + \frac{200}{v} = \frac{25}{6} \quad (2)$$

Putting $\frac{1}{u} = p$ and $\frac{1}{v} = q$ in these equations, we obtain

$$60p + 240q = 4 \quad (3)$$

$$100p + 200q = \frac{25}{6}$$

$$600p + 1200q = 25 \quad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p + 2400q = 40 \quad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q = 15$$

$$q = \frac{15}{1200} = \frac{1}{80} \quad (6)$$

Substituting in equation (3), we obtain

$$60p + 3 = 4$$

$$60p = 1$$

$$p = \frac{1}{60}$$

$$p = \frac{1}{u} = \frac{1}{60} \quad \text{and} \quad q = \frac{1}{v} = \frac{1}{80}$$

$$u = 60 \text{ km/h} \quad \text{and} \quad v = 80 \text{ km/h}$$

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h

Exercise 3.7 : Solutions of Questions on Page Number : 68

Q1 :

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer :

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = $2 \times x = 2x$ years

And age of Biju's sister Cathy = $\frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years,

$$x - y = 3 \quad (i)$$

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad (ii)$$

Subtracting (i) from (ii), we obtain

$$3x = 60 - 3 = 57$$

$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = $19 - 3 = 16$ years

Case (II) When Biju is older than Ani,

$$y - x = 3 \quad (i)$$

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad (ii)$$

Adding (i) and (ii), we obtain

$$3x = 63$$

$$x = 21$$

Therefore, age of Ani = 21 years

And age of Biju = $21 + 3 = 24$ years

Q2 :

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?

[From the Bijaganita of Bhaskara II]

[Hint: $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer :

Let those friends were having Rs x and y with them.

Using the information given in the question, we obtain

$$x + 100 = 2(y - 100)$$

$$x + 100 = 2y - 200$$

$$x - 2y = -300 \text{ (i)}$$

And, $6(x - 10) = (y + 10)$

$$6x - 60 = y + 10$$

$$6x - y = 70 \text{ (ii)}$$

Multiplying equation (ii) by 2, we obtain

$$12x - 2y = 140 \text{ (iii)}$$

Subtracting equation (i) from equation (iii), we obtain

$$11x = 140 + 300$$

$$11x = 440$$

$$x = 40$$

Using this in equation (i), we obtain

$$40 - 2y = -300$$

$$40 + 300 = 2y$$

$$2y = 340$$

$$y = 170$$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Q3 :

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer :

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ (i)

Using the information given in the question, we obtain

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (i), we obtain

$$-2x + 10t = 20 \text{ (ii)}$$

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (i), we obtain

$$3x - 10t = 30 \text{ (iii)}$$

Adding equations (ii) and (iii), we obtain

$$x = 50$$

Using equation (ii), we obtain

$$(-2) \times (50) + 10t = 20$$

$$-100 + 10t = 20$$

$$10t = 120$$

$$t = 12 \text{ hours}$$

From equation (i), we obtain

$$\text{Distance to travel} = d = xt$$

$$= 50 \times 12$$

$$= 600 \text{ km}$$

Hence, the distance covered by the train is 600 km.

Q4 :

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer :

Let the number of rows be x and number of students in a row be y .

Total students of the class

$$= \text{Number of rows} \times \text{Number of students in a row}$$

$$= xy$$

Using the information given in the question,

Condition 1

$$\text{Total number of students} = (x - 1)(y + 3)$$

$$xy = (x - 1)(y + 3) = xy - y + 3x - 3$$

$$3x - y - 3 = 0$$

$$3x - y = 3 \quad (i)$$

Condition 2

$$\text{Total number of students} = (x + 2)(y - 3)$$

$$xy = xy + 2y - 3x - 6$$

$$3x - 2y = -6 \quad (ii)$$

Subtracting equation (ii) from (i),

$$(3x - y) - (3x - 2y) = 3 - (-6)$$

$$-y + 2y = 3 + 6$$

$$y = 9$$

By using equation (i), we obtain

$$3x - 9 = 3$$

$$3x = 9 + 3 = 12$$

$$x = 4$$

$$\text{Number of rows} = x = 4$$

$$\text{Number of students in a row} = y = 9$$

$$\text{Number of total students in a class} = xy = 4 \times 9 = 36$$

Q5 :

In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Answer :

Given that,

$$\angle C = 3\angle B = 2(\angle A + \angle B)$$

$$3\angle B = 2(\angle A + \angle B)$$

$$3\angle B = 2\angle A + 2\angle B$$

$$\angle B = 2\angle A$$

$$2\angle A - \angle B = 0 \quad \dots (i)$$

We know that the sum of the measures of all angles of a triangle is 180° . Therefore,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + 3 \angle B = 180^\circ$$

$$\angle A + 4 \angle B = 180^\circ \dots (i)$$

Multiplying equation (i) by 4, we obtain

$$8 \angle A - 4 \angle B = 0 \dots (iii)$$

Adding equations (i) and (iii), we obtain

$$9 \angle A = 180^\circ$$

$$\angle A = 20^\circ$$

From equation (i), we obtain

$$20^\circ + 4 \angle B = 180^\circ$$

$$4 \angle B = 160^\circ$$

$$\angle B = 40^\circ$$

$$\angle C = 3 \angle B$$

$$= 3 \times 40^\circ = 120^\circ$$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20° , 40° , and 120° respectively.

Q6 :

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Answer :

$$5x - y = 5$$

$$\text{Or, } y = 5x - 5$$

The solution table will be as follows.

x	0	1	2
y	- 5	0	5

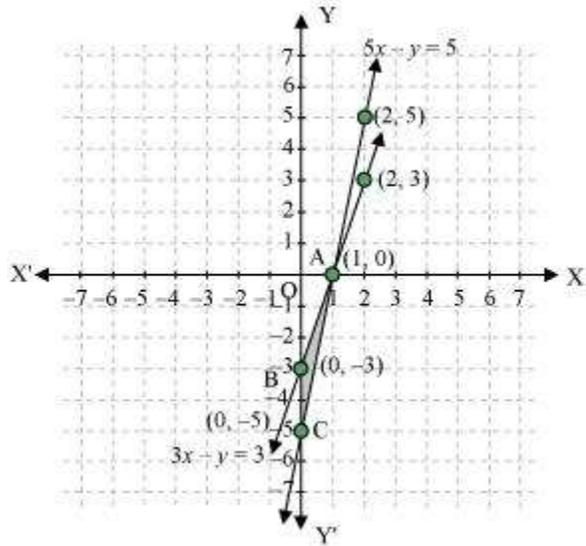
$$3x - y = 3$$

$$\text{Or, } y = 3x - 3$$

The solution table will be as follows.

x	0	1	2
y	- 3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is $\triangle ABC$ formed by these lines and y -axis.

The coordinates of vertices are A (1, 0), B (0, -3), C (0, -5).

Q7 :

Solve the following pair of linear equations.

(i) $px + qy = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Answer :

(i) $px + qy = p - q \dots (1)$

$qx - py = p + q \dots (2)$

Multiplying equation (1) by p and equation (2) by q , we obtain

$p^2x + pqy = p^2 - pq \dots (3)$

$$q^2x - pqy = pq + q^2 \dots (4)$$

Adding equations (3) and (4), we obtain

$$p^2x + q^2x = p^2 + q^2$$

$$(p^2 + q^2)x = p^2 + q^2$$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (1), we obtain

$$p(1) + qy = p - q$$

$$qy = -q$$

$$y = -1$$

$$(ii) ax + by = c \dots (1)$$

$$bx + ay = 1 + c \dots (2)$$

Multiplying equation (1) by a and equation (2) by b , we obtain

$$a^2x + aby = ac \dots (3)$$

$$b^2x + aby = b + bc \dots (4)$$

Subtracting equation (4) from equation (3),

$$(a^2 - b^2)x = ac - bc - b$$

$$x = \frac{c(a-b) - b}{a^2 - b^2}$$

From equation (1), we obtain

$$ax + by = c$$

$$a \left\{ \frac{c(a-b) - b}{a^2 - b^2} \right\} + by = c$$

$$\frac{ac(a-b) - ab}{a^2 - b^2} + by = c$$

$$by = c - \frac{ac(a-b) - ab}{a^2 - b^2}$$

$$by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2 - b^2}$$

$$by = \frac{abc - b^2c + ab}{a^2 - b^2}$$

$$by = \frac{bc(a-b) + ab}{a^2 - b^2}$$

$$y = \frac{c(a-b) + a}{a^2 - b^2}$$

$$(iii) \frac{x}{a} - \frac{y}{b} = 0$$

Or, $bx - ay = 0 \dots (1)$

$$ax + by = a^2 + b^2 \dots (2)$$

Multiplying equation (1) and (2) by b and a respectively, we obtain

$$b^2x - aby = 0 \dots (3)$$

$$a^2x + aby = a^3 + ab^2 \dots (4)$$

Adding equations (3) and (4), we obtain

$$b^2x + a^2x = a^3 + ab^2$$

$$x(b^2 + a^2) = a(a^2 + b^2)$$

$$x = a$$

By using (1), we obtain

$$b(a) - ay = 0$$

$$ab - ay = 0$$

$$ay = ab$$

$$y = b$$

$$(iv) (a - b)x + (a + b)y = a^2 - 2ab - b^2 \dots (1)$$

$$(a + b)(x + y) = a^2 + b^2$$

$$(a + b)x + (a + b)y = a^2 + b^2 \dots (2)$$

Subtracting equation (2) from (1), we obtain

$$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$$

$$(a - b - a - b)x = -2ab - 2b^2$$

$$-2bx = -2b(a + b)$$

$$x = a + b$$

Using equation (1), we obtain

$$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$$

$$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$$

$$(a + b)y = -2ab$$

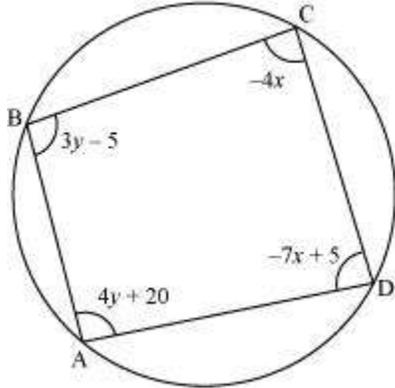
$$y = \frac{-2ab}{a + b}$$

$$(v) 152x - 378y = -74$$

$$76x - 189y = -37$$

Q8 :

ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Answer :

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180° .

Therefore, $\angle A + \angle C = 180$

$$4y + 20 - 4x = 180$$

$$-4x + 4y = 160$$

$$x - y = -40 \quad (i)$$

Also, $\angle B + \angle D = 180$

$$3y - 5 - 7x + 5 = 180$$

$$-7x + 3y = 180 \quad (ii)$$

Multiplying equation (i) by 3, we obtain

$$3x - 3y = -120 \quad (iii)$$

Adding equations (ii) and (iii), we obtain

$$-7x + 3x = 180 - 120$$

$$-4x = 60$$

$$x = -15$$

By using equation (i), we obtain

$$x - y = -40$$

$$-15 - y = -40$$

$$y = -15 + 40 = 25$$

$$\angle A = 4y + 20 = 4(25) + 20 = 120^\circ$$

$$\angle B = 3y - 5 = 3(25) - 5 = 70^\circ$$

$$\angle C = -4x = -4(-15) = 60^\circ$$

$$\angle D = -7x + 5 = -7(-15) + 5 = 110^\circ$$

NCERT Solutions for Class 10 Maths Unit 12

Areas Related to Circles Class 10

Unit 12 Areas Related to Circles Exercise 12.1, 12.2, 12.3 Solutions

Exercise 12.1 : Solutions of Questions on Page Number : 225

Q1 :

The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer :

Radius (r_1) of 1st circle = 19 cm

Radius (r_2) of 2nd circle = 9 cm

Let the radius of 3rd circle be r .

Circumference of 1st circle = $2\pi r_1 = 2\pi (19) = 38\pi$

Circumference of 2nd circle = $2\pi r_2 = 2\pi (9) = 18\pi$

Circumference of 3rd circle = $2\pi r$

Given that,

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

$$2\pi r = 38\pi + 18\pi = 56\pi$$

$$r = \frac{56\pi}{2\pi} = 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Q2 :

The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer :

Radius (r_1) of 1st circle = 8 cm

Radius (r_2) of 2nd circle = 6 cm

Let the radius of 3rd circle be r .

$$\text{Area of 1st circle} = \pi r_1^2 = \pi (8)^2 = 64\pi$$

$$\text{Area of 2nd circle} = \pi r_2^2 = \pi (6)^2 = 36\pi$$

Given that,

Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi r^2 = 64\pi + 36\pi$$

$$\pi r^2 = 100\pi$$

$$r^2 = 100$$

$$r = \pm 10$$

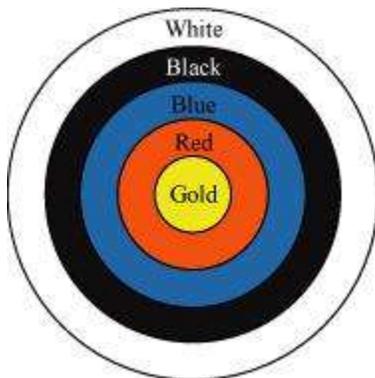
However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Q3 :

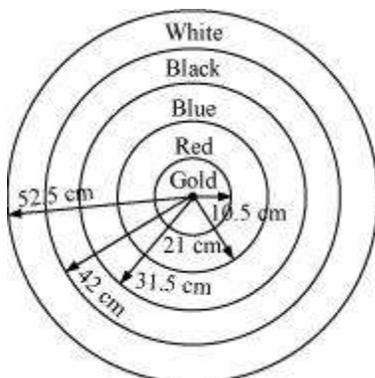
Given figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

other bands is 10.5 cm wide. Find the area of each of the five scoring regions.



Answer :



Radius (r) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5 \text{ cm}$

Given that each circle is 10.5 cm wider than the previous circle.

Therefore, radius (r_2) of 2nd circle = 10.5 + 10.5

21 cm

Radius (r_3) of 3rd circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5

= 42 cm

Radius (r_5) of 5th circle = 42 + 10.5

= 52.5 cm

Area of gold region = Area of 1st circle = $\pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of red region = Area of 2nd circle - Area of 1st circle

$$= \pi r_2^2 - \pi r_1^2$$

$$= \pi(21)^2 - \pi(10.5)^2$$

$$= 441\pi - 110.25\pi = 330.75\pi$$

$$= 1039.5 \text{ cm}^2$$

Area of blue region = Area of 3rd circle - Area of 2nd circle

$$= \pi r_3^2 - \pi r_2^2$$

$$= \pi(31.5)^2 - \pi(21)^2$$

$$= 992.25\pi - 441\pi = 551.25\pi$$

$$= 1732.5 \text{ cm}^2$$

Area of black region = Area of 4th circle - Area of 3rd circle

$$= \pi r_4^2 - \pi r_3^2$$

$$= \pi(42)^2 - \pi(31.5)^2$$

$$= 1764\pi - 992.25\pi$$

$$= 771.75\pi = 2425.5 \text{ cm}^2$$

Area of white region = Area of 5th circle - Area of 4th circle

$$= \pi r_5^2 - \pi r_4^2$$

$$= \pi(52.5)^2 - \pi(42)^2$$

$$= 2756.25\pi - 1764\pi$$

$$= 992.25\pi = 3118.5 \text{ cm}^2$$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm², 1039.5 cm², 1732.5 cm², 2425.5 cm², and 3118.5 cm² respectively.

Q4 :

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour? $\left[\text{Use } \pi = \frac{22}{7} \right]$

Answer :

Diameter of the wheel of the car = 80 cm

Radius (r) of the wheel of the car = 40 cm

Circumference of wheel = $2\pi r$

$$= 2\pi (40) = 80\pi \text{ cm}$$

Speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{ cm/min}$$

$$= 110000 \text{ cm/min}$$

Distance travelled by the car in 10 minutes

$$= 110000 \times 10 = 1100000 \text{ cm}$$

Let the number of revolutions of the wheel of the car be n .

$n \times$ Distance travelled in 1 revolution (i.e., circumference)

= Distance travelled in 10 minutes

$$n \times 80\pi = 1100000$$

$$n = \frac{1100000 \times 7}{80 \times 22}$$

$$= \frac{35000}{8} = 4375$$

Therefore, each wheel of the car will make 4375 revolutions.

Q5 :

Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D) 7 units

Answer :

Let the radius of the circle be r .

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle are equal.

This implies $2\pi r = \pi r^2$

$$2 = r$$

Therefore, the radius of the circle is 2 units.

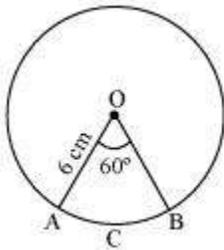
Hence, the correct answer is A.

Exercise 12.2 : Solutions of Questions on Page Number : 230

Q1 :

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° . [Use $\pi = \frac{22}{7}$]

Answer :



Let OACB be a sector of the circle making 60° angle at centre O of the circle.

$$\text{Area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Area of sector OACB} = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$$

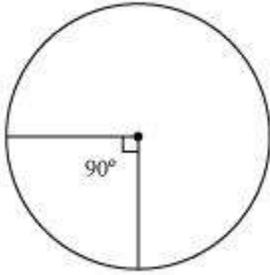
$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$

Therefore, the area of the sector of the circle making 60° at the centre of the circle is $\frac{132}{7} \text{ cm}^2$

Q2 :

Find the area of a quadrant of a circle whose circumference is 22 cm. [Use $\pi = \frac{22}{7}$]

Answer :



Let the radius of the circle be r .

Circumference = 22 cm

$$2\pi r = 22$$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend 90° angle at the centre of the circle.

$$\text{Area of such quadrant of the circle} = \frac{90^\circ}{360^\circ} \times \pi \times r^2$$

$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

Q3 :

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5

minutes. $\left[\text{Use } \pi = \frac{22}{7} \right]$

Answer :



We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360° .

$$\frac{360^\circ}{60} \times 5 = 30^\circ$$

In 5 minutes, minute hand will rotate =

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

$$\text{Area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Area of sector of } 30^\circ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{11 \times 14}{3}$$

$$= \frac{154}{3} \text{ cm}^2$$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3} \text{ cm}^2$.

Q4 :

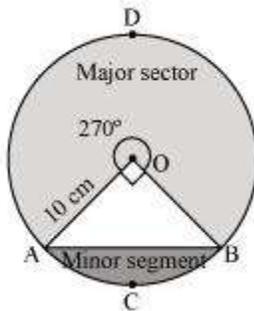
A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

[Use $\pi = 3.14$]

Answer :



Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

$$\text{Area of major sector OADB} = \left(\frac{360^\circ - 90^\circ}{360^\circ} \right) \times \pi r^2 = \left(\frac{270^\circ}{360^\circ} \right) \pi r^2$$

$$= \frac{3}{4} \times 3.14 \times 10 \times 10$$

$$= 235.5 \text{ cm}^2$$

$$\text{Area of minor sector OACB} = \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$

$$= 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ cm}^2$$

Area of minor segment ACB = Area of minor sector OACB -

$$\text{Area of } \triangle OAB = 78.5 - 50 = 28.5 \text{ cm}^2$$

Q5 :

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment formed by the corresponding chord

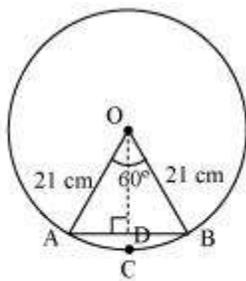
$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

Answer :

Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60°

$$\text{Length of an arc of a sector of angle } \theta = \frac{\theta}{360^\circ} \times 2\pi r$$



$$\begin{aligned} \text{Length of arc ACB} &= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \\ &= \frac{1}{6} \times 2 \times 22 \times 3 \end{aligned}$$

$$= 22 \text{ cm}$$

$$\begin{aligned} \text{Area of sector OACB} &= \frac{60^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \\ &= 231 \text{ cm}^2 \end{aligned}$$

In ΔOAB ,

$$\angle OAB = \angle OBA \text{ (As } OA = OB)$$

$$\angle OAB + \angle AOB + \angle OBA = 180^\circ$$

$$2\angle OAB + 60^\circ = 180^\circ$$

$$\angle OAB = 60^\circ$$

Therefore, ΔOAB is an equilateral triangle.

$$\begin{aligned} \text{Area of } \Delta OAB &= \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\ &= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2 \end{aligned}$$

Area of segment ACB = Area of sector OACB - Area of ΔOAB

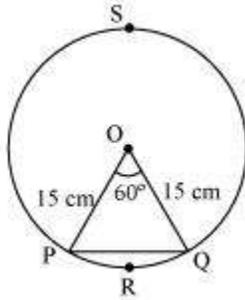
$$= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

Q6 :

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

[Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Answer :



Radius (r) of circle = 15 cm

$$\begin{aligned} \text{Area of sector OPRQ} &= \frac{60^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{6} \times 3.14 \times (15)^2 \\ &= 117.75 \text{ cm}^2 \end{aligned}$$

In $\triangle OPQ$,

$\angle OPQ = \angle OQP$ (As $OP = OQ$)

$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$2 \angle OPQ = 120^\circ$

$\angle OPQ = 60^\circ$

$\triangle OPQ$ is an equilateral triangle.

$$\begin{aligned} \text{Area of } \triangle OPQ &= \frac{\sqrt{3}}{4} \times (\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2 \\ &= 56.25\sqrt{3} \\ &= 97.3125 \text{ cm}^2 \end{aligned}$$

Area of segment PRQ = Area of sector OPRQ - Area of $\triangle OPQ$

= 117.75 - 97.3125

= 20.4375 cm^2

Area of major segment PSQ = Area of circle - Area of segment PRQ

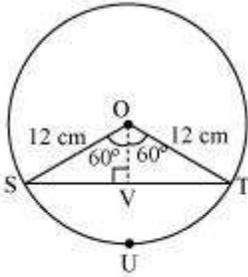
$$\begin{aligned} &= \pi(15)^2 - 20.4375 \\ &= 3.14 \times 225 - 20.4375 \\ &= 706.5 - 20.4375 \\ \bullet &= 686.0625 \text{ cm}^2 \end{aligned}$$

Q7 :

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

[Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Answer :



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

$$SV = VT$$

In ΔOVS ,

$$\frac{OV}{OS} = \cos 60^\circ$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

$$\text{Area of } \Delta OST = \frac{1}{2} \times ST \times OV$$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

$$\text{Area of sector OSUT} = \frac{120^\circ}{360^\circ} \times \pi (12)^2$$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

$$\begin{aligned}
 \text{Area of segment SUT} &= \text{Area of sector OSUT} - \text{Area of } \triangle OST \\
 &= 150.72 - 62.28 \\
 &= 88.44 \text{ cm}^2
 \end{aligned}$$

Q8 :

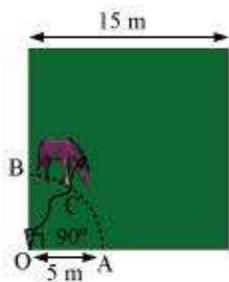
A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). Find

- (i) The area of that part of the field in which the horse can graze.
- (ii) The increase in the grazing area of the rope were 10 m long instead of 5 m.

[Use $\pi = 3.14$]



Answer :



From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse = Area of sector OACB

$$\begin{aligned}
 &= \frac{90^\circ}{360^\circ} \pi r^2 \\
 &= \frac{1}{4} \times 3.14 \times (5)^2 \\
 &= 19.625 \text{ m}^2
 \end{aligned}$$

Area that can be grazed by the horse when length of rope is 10 m long

$$\begin{aligned}
 &= \frac{90^\circ}{360^\circ} \times \pi \times (10)^2 \\
 &= \frac{1}{4} \times 3.14 \times 100 \\
 &= 78.5 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Increase in grazing area} &= (78.5 - 19.625) \text{ m}^2 \\
 &= 58.875 \text{ m}^2
 \end{aligned}$$

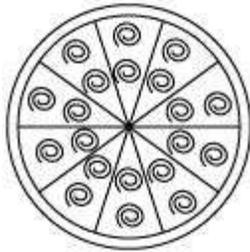
Q9 :

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$



Answer :

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

$$\text{Radius of circle} = \frac{35}{2} \text{ mm}$$

$$\text{Circumference of brooch} = 2\pi r$$

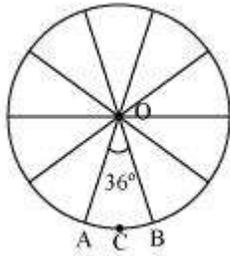
$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2} \right)$$

$$= 110 \text{ mm}$$

$$\text{Length of wire required} = 110 + 5 \times 35$$

$$= 110 + 175 = 285 \text{ mm}$$

It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



Therefore, area of each sector = $\frac{36^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

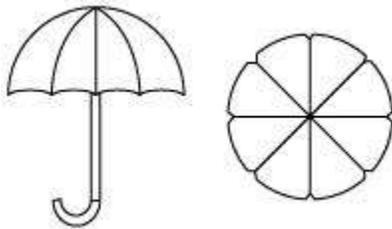
$$= \frac{385}{4} \text{ mm}^2$$

Q10 :

An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius

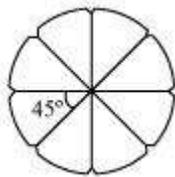
$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

45 cm, find the area between the two consecutive ribs of the umbrella.



Answer :

There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending $\frac{360^\circ}{8} = 45^\circ$ at the centre of the assumed flat circle.



Area between two consecutive ribs of circle = $\frac{45^\circ}{360^\circ} \times \pi r^2$

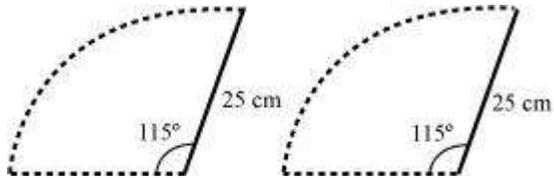
$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

Q11 :

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades. $\left[\text{Use } \pi = \frac{22}{7} \right]$

Answer :



It can be observed from the figure that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

$$\text{Area of such sector} = \frac{115^\circ}{360^\circ} \times \pi \times (25)^2$$

$$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$$

$$= \frac{158125}{252} \text{ cm}^2$$

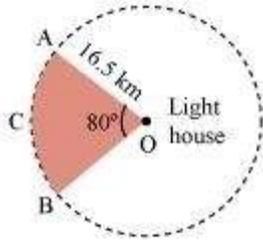
$$\text{Area swept by 2 blades} = 2 \times \frac{158125}{252}$$

$$= \frac{158125}{126} \text{ cm}^2$$

Q12 :

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use $\pi = 3.14$]

Answer :

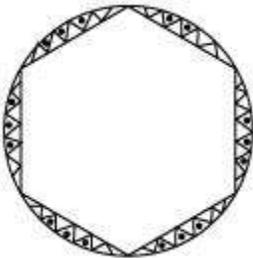


It can be observed from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius.

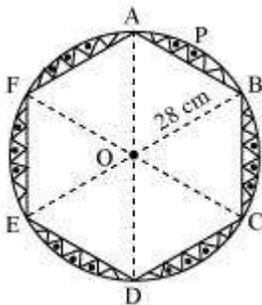
$$\begin{aligned} \text{Area of sector OACB} &= \frac{80^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5 \\ &= 189.97 \text{ km}^2 \end{aligned}$$

Q13 :

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per cm^2 . [Use $\sqrt{3} = 1.7$]



Answer :



It can be observed that these designs are segments of the circle.

Consider segment APB. Chord AB is a side of the hexagon. Each chord will subtend $\frac{360^\circ}{6} = 60^\circ$ at the centre of the circle.

In ΔOAB ,

$$\angle OAB = \angle OBA \text{ (As } OA = OB)$$

$$\angle AOB = 60^\circ$$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2\angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\angle OAB = 60^\circ$$

Therefore, ΔOAB is an equilateral triangle.

$$\begin{aligned} \text{Area of } \Delta OAB &= \frac{\sqrt{3}}{4} \times (\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} = 196 \times 1.7 \\ &= 333.2 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of sector OAPB} &= \frac{60^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 \\ &= \frac{1232}{3} \text{ cm}^2 \end{aligned}$$

Area of segment APB = Area of sector OAPB - Area of ΔOAB

$$= \left(\frac{1232}{3} - 333.2 \right) \text{ cm}^2$$

$$\begin{aligned} \text{Therefore, area of designs} &= 6 \times \left(\frac{1232}{3} - 333.2 \right) \text{ cm}^2 \\ &= (2464 - 1999.2) \text{ cm}^2 \\ &= 464.8 \text{ cm}^2 \end{aligned}$$

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8×0.35 = Rs 162.68

Therefore, the cost of making such designs is Rs 162.68.

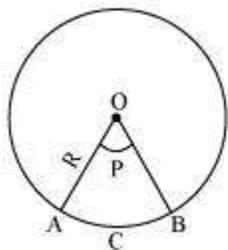
Q14 :

Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$, (B) $\frac{p}{180} \times \pi R^2$, (C) $\frac{p}{360} \times 2\pi R$, (D) $\frac{p}{720} \times 2\pi R^2$

Answer :



We know that area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi R^2$

$$\text{Area of sector of angle } P = \frac{P}{360^\circ} (\pi R^2)$$

$$= \left(\frac{P}{720^\circ} \right) (2\pi R^2)$$

Hence, (D) is the correct answer.

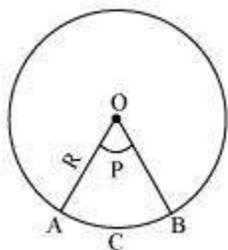
Q15 :

Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$, (B) $\frac{p}{180} \times \pi R^2$, (C) $\frac{p}{360} \times 2\pi R$, (D) $\frac{p}{720} \times 2\pi R^2$

Answer :



We know that area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi R^2$

$$\text{Area of sector of angle } P = \frac{P}{360^\circ} (\pi R^2)$$

$$= \left(\frac{P}{720^\circ} \right) (2\pi R^2)$$

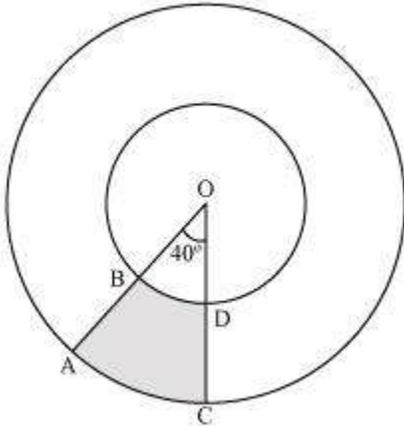
Hence, (D) is the correct answer.

Exercise 12.3 : Solutions of Questions on Page Number : 235

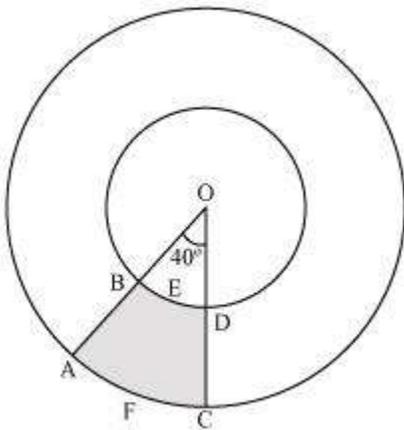
Q1 :

Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7

cm and 14 cm respectively and $\angle AOC = 40^\circ$. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



Radius of inner circle = 7 cm

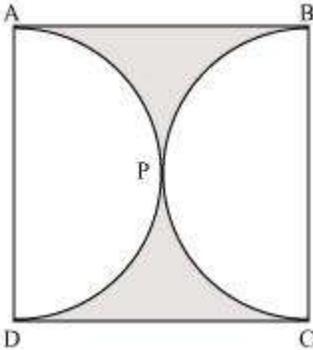
Radius of outer circle = 14 cm

Area of shaded region = Area of sector OAFC - Area of sector OBED

$$= 40^\circ \frac{360^\circ}{\pi} \times \pi (14)^2 - 40^\circ$$

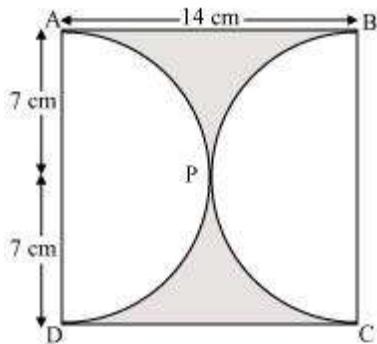
Q2 :

Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :

It can be observed from the figure that the radius of each semi-circle is 7 cm.



$$\text{Area of each semi-circle} = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 \text{ cm}^2$$

$$\text{Area of square ABCD} = (\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$$

Area of the shaded region

$$= \text{Area of square ABCD} - \text{Area of semi-circle APD} - \text{Area of semi-circle BPC}$$

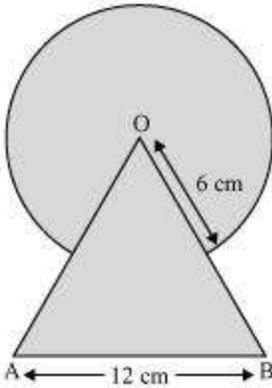
$$= 196 - 77 - 77 = 196 - 154 = 42 \text{ cm}^2$$

Q3 :

Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn

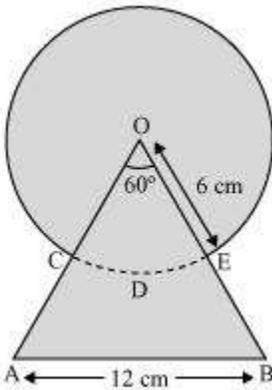
$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Answer :

We know that each interior angle of an equilateral triangle is of measure 60° .



$$\text{Area of sector OCDE} = \frac{60^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} \text{ cm}^2$$

$$\text{Area of } \Delta OAB = \frac{\sqrt{3}}{4} (12)^2 = \frac{\sqrt{3} \times 12 \times 12}{4} = 36\sqrt{3} \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7} \text{ cm}^2$$

Area of shaded region = Area of ΔOAB + Area of circle - Area of sector OCDE

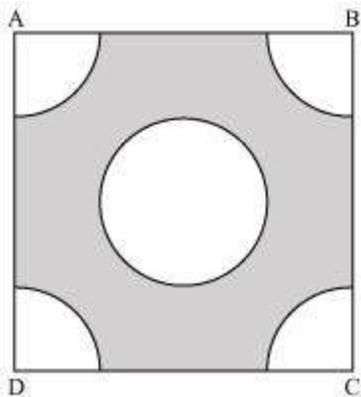
$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$

$$= \left(36\sqrt{3} + \frac{660}{7} \right) \text{cm}^2$$

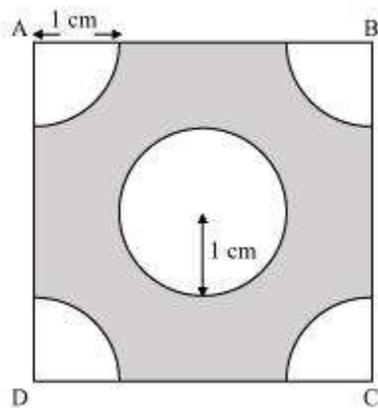
Q4 :

From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the

square. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



Each quadrant is a sector of 90° in a circle of 1 cm radius.

$$\text{Area of each quadrant} = \frac{90^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28} \text{cm}^2$$

Area of square = (Side)² = (4)² = 16 cm²

Area of circle = $\pi r^2 = \pi (1)^2$

= $\frac{22}{7}$ cm²

Area of the shaded region = Area of square - Area of circle - 4 × Area of quadrant

= $16 - \frac{22}{7} - 4 \times \frac{22}{28}$

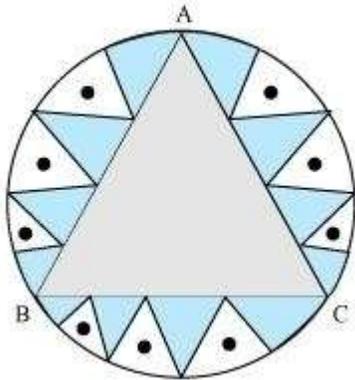
= $16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$

= $\frac{112 - 44}{7} = \frac{68}{7}$ cm²

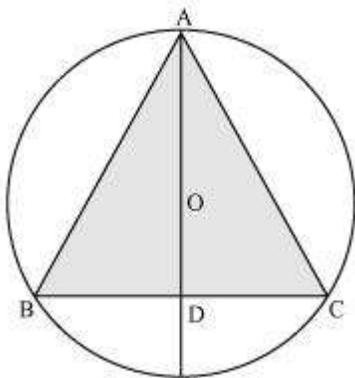
Q5 :

In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle

as shown in the given figure. Find the area of the design (Shaded region). [Use $\pi = \frac{22}{7}$]



Answer :



Radius (r) of circle = 32 cm

AD is the median of $\triangle ABC$.

$$AO = \frac{2}{3} AD = 32$$

AD = 48 cm

In $\triangle ABD$,

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}} \\ = 32\sqrt{3} \text{ cm}$$

Area of equilateral triangle, $\triangle ABC = \frac{\sqrt{3}}{4} (32\sqrt{3})^2$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3} \\ = 768\sqrt{3} \text{ cm}^2$$

Area of circle = πr^2

$$= \frac{22}{7} \times (32)^2 \\ = \frac{22}{7} \times 1024 \\ = \frac{22528}{7} \text{ cm}^2$$

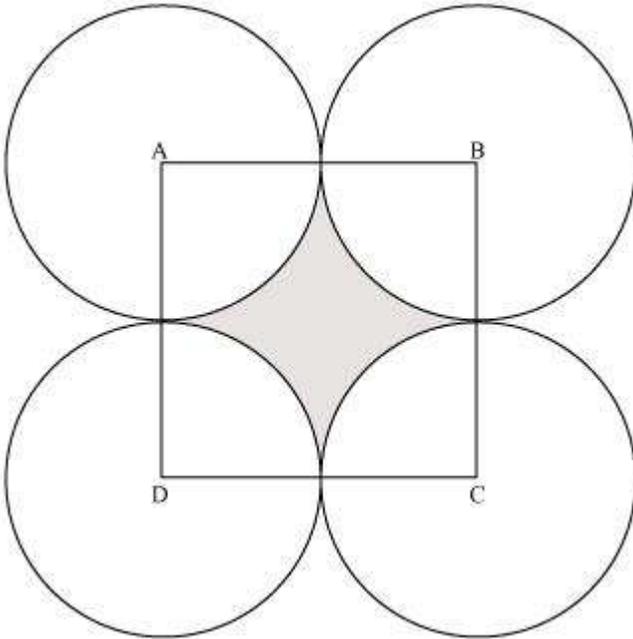
Area of design = Area of circle - Area of $\triangle ABC$

$$= \left(\frac{22528}{7} - 768\sqrt{3} \right) \text{ cm}^2$$

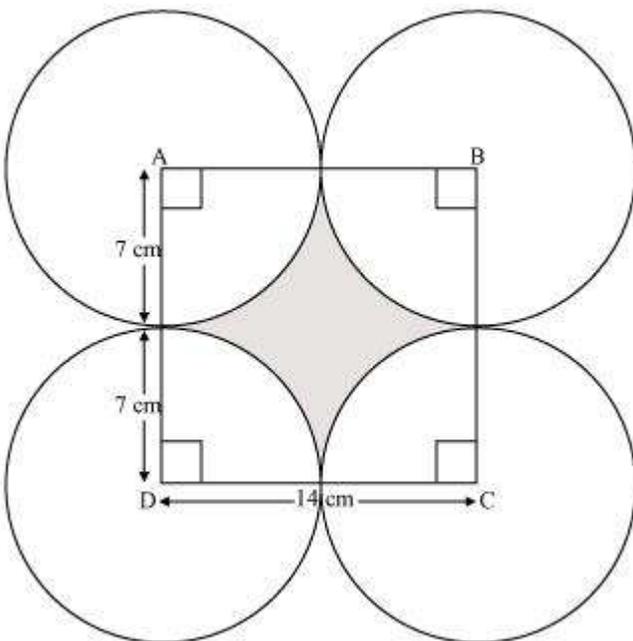
Q6 :

In the given figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded

region. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius.

$$\begin{aligned} \text{Area of each sector} &= \frac{90^\circ}{360^\circ} \times \pi(7)^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \\ &= \frac{77}{2} \text{ cm}^2 \end{aligned}$$

$$\text{Area of square ABCD} = (\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$$

Area of shaded portion = Area of square ABCD - 4 × Area of each sector

$$\begin{aligned} &= 196 - 4 \times \frac{77}{2} = 196 - 154 \\ &= 42 \text{ cm}^2 \end{aligned}$$

Therefore, the area of shaded portion is 42 cm².

Q7 :

The given figure depicts a racing track whose left and right ends are semicircular.

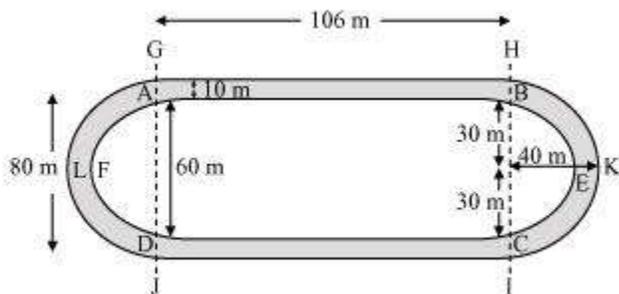


The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

- (i) The distance around the track along its inner edge
- (ii) The area of the track

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

Answer :



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$\begin{aligned}
&= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r \\
&= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 \\
&= 212 + 2 \times \frac{22}{7} \times 30 \\
&= 212 + \frac{1320}{7} \\
&= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}
\end{aligned}$$

Area of the track = (Area of GHIJ - Area of ABCD) + (Area of semi-circle HKI - Area of semi-circle BEC) + (Area of semi-circle GLJ - Area of semi-circle

$$\begin{aligned}
&= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 \\
&= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2 \\
&= 106(20) + \frac{22}{7} [(40)^2 - (30)^2] \\
&= 2120 + \frac{22}{7} (40 - 30)(40 + 30) \\
&= 2120 + \left(\frac{22}{7}\right)(10)(70) \\
&= 2120 + 2200
\end{aligned}$$

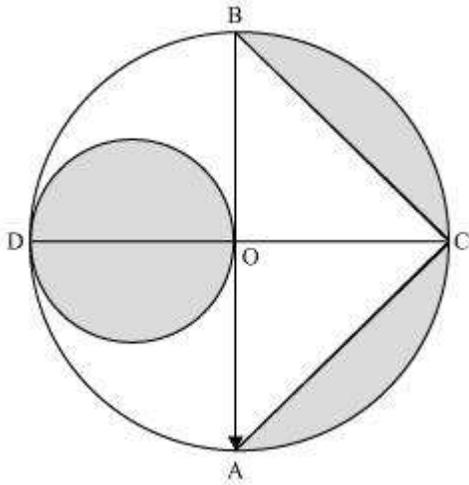
AFD) = 4320 m²

Therefore, the area of the track is 4320 m².

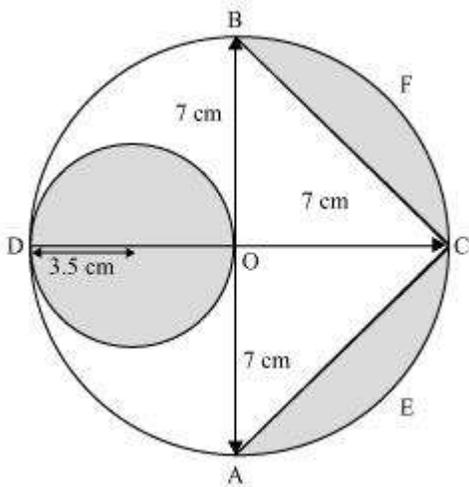
Q8 :

In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and

OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Answer :



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle = $\frac{7}{2}$ cm

Area of smaller circle = πr_1^2

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of semi-circle AECFB of larger circle = $\frac{1}{2} \pi r_2^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 \text{ cm}^2$$

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

Area of the shaded region

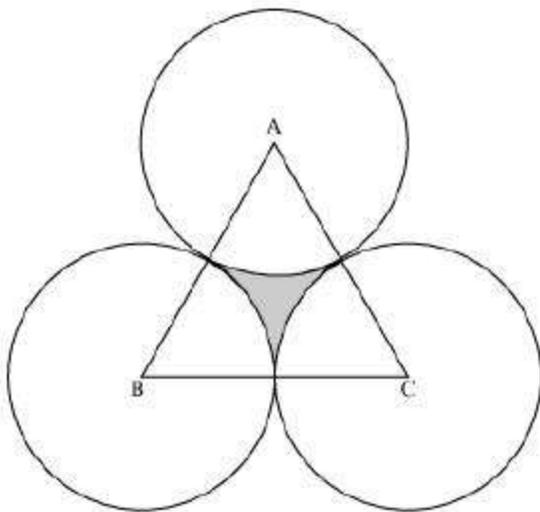
= Area of smaller circle + Area of semi-circle AECFB - Area of ΔABC

$$= \frac{77}{2} + 77 - 49$$

$$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5 \text{ cm}^2$$

Q9 :

The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$]



Answer :

Let the side of the equilateral triangle be a .

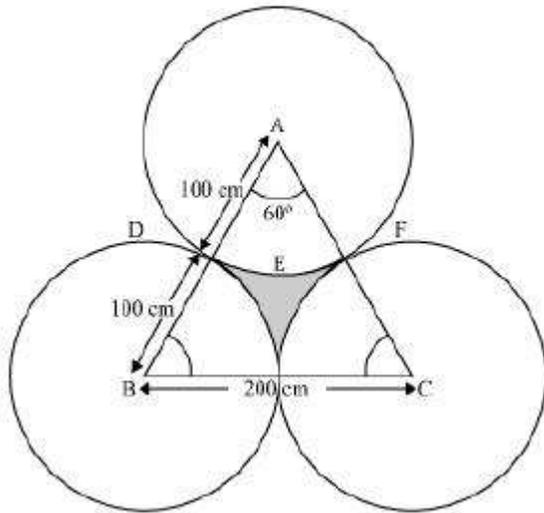
Area of equilateral triangle = 17320.5 cm^2

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$

$$\frac{1.73205}{4}a^2 = 17320.5$$

$$a^2 = 4 \times 10000$$

$$a = 200 \text{ cm}$$



Each sector is of measure 60° .

$$\text{Area of sector ADEF} = \frac{60^\circ}{360^\circ} \times \pi \times r^2$$

$$= \frac{1}{6} \times \pi \times (100)^2$$

$$= \frac{3.14 \times 10000}{6}$$

$$= \frac{15700}{3} \text{ cm}^2$$

Area of shaded region = Area of equilateral triangle - $3 \times$ Area of each sector

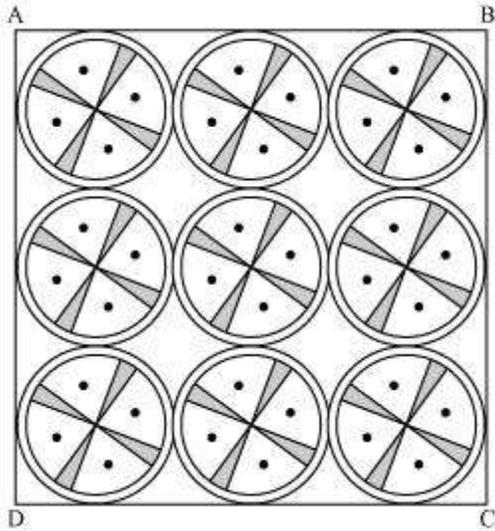
$$= 17320.5 - 3 \times \frac{15700}{3}$$

$$= 17320.5 - 15700 = 1620.5 \text{ cm}^2$$

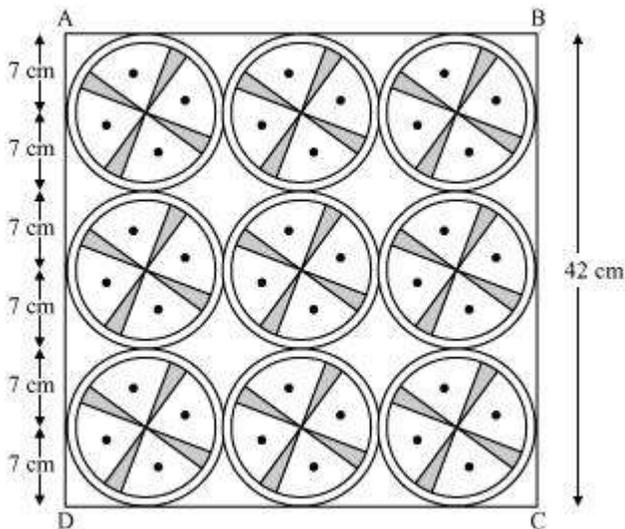
Q10 :

On a square handkerchief, nine circular designs each of radius 7 cm are made (see the given figure). Find the

area of the remaining portion of the handkerchief. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



From the figure, it can be observed that the side of the square is 42 cm.

Area of square = (Side)² = (42)² = 1764 cm²

Area of each circle = πr^2
 $= \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$

Area of 9 circles = 9 × 154 = 1386 cm²

Area of the remaining portion of the handkerchief = 1764 - 1386 = 378 cm²

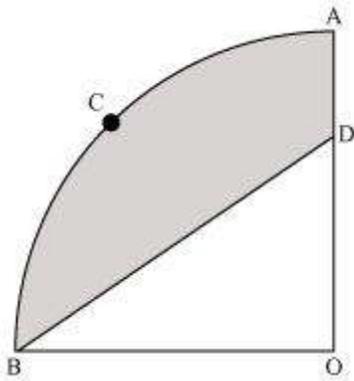
Q11 :

In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

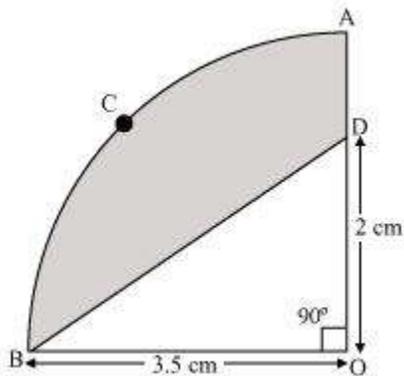
(i) Quadrant OACB

(ii) Shaded region

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$



Answer :



(i) Since OACB is a quadrant, it will subtend 90° angle at O.

$$\text{Area of quadrant OACB} = \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2$$

$$\text{(ii) Area of } \triangle OBD = \frac{1}{2} \times OB \times OD$$

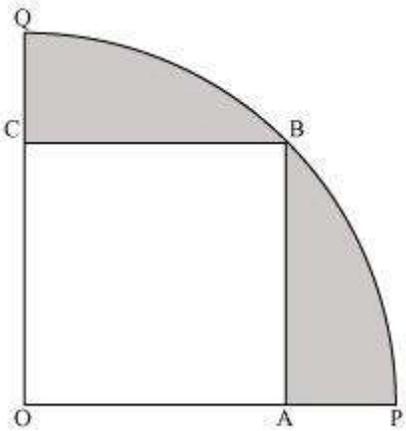
$$\begin{aligned}
 &= \frac{1}{2} \times 3.5 \times 2 \\
 &= \frac{1}{2} \times \frac{7}{2} \times 2 \\
 &= \frac{7}{2} \text{ cm}^2
 \end{aligned}$$

Area of the shaded region = Area of quadrant OACB - Area of ΔOBD

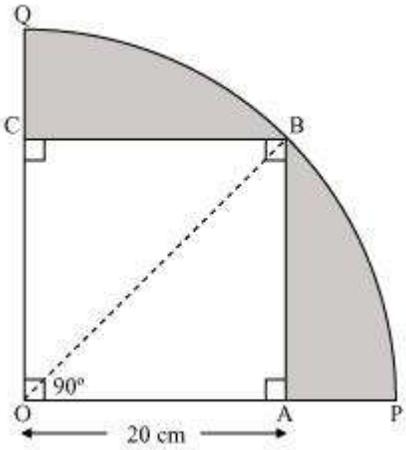
$$\begin{aligned}
 &= \frac{77}{8} - \frac{7}{2} \\
 &= \frac{77 - 28}{8} \\
 &= \frac{49}{8} \text{ cm}^2
 \end{aligned}$$

Q12 :

In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [Use $\pi = 3.14$]



Answer :



In $\triangle OAB$,

$$OB^2 = OA^2 + AB^2$$

$$= (20)^2 + (20)^2$$

$$OB = 20\sqrt{2}$$

$$\text{Radius (r) of circle} = 20\sqrt{2} \text{ cm}$$

$$\text{Area of quadrant OPBQ} = \frac{90^\circ}{360^\circ} \times 3.14 \times (20\sqrt{2})^2$$

$$= \frac{1}{4} \times 3.14 \times 800$$

$$= 628 \text{ cm}^2$$

$$\text{Area of OACB} = (\text{Side})^2 = (20)^2 = 400 \text{ cm}^2$$

$$\text{Area of shaded region} = \text{Area of quadrant OPBQ} - \text{Area of OACB}$$

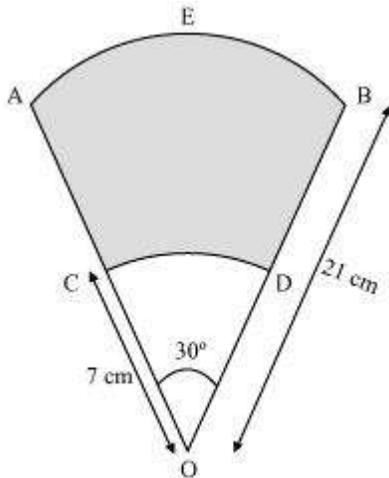
$$= (628 - 400) \text{ cm}^2$$

$$= 228 \text{ cm}^2$$

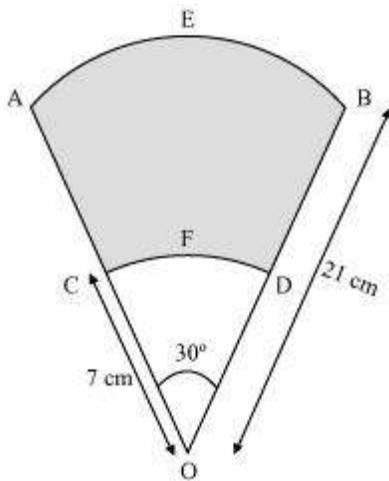
Q13 :

AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see the

given figure). If $\angle AOB = 30^\circ$, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :

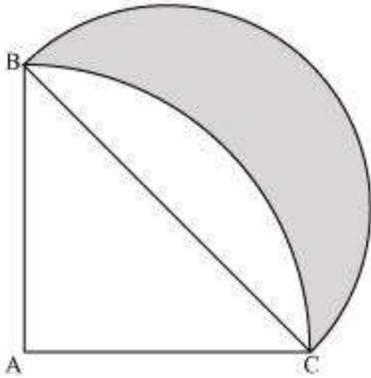


Area of the shaded region = Area of sector OAEB - Area of sector OCFD

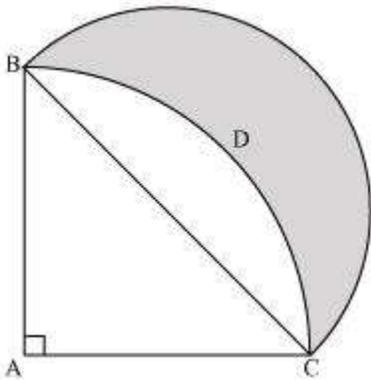
$$\begin{aligned}
 &= \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times (7)^2 \\
 &= \frac{1}{12} \times \pi [(21)^2 - (7)^2] \\
 &= \frac{1}{12} \times \frac{22}{7} \times [(21-7)(21+7)] \\
 &= \frac{22 \times 14 \times 28}{12 \times 7} \\
 &= \frac{308}{3} \text{ cm}^2
 \end{aligned}$$

Q14 :

In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$



Answer :



As ABC is a quadrant of the circle, $\angle BAC$ will be of measure 90° .

In $\triangle ABC$,

$$BC^2 = AC^2 + AB^2$$

$$= (14)^2 + (14)^2$$

$$BC = 14\sqrt{2}$$

Radius (r) of semi-circle drawn on

$$BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2} \text{ cm}$$

Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 14 \times 14$$

$$= 98 \text{ cm}^2$$

$$\text{Area of sector ABDC} = \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 154 \text{ cm}^2$$

$$\text{Area of semi-circle drawn on BC} = \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$$

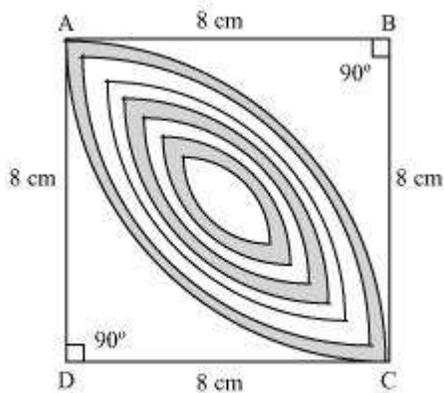
$$\text{Area of shaded region} = \text{Area of semi-circle} - (\text{Area of sector ABDC} - \text{Area of } \triangle ABC) = 154 - (154 - 98)$$

$$= 98 \text{ cm}^2$$

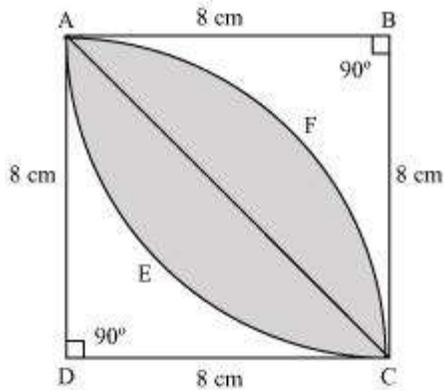
Q15 :

Calculate the area of the designed region in the given figure common between the two quadrants of circles of

radius 8 cm each. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



The designed area is the common region between two sectors BAEC and DAFC.

$$\begin{aligned} \text{Area of sector BAEC} &= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 64 \\ &= \frac{22 \times 16}{7} \\ &= \frac{352}{7} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle BAC &= \frac{1}{2} \times BA \times BC \\ &= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2 \end{aligned}$$

Area of the designed portion = 2 × (Area of segment AEC)

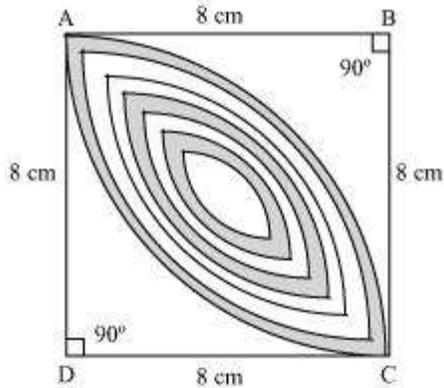
= 2 × (Area of sector BAEC - Area of $\triangle BAC$)

$$\begin{aligned} &= 2 \times \left(\frac{352}{7} - 32 \right) = 2 \times \left(\frac{352 - 224}{7} \right) \\ &= \frac{2 \times 128}{7} \\ &= \frac{256}{7} \text{ cm}^2 \end{aligned}$$

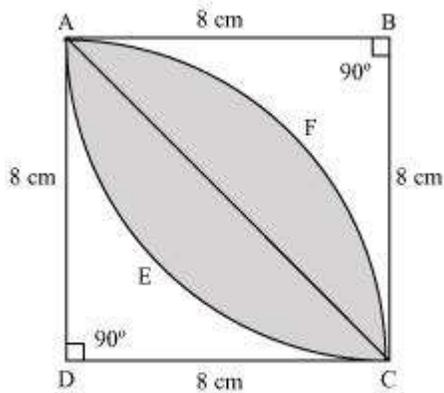
Q16 :

Calculate the area of the designed region in the given figure common between the two quadrants of circles of

radius 8 cm each. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer :



The designed area is the common region between two sectors BAEC and DAFC.

$$\begin{aligned} \text{Area of sector BAEC} &= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 64 \\ &= \frac{22 \times 16}{7} \\ &= \frac{352}{7} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle BAC &= \frac{1}{2} \times BA \times BC \\ &= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2 \end{aligned}$$

Area of the designed portion = 2 × (Area of segment AEC)

= 2 × (Area of sector BAEC - Area of $\triangle BAC$)

$$\begin{aligned} &= 2 \times \left(\frac{352}{7} - 32 \right) = 2 \left(\frac{352 - 224}{7} \right) \\ &= \frac{2 \times 128}{7} \\ &= \frac{256}{7} \text{ cm}^2 \end{aligned}$$



DELHI PUBLIC SCHOOL, GANDHINAGAR
CHAPTER – 12 AREAS RELATED TO CIRCLES

MIND MAP

This chapter consists of three different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 **CIRCUMFERENCE AND AREA OF A CIRCLE**

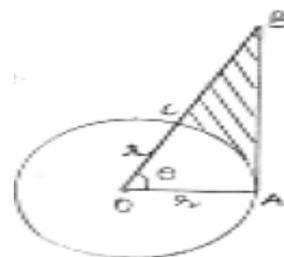
- Q.1. The area enclosed between the concentric circles is 770cm^2 . If the radius of outer circle is 21 cm, find the radius of the inner circle.
- Q.2. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?
- Q.3. The outer circumference of a circular race track is 528m. The track is everywhere 14m wide. Calculate the cost of leveling the track at rate of 50 paise per square metre.

TYPE: 2 **AREAS OF SECTOR AND SEGMENT OF A CIRCLE**

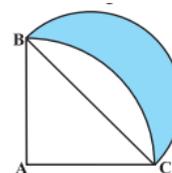
- Q.1. A chord AB of circle, of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.
- Q.2. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments.
- Q.3. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

TYPE: 3 **AREAS OF COMBINATIONS OF PLANE FIGURES**

- Q.1. The adjoining figure shows a sector of a circle, centre O containing an angle θ° . Prove that:
(i) Perimeter of shaded region is $(\tan \theta + \sec \theta + \pi \theta / 180 - 1)$
(ii) Area of shaded region is $\frac{r^2}{2} (\tan \theta - \pi \theta / 180)$



- Q.2. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



NCERT Solutions for Class 10 Maths Unit 15

Probability Class 10

Unit 15 Probability Exercise 15.1, 15.1 15.2, 15.2 Solutions

Exercise 15.1 : Solutions of Questions on Page Number : 308

Q1 :

Complete the following statements:

- (i) Probability of an event E + Probability of the event 'not E' = _____.
- (ii) The probability of an event that cannot happen is _____. Such as event is called _____.
- (iii) The probability of an event that is certain to happen is _____. Such as event is called _____.
- (iv) The sum of the probabilities of all the elementary events of an experiment is _____.
- (v) The probability of an event is greater than or equal to _____ and less than or equal to _____.

Answer :

- (i) 1
- (ii) 0, impossible event
- (iii) 1, sure event or certain event
- (iv) 1
- (v) 0, 1

Q2 :

Which of the following experiments have equally likely outcomes? Explain.

- (i) A driver attempts to start a car. The car starts or does not start.
- (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
- (iii) A trial is made to answer a true-false question. The answer is right or wrong.
- (iv) A baby is born. It is a boy or a girl.

Answer :

- (i) It is not an equally likely event, as it depends on various factors such as whether the car will start or not. And factors for both the conditions are not the same.
- (ii) It is not an equally likely event, as it depends on the player's ability and there is no information given about that.
- (iii) It is an equally likely event.
- (iv) It is an equally likely event.

Q3 :

Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Answer :

When we toss a coin, the possible outcomes are only two, head or tail, which are equally likely outcomes. Therefore, the result of an individual toss is completely unpredictable.

Q4 :

Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

Answer :

Probability of an event (E) is always greater than or equal to 0. Also, it is always less than or equal to one. This implies that the probability of an event cannot be negative or greater than 1. Therefore, out of these alternatives, -1.5 cannot be a probability of an event.

Hence, (B)

Q5 :

If $P(E) = 0.05$, what is the probability of 'not E'?

Answer :

We know that,

$$P(\bar{E}) = 1 - P(E)$$

$$\begin{aligned} P(\bar{E}) &= 1 - 0.05 \\ &= 0.95 \end{aligned}$$

Therefore, the probability of 'not E' is 0.95.

Q6 :

A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Answer :

(i) The bag contains lemon flavoured candies only. It does not contain any orange flavoured candies. This implies that every time, she will take out only lemon flavoured candies. Therefore, event that Malini will take out an orange flavoured candy is an impossible event.

Hence, $P(\text{an orange flavoured candy}) = 0$

(ii) As the bag has lemon flavoured candies, Malini will take out only lemon flavoured candies. Therefore, event that Malini will take out a lemon flavoured candy is a sure event.

$P(\text{a lemon flavoured candy}) = 1$

Q7 :

It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Answer :

Probability that two students are not having same birthday $P(\bar{E}) = 0.992$

Probability that two students are having same birthday $P(E) = 1 - P(\bar{E})$

$= 1 - 0.992$

$= 0.008$

Q8 :

A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

Answer :

(i) Total number of balls in the bag = 8

$$\begin{aligned} \text{Probability of getting a red ball} &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{3}{8} \end{aligned}$$

(ii) Probability of not getting red ball

$= 1 - \text{Probability of getting a red ball}$

$$= 1 - \frac{3}{8}$$
$$= \frac{5}{8}$$

Q9 :

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

Answer :

Total number of marbles = 5 + 8 + 4

= 17

(i) Number of red marbles = 5

$$\text{Probability of getting a red marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$
$$= \frac{5}{17}$$

(ii) Number of white marbles = 8

$$\text{Probability of getting a white marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$
$$= \frac{8}{17}$$

(iii) Number of green marbles = 4

$$\text{Probability of getting a green marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$
$$= \frac{4}{17}$$

$$\text{Probability of not getting a green marble} = 1 - \frac{4}{17} = \frac{13}{17}$$

Q10 :

A piggy bank contains hundred 50 p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) Will be a 50 p coin?

(ii) Will not be a Rs.5 coin?

Answer :

Total number of coins in a piggy bank = 100 + 50 + 20 + 10

= 180

(i) Number of 50 p coins = 100

$$\begin{aligned}\text{Probability of getting a 50 p coin} &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{100}{180} = \frac{5}{9}\end{aligned}$$

(ii) Number of Rs 5 coins = 10

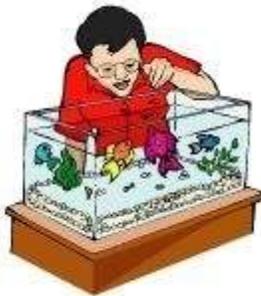
$$\begin{aligned}\text{Probability of getting a Rs 5 coin} &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{10}{180} = \frac{1}{18}\end{aligned}$$

Probability of not getting a Rs 5 coin = $1 - \frac{1}{18}$

$$= \frac{17}{18}$$

Q11 :

Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see the given figure). What is the probability that the fish taken out is a male fish?



Answer :

Total number of fishes in a tank

= Number of male fishes + Number of female fishes

= 5 + 8 = 13

$$\begin{aligned} \text{Probability of getting a male fish} &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{5}{13} \end{aligned}$$

Q12 :

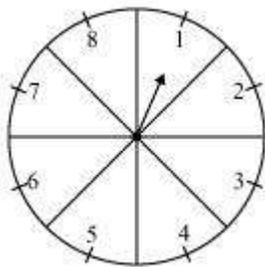
A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see the given figure), and these are equally likely outcomes. What is the probability that it will point at

(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?



Answer :

Total number of possible outcomes = 8

$$\text{(i) Probability of getting 8} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{1}{8}$$

(ii) Total number of odd numbers on spinner = 4

$$\begin{aligned} \text{Probability of getting an odd number} &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{4}{8} = \frac{1}{2} \end{aligned}$$

(iii) The numbers greater than 2 are 3, 4, 5, 6, 7, and 8.

Therefore, total numbers greater than 2 = 6

$$\begin{aligned} \text{Probability of getting a number greater than 2} \\ &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{6}{8} = \frac{3}{4} \end{aligned}$$

(iv) The numbers less than 9 are 1, 2, 3, 4, 6, 7, and 8.

Therefore, total numbers less than 9 = 8

$$= \frac{8}{8} = 1$$

Probability of getting a number less than 9

Q13 :

A die is thrown once. Find the probability of getting

(i) a prime number;

(ii) a number lying between 2 and 6;

(iii) an odd number.

Answer :

The possible outcomes when a dice is thrown = {1, 2, 3, 4, 5, 6}

Number of possible outcomes of a dice = 6

(i) Prime numbers on a dice are 2, 3, and 5.

Total prime numbers on a dice = 3

Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$

(ii) Numbers lying between 2 and 6 = 3, 4, 5

Total numbers lying between 2 and 6 = 3

Probability of getting a number lying between 2 and 6 = $\frac{3}{6} = \frac{1}{2}$

(iii) Odd numbers on a dice = 1, 3, and 5

Total odd numbers on a dice = 3

Probability of getting an odd number = $\frac{3}{6} = \frac{1}{2}$

Q14 :

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds

Answer :

Total number of cards in a well-shuffled deck = 52

(i) Total number of kings of red colour = 2

$$\begin{aligned} P(\text{getting a king of red colour}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{2}{52} = \frac{1}{26} \end{aligned}$$

(ii) Total number of face cards = 12

$$\begin{aligned} P(\text{getting a face card}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{12}{52} = \frac{3}{13} \end{aligned}$$

(iii) Total number of red face cards = 6

$$\begin{aligned} P(\text{getting a red face card}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{6}{52} = \frac{3}{26} \end{aligned}$$

(iv) Total number of Jack of hearts = 1

$$\begin{aligned} P(\text{getting a Jack of hearts}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{1}{52} \end{aligned}$$

(v) Total number of spade cards = 13

$$\begin{aligned} P(\text{getting a spade card}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{13}{52} = \frac{1}{4} \end{aligned}$$

(vi) Total number of queen of diamonds = 1

$$\begin{aligned} P(\text{getting a queen of diamond}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{1}{52} \end{aligned}$$

Q15 :

Five cards--the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace?
(b) a queen?

Answer :

(i) Total number of cards = 5

Total number of queens = 1

$$\begin{aligned} P(\text{getting a queen}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{1}{5} \end{aligned}$$

(ii) When the queen is drawn and put aside, the total number of remaining cards will be 4.

(a) Total number of aces = 1

$$P(\text{getting an ace}) = \frac{1}{4}$$

(b) As queen is already drawn, therefore, the number of queens will be 0.

$$P(\text{getting a queen}) = \frac{0}{4} = 0$$

Q16 :

12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Answer :

Total number of pens = 12 + 132 = 144

Total number of good pens = 132

$$P(\text{getting a good pen}) = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{132}{144} = \frac{11}{12}$$

Q17 :

(i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Answer :

(i) Total number of bulbs = 20

Total number of defective bulbs = 4

$$\begin{aligned} P(\text{getting a defective bulb}) &= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} \\ &= \frac{4}{20} = \frac{1}{5} \end{aligned}$$

(ii) Remaining total number of bulbs = 19

Remaining total number of non-defective bulbs = 16 - 1 = 15

$$P(\text{getting a not defective bulb}) = \frac{15}{19}$$

Q18 :

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

Answer :

Total number of discs = 90

(i) Total number of two-digit numbers between 1 and 90 = 81

$$P(\text{getting a two-digit number}) = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect squares between 1 and 90 are 1, 4, 9, 16, 25, 36, 49, 64, and 81. Therefore, total number of perfect squares between 1 and 90 is 9.

$$P(\text{getting a perfect square}) = \frac{9}{90} = \frac{1}{10}$$

(iii) Numbers that are between 1 and 90 and divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, and 90. Therefore, total numbers divisible by 5 = 18

$$P(\text{getting a number divisible by 5}) = \frac{18}{90} = \frac{1}{5}$$

Q19 :

A child has a die whose six faces shows the letters as given below:



The die is thrown once. What is the probability of getting (i) A? (ii) D?

Answer :

Total number of possible outcomes on the dice = 6

(i) Total number of faces having A on it = 2

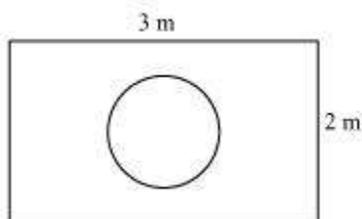
$$P(\text{getting A}) = \frac{2}{6} = \frac{1}{3}$$

(ii) Total number of faces having D on it = 1

$$P(\text{getting D}) = \frac{1}{6}$$

Q20 :

Suppose you drop a die at random on the rectangular region shown in the given figure. What is the probability that it will land inside the circle with diameter 1 m?



Answer :

Area of rectangle = $l \times b = 3 \times 2 = 6 \text{ m}^2$

Area of circle (of diameter 1 m)

$$= \pi r^2 = \pi \left(\frac{1\pi}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

P (die will land inside the circle)

$$= \frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$$

Q21 :

A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it?

(ii) She will not buy it?

Answer :

Total number of pens = 144

Total number of defective pens = 20

Total number of good pens = $144 - 20 = 124$

(i) Probability of getting a good pen

$$= \frac{124}{144} = \frac{31}{36}$$

P (Nuri buys a pen)

$$= \frac{31}{36}$$

(ii) P (Nuri will not buy a pen)

$$= 1 - \frac{31}{36} = \frac{5}{36}$$

Q22 :

Two dice, one blue and one grey, are thrown at the same time.

(i) Write down all the possible outcomes and complete the following table:

Event:												
Sum of two dice	2	3	4	5	6	7	8	9	10	11	12	
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$	

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$ '

Answer :

(i) It can be observed that,

To get the sum as 2, possible outcomes = (1, 1)

To get the sum as 3, possible outcomes = (2, 1) and (1, 2)

To get the sum as 4, possible outcomes = (3, 1), (1, 3), (2, 2)

To get the sum as 5, possible outcomes = (4, 1), (1, 4), (2, 3), (3, 2)

To get the sum as 6, possible outcomes = (5, 1), (1, 5), (2, 4), (4, 2), (3, 3)

To get the sum as 7, possible outcomes = (6, 1), (1, 6), (2, 5), (5, 2), (3, 4), (4, 3)

To get the sum as 8, possible outcomes = (6, 2), (2, 6), (3, 5), (5, 3), (4, 4)

To get the sum as 9, possible outcomes = (3, 6), (6, 3), (4, 5), (5, 4)

To get the sum as 10, possible outcomes = (4, 6), (6, 4), (5, 5)

To get the sum as 11, possible outcomes = (5, 6), (6, 5)

To get the sum as 12, possible outcomes = (6, 6)

Event:											
Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) Probability of each of these sums will not be $\frac{1}{11}$ as these sums are not equally likely.

Q23 :

A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Answer :

The possible outcomes are

{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

Number of total possible outcomes = 8

Number of favourable outcomes = 2 {i.e., TTT and HHH}

$$P(\text{Hanif will win the game}) = \frac{2}{8} = \frac{1}{4}$$

$$P(\text{Hanif will lose the game}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Q24 :

A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

[Hint: Throwing a die twice and throwing two dice simultaneously are treated as the same experiment].

Answer :

Total number of outcomes = 6×6

= 36

(i) Total number of outcomes when 5 comes up on either time are (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)

Hence, total number of favourable cases = 11

$$P(\text{5 will come up either time}) = \frac{11}{36}$$

$$P(\text{5 will not come up either time}) = 1 - \frac{11}{36} = \frac{25}{36}$$

(ii) Total number of cases, when 5 can come at least once = 11

$$P(\text{5 will come at least once}) = \frac{11}{36}$$

Exercise 15.1 15.2 : Solutions of Questions on Page Number : 311

Q1 :

Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on

(i) the same day? (ii) consecutive days? (iii) different days?

Answer :

There are a total of 5 days. Shyam can go to the shop in 5 ways and Ekta can go to the shop in 5 ways.

Therefore, total number of outcomes = $5 \times 5 = 25$

(i) They can reach on the same day in 5 ways.

i.e., (t, t), (w, w), (th, th), (f, f), (s, s)

$$P(\text{both will reach on same day}) = \frac{5}{25} = \frac{1}{5}$$

(ii) They can reach on consecutive days in these 8 ways - (t, w), (w, th), (th, f), (f, s), (w, t), (th, w), (f, th), (s, f).

$$\text{Therefore, } P(\text{both will reach on consecutive days}) = \frac{8}{25}$$

$$\text{(iii) } P(\text{both will reach on same day}) = \frac{1}{5} \quad \text{[[From (i)]]}$$

$$P(\text{both will reach on different days}) = 1 - \frac{1}{5} = \frac{4}{5}$$

Q2 :

A die is numbered in such a way that its faces show the number 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

		Number in first throw					
		1	2	2	3	3	6
Number in second throw	+	1	2	2	3	3	6
	1	1	2	2	3	3	6
	2	3	4	4	5	5	8
	2					5	
	3						
	3			5			9
6	7	8	8	9	9	12	

What is the probability that the total score is

(i) even? (ii) 6? (iii) at least 6?

Answer :

+	1 2 2 3 3 6
1	2 3 3 4 4 7

2	3 4 4 5 5 8
2	3 4 4 5 5 8
3	4 5 5 6 6 9
3	4 5 5 6 6 9
6	7 8 8 9 9 12

Total number of possible outcomes when two dice are thrown = $6 \times 6 = 36$

(i) Total times when the sum is even = 18

$$P(\text{getting an even number}) = \frac{18}{36} = \frac{1}{2}$$

(ii) Total times when the sum is 6 = 4

$$P(\text{getting sum as 6}) = \frac{4}{36} = \frac{1}{9}$$

(iii) Total times when the sum is at least 6 (i.e., greater than 5) = 15

$$P(\text{getting sum at least 6}) = \frac{15}{36} = \frac{5}{12}$$

Q3 :

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.

Answer :

Let the number of blue balls be x .

Number of red balls = 5

Total number of balls = $x + 5$

$$P(\text{getting a red ball}) = \frac{5}{x+5}$$

$$P(\text{getting a blue ball}) = \frac{x}{x+5}$$

Given that,

$$2\left(\frac{5}{x+5}\right) = \frac{x}{x+5}$$

$$10(x+5) = x^2 + 5x$$

$$x^2 - 5x - 50 = 0$$

$$x^2 - 10x + 5x - 50 = 0$$

$$x(x-10) + 5(x-10) = 0$$

$$(x-10)(x+5) = 0$$

$$\text{Either } x-10 = 0 \text{ or } x+5 = 0$$

$$x = 10 \text{ or } x = -5$$

However, the number of balls cannot be negative.

Hence, number of blue balls = 10

Q4 :

A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x .

Answer :

Total number of balls = 12

Total number of black balls = x

$$P(\text{getting a black ball}) = \frac{x}{12}$$

If 6 more black balls are put in the box, then

Total number of balls = $12 + 6 = 18$

Total number of black balls = $x + 6$

$$P(\text{getting a black ball now}) = \frac{x+6}{18}$$

According to the condition given in the question,

$$2\left(\frac{x}{12}\right) = \frac{x+6}{18}$$

$$3x = x + 6$$

$$2x = 6$$

$$x = 3$$

Q5 :

Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes - - two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes - - an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Answer :

(i) Incorrect

When two coins are tossed, the possible outcomes are (H, H), (H, T), (T, H), and (T, T). It can be observed that there can be one of each in two possible ways - (H, T), (T, H).

Therefore, the probability of getting two heads is $\frac{1}{4}$, the probability of getting two tails is $\frac{1}{4}$, and the probability of getting one of each is $\frac{1}{2}$.

It can be observed that for each outcome, the probability is not $\frac{1}{3}$.

(ii) Correct

When a dice is thrown, the possible outcomes are 1, 2, 3, 4, 5, and 6. Out of these, 1, 3, 5 are odd and 2, 4, 6 are even numbers.

Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Exercise 15.2 : Solutions of Questions on Page Number : 312

Q1 :

A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

Answer :

Total number of marbles = 24

Let the total number of green marbles be x.

Then, total number of blue marbles = $24 - x$

$$P(\text{getting a given marble}) = \frac{x}{24}$$

According to the condition given in the question,

$$\frac{x}{24} = \frac{2}{3}$$
$$x = 16$$

Therefore, total number of green marbles in the jar = 16

Hence, total number of blue marbles = $24 - x = 24 - 16 = 8$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER 12 PROBABILITY

MIND MAP

The most probable questions from the examination point of view are given below.

- Q.1. Red queens and black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining card, after reshuffling them. Find the probability that the drawn card is (i) a king (ii) of red colour (iii) a face card (iv) queen.
- Q.2. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards after reshuffling them. Find the probability that the card drawn is (i) of red colour (ii) a queen (iii) an ace (iv) a face card.
- Q.3. In a family of 3 children, find the probability of having at least 1 boy.
- Q.4. Three unbiased coins are thrown simultaneously. Find the probability of getting
(i) Exactly two heads. (ii). At least two heads.
(iii) At most two heads.
- Q.5. JKLM is a square with sides of length 6 units. Points A and B are the mid- points of sides KL and LM respectively. If a point is selected at random from the interior of the square, find the probability that the point will be chosen from the interior of ΔJAB .
- Q.6. A square dart board is shown. The length of a side of the larger square is 1.5 times the length of a side of the smaller square. If a dart is thrown and lands on the larger square. What is the probability that it will land in the interior of the smaller square?
- Q.7. From a pack of 52 playing cards Jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is
(i) a black queen. (ii) a red card. (iii) a black jack .
(iv) a picture card (Jacks, queens and kings are picture cards).