DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.1 NUMBER SYSTEMS

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

TYPE: 1 RATIONAL AND IRRATIONAL NUMBERS

Q.1. Find 5 rational numbers between $\frac{3}{4}$ and $\frac{5}{8}$.

Q.2 Find two irrational numbers between 1.5 and 1.6.

Q.3 Represent $\sqrt{11}$, $\sqrt{13}$ and $\sqrt{5.6}$ on the number line.

Q.4 Express $0.56\overline{28}$ in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$

TYPE: 2 POWERS AND EXPONENTS

Q.1 Find the value of $\frac{3^{49}+3^{50}-9^{24}}{3^{48}+3^{47}+9^{23}}$

Q.2 Prove that $\frac{2}{1+x^{2a-2b}}+\frac{2}{1+x^{2b-2a}}=2$

Q.3 Prove that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Q.4 Simplify: $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

TYPE: 3 RATIONALIZING THE DENOMINATOR

Q.1. Find the value of a and b in $\frac{7+3\sqrt{5}}{7-3\sqrt{5}} = \frac{a}{2} + \frac{b\sqrt{5}}{2}$

Q.2 If $x = 2+\sqrt{3}$, find the value of $x^2 + \frac{1}{x^2}$

Q.3 If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x^2+y^2

Ch. 1 Number Systems

Exercise 1.1

Question 1 :

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q≠0

Answer:

Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2:

Find six rational numbers between 3 and 4.

Answer:

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$ respectively.

Therefore, rational numbers between 3 and 4 are

$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$$

Therefore, the required rational numbers are
$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer:

(i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: $\frac{1}{5}$ is a rational number but not a whole number.

Exercise 1.2 Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer:

(i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer:

If numbers such as $\sqrt{4} = 2,\ \sqrt{9} = 3$ are considered,
Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

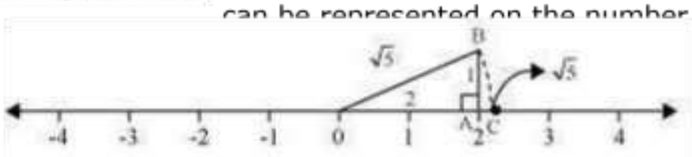
Question 3:

Show how $\sqrt{5}$ can be represented on the number line.

Answer:

We know that, $\sqrt{4} = 2$

And, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$



Mark a point 'A' representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw an arc intersecting number line at C.

C is representing $\sqrt{5}$.

has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909......$ $= 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769....$ $= 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818........$ $= 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

You know that $\frac{1}{7} = 0.\overline{142857}$ Question 2:

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

Exercise 1.3 Question 1:

Write the following in decimal form and say what kind of decimal expansion each . Can you predict what the decimal expansion of are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.] Answer:

Yes. It can be done as follows.

$$\frac{2}{7}=2\times\frac{1}{7}=2\times0.\overline{142857}=0.\overline{285714}$$

$$\frac{3}{7}=3\times\frac{1}{7}=3\times0.\overline{142857}=0.\overline{428571}$$

$$\frac{4}{7}=4\times\frac{1}{7}=4\times0.\overline{142857}=0.\overline{571428}$$

$$\frac{5}{7}=5\times\frac{1}{7}=5\times0.\overline{142857}=0.\overline{714285}$$

$$\frac{6}{7}=6\times\frac{1}{7}=6\times0.\overline{142857}=0.\overline{857142}$$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6}=0.666...$

Let x = 0.666...

10x = 6.666...

10x = 6 + x

9x = 6

$x=\frac{2}{3}$

(ii) $0.4\overline{7}=0.4777.....$

$=\frac{4}{10}+\frac{0.777}{10}$

Let x = 0.777...

10x = 7.777...

10x = 7 + x

$x=\frac{7}{9}$

$$\frac{4}{10}+\frac{0.777...}{10}=\frac{4}{10}+\frac{7}{90}$$
$$=\frac{36+7}{90}=\frac{43}{90}$$

(iii) $0.\overline{001}=0.001001...$

Let x = 0.001001...

1000x = 1.001001...

1000x = 1 + x

999x = 1

$x=\frac{1}{999}$

Question 4:

Express 0.99999...in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer:

Let x = 0.9999...

10x = 9.9999...

10x = 9 + x

9x = 9 x =

1

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$
$$\frac{11}{8} = 1.375$$
$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.
Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005…

0.7207200720007200007200000… 0.08008000800008000008000008…

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
Answer:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are as follows.

0.73073007300073000073…

0.75075007500075000075… 0.79079007900079000079…

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001…

(i) $\sqrt{23} = 4.79583152331\ ...$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 … $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 …

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

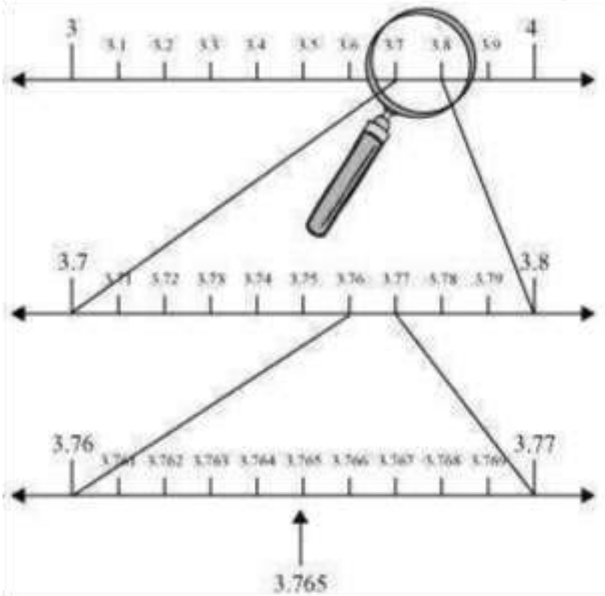
Exercise 1.4 Question

1:

Visualise 3.765 on the number line using successive magnification.

Answer:

3.765 can be visualised as in the following steps.



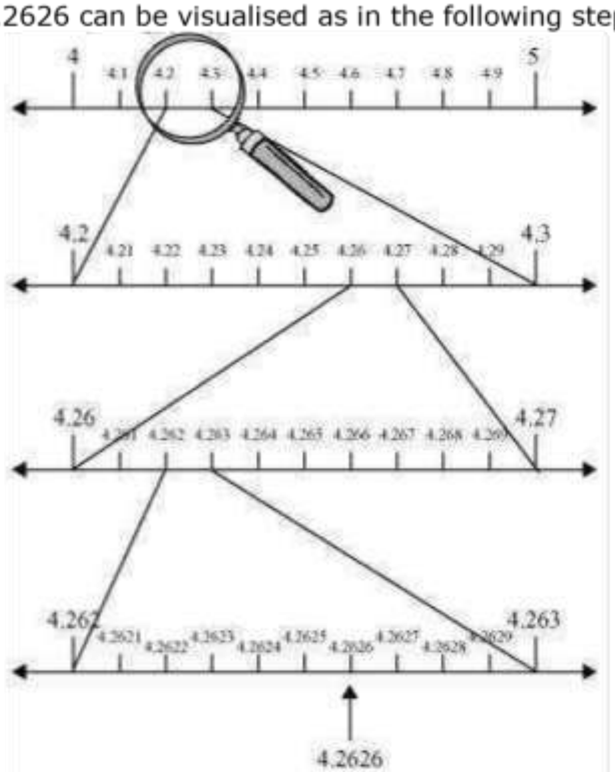
Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer:

$4.\overline{26}$ = 4.2626…

4.2626 can be visualised as in the following steps.



Exercise 1.5 Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ = 2 − 2.2360679...

= − 0.2360679...

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

form, therefore, it is a rational

number. form, therefore, it is a

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$ rational number.

As it can be represented in $\frac{p}{q}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$

(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811...$ As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(v) 2π = 2(3.1415 …)

= 6.2830 …

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ (iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Answer:

(i) $(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$

= 9 − 3 = 6

(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$

$= 5+2+2\sqrt{10} = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$

= 5 − 2 = 3

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore, the $\frac{c}{d}$ fraction is irrational. Hence, π is irrational.

Question 4:

Represent $\sqrt{9.3}$ on the number line. Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the midpoint

D of OC and draw a semi-circle on OC while taking D as its centre. Draw a

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

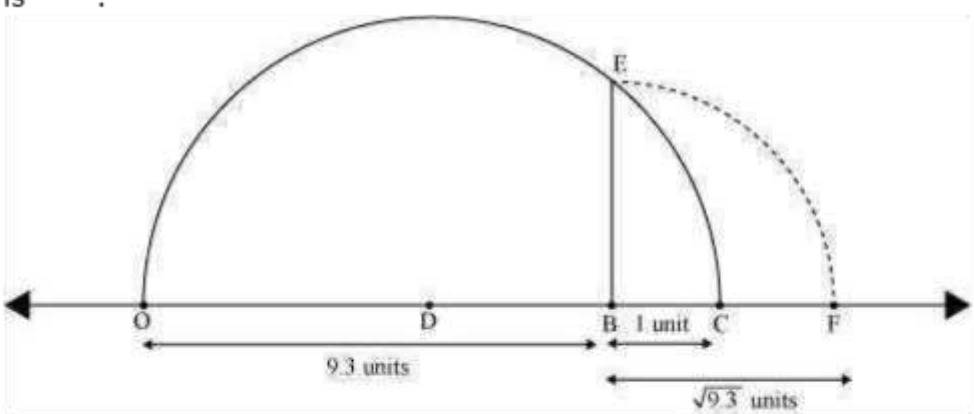
(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

perpendicular to line OC passing through point B. Let it intersect the semi-circle at E.

Taking B as centre and BE as radius, draw an arc in tersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(ii)
$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)}\frac{\left(\sqrt{7}+\sqrt{6}\right)}{\left(\sqrt{7}+\sqrt{6}\right)}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

(iii)
$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\left(\sqrt{5}+\sqrt{2}\right)}\frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv)
$$\frac{1}{\sqrt{7}-2}=\frac{1}{\left(\sqrt{7}-2\right)}\frac{\left(\sqrt{7}+2\right)}{\left(\sqrt{7}+2\right)}$$

$$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$$

Exercise 1.6 Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:
Answer:

(i)

$$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}}$$
$$= 2^{6\times\frac{1}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$
$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^1 = 2$$

(iii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}$$
$$= 5^{3\times\frac{1}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 5^1 = 5$$

Question 2:

(i)

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}}$$
$$= 3^{2\times\frac{3}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 3^3 = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}}$$
$$= 2^{5\times\frac{2}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^2 = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}}$$
$$= 2^{4\times\frac{3}{4}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$
$$= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}$$
$$= \frac{1}{5^{3\times\frac{1}{3}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \frac{1}{5}$$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer:

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$
$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times 7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \frac{1}{3^{21}}$$
$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$
$$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{\frac{1}{2}} \qquad [a^m.b^m = (ab)^m]$$
$$= (56)^{\frac{1}{2}}$$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH. 2 POLYNOMIALS

MIND MAP

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

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Q.1. Find 5 rational numbers between $\frac{3}{4}$ and $\frac{5}{8}$.

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TYPE: 2 POWERS AND EXPONENTS

Q.1 Find the value of $\frac{3^{49}+3^{50}-9^{24}}{3^{48}+\ 3^{47}+9^{23}}$

Q.2 Prove that $\frac{2}{1+x^{2a-2b}} + \frac{2}{1+x^{2b-2a}} = 2$

Q.3 Prove that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Q.4 Simplify: $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

TYPE: 3 RATIONALIZING THE DENOMINATOR

Q.1. Find the value of $a \ and \ b \ in \ \frac{7+3\sqrt{5}}{7-3\sqrt{5}} = \frac{a}{2} + \frac{b\sqrt{5}}{2}$

Q.2 If $x = 2 + \sqrt{3}$, find the value of $x^2 + \frac{1}{x^2}$

Q.3 If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find $x^2 + y^2$

Exercise 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

Solution:

The equation $4x^2 - 3x + 7$ can be written as $4x^2 - 3x^1 + 7x^0$

Since x is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation $y^2 + \sqrt{2}$ can be written as $y^2 + \sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{\frac{1}{2}} + \sqrt{2}t$

Though, t is the only variable in the given equation, the powers of t (i.e., $\frac{1}{2}$) is not a whole number.

Hence, we can say that the expression $3\sqrt{t} + t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) $y + \frac{2}{y}$

Solution:

The equation $y + \frac{2}{y}$ can be written as $y+2y^{-1}$

Though, y is the only variable in the given equation, the powers of y (i.e.,-1) is not a whole number.

Hence, we can say that the expression $y + \frac{2}{y}$ is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)\,x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

∴, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 - x^2 + x^3$

Solution:

The equation $2 - x^2 + x^3$ can be written as $2 + (-1)\,x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

∴, the coefficients of x^2 in $2 - x^2 + x^3$ is -1.

(iii) $\frac{\pi}{2}x^2 + x$

Solution:

The equation $\frac{\pi}{2}x^2 + x$ can be written as $(\frac{\pi}{2})\,x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\frac{\pi}{2}$

∴, the coefficients of x^2 in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

(iv) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2 + \sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

∴, the coefficients of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

∴, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 - y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 - y^2$,

The power of the variable y is: 2

∴, the degree of $4 - y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t - \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t - \sqrt{7}$,

The power of the variable y is: 1

∴, the degree of $5t - \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3 = 3 \times 1 = 3 \times x^0$

The power of the variable here is: 0

∴, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

∴, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) **$x - x^3$**

Solution:

The highest power of $x - x^3$ is 3

∴, the degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) **$y + y^2 + 4$**

Solution:

The highest power of $y + y^2 + 4$ is 2

∴, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) **$1 + x$**

Solution:

The highest power of $1 + x$ is 1

∴, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) **$3t$**

Solution:

The highest power of $3t$ is 1

∴, the degree is 1

Hence, $3t$ is a linear polynomial

(vi) **r^2**

Solution:

The highest power of r^2 is 2

∴, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) **$7x^3$**

Solution:

The highest power of $7x^3$ is 3

∴, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2

1. Find the value of the polynomial (x)=$5x-4x^2+3$

(i) x= 0

(ii) x = – 1

(iii) x = 2

Solution:

Let f(x)= $5x-4x^2+3$

(i) When x=0

$f(0)=5(0)+4(0)^2+3$
$=3$

(ii) When x= -1

$f(x)=5x-4x^2+3$
$f(-1)=5(-1)-4(-1)^2+3$
$=-5-4+3$
$=-6$

(iii) When x=2

$f(x)=5x-4x^2+3$
$f(2)=5(2)-4(2)^2+3$
$=10-16+3$
$=-3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$
$\therefore p(0)=(0)^2-(0)+1=1$
$p(1)=(1)^2-(1)+1=1$
$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$
$\therefore p(0)=2+0+2(0)^2-(0)^3=2$
$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$
$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$
$\therefore p(0)=(0)^3=0$
$p(1)=(1)^3=1$
$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1$, $x=-\frac{1}{3}$

Solution:

For, $x=-\frac{1}{3}$, $p(x)=3x+1$

$\therefore p(-\frac{1}{3})=3(-\frac{1}{3})+1=-1+1=0$

$\therefore -\frac{1}{3}$ is a zero of p(x).

(ii) $p(x)=5x-\pi$, $x=\frac{4}{5}$

Solution:

For, $x=\frac{4}{5}$ $p(x)=5x-\pi$

$\therefore p(\frac{4}{5})=5(\frac{4}{5})-\pi=4-\pi$

$\therefore \frac{4}{5}$ is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x)=(x+1)(x-2)$, $x=-1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, x=0 p(x)= x^2
p(0)=0^2=0
∴0 is a zero of p(x).

(vi)p(x)=lx+m, x=$-\frac{m}{l}$

Solution:

For, x=$-\frac{m}{l}$; p(x)=lx+m
∴p($-\frac{m}{l}$)=l($-\frac{m}{l}$)+m=−m+m=0
∴$-\frac{m}{l}$is a zero of p(x).

(vii) p(x)=3x²−1,x=$-\frac{1}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$

Solution:

For, x=$-\frac{1}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$; p(x)=$3x^2-1$
∴p($-\frac{1}{\sqrt{3}}$)=3($-\frac{1}{\sqrt{3}}$)^2−1=3($\frac{1}{3}$)−1=1−1=0
∴p($\frac{2}{\sqrt{3}}$)=3($\frac{2}{\sqrt{3}}$)^2−1=3($\frac{4}{3}$)−1=4−1=3≠0
∴$-\frac{1}{\sqrt{3}}$is a zero of p(x) but $\frac{2}{\sqrt{3}}$is not a zero of p(x).

(viii) p(x)=2x+1,x=$\frac{1}{2}$

Solution:

For, x=$\frac{1}{2}$ p(x)=2x+1
∴p($\frac{1}{2}$)=2($\frac{1}{2}$)+1=1+1=2≠0
∴$\frac{1}{2}$is not a zero of p(x).

4. Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

Solution:

p(x)=x+5
⇒x+5=0
⇒x=−5
∴-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

p(x)=x−5
⇒x−5=0

$\Rightarrow x=5$

$\therefore 5$ is a zero polynomial of the polynomial p(x).

(iii)p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

$\therefore x=-\frac{5}{2}$ is a zero polynomial of the polynomial p(x).

(iv)p(x) = 3x – 2

Solution:

$p(x)=3x-2$

$\Rightarrow 3x-2=0$

$\Rightarrow 3x=2$

$\Rightarrow x=\frac{2}{3}$

$\therefore x=\frac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x)=3x$

$\Rightarrow 3x=0$

$\Rightarrow x=0$

$\therefore 0$ is a zero polynomial of the polynomial p(x).

(vi)p(x) = ax, a≠0

Solution:

$p(x)=ax$

$\Rightarrow ax=0$

$\Rightarrow x=0$

$\therefore x=0$ is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

$p(x)= cx + d$

$\Rightarrow cx + d =0$

$\Rightarrow x=\frac{-d}{c}$

$\therefore x=\frac{-d}{c}$ is a zero polynomial of the polynomial p(x).

Exercise 2.3

1. **Find the remainder when x^3+3x^2+3x+1 is divided by**

(i) $x+1$

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-\frac{1}{2}$

Solution:

$x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{2}$

$\therefore$Remainder:

$p(\frac{1}{2})=(\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1$

$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$=\frac{27}{8}$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) $5+2x$

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

∴Remainder:

$(-\frac{5}{2})^3+3(-\frac{5}{2})^2+3(-\frac{5}{2})+1=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$

$=-\frac{27}{8}$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a=5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=\frac{-7}{3}$

∴Remainder:

$3(\frac{-7}{3})^3+7(\frac{-7}{3})=-\frac{-343}{9}+\frac{-49}{3}$

$=\frac{-343-(49)3}{9}$

$=\frac{-343-147}{9}$

$=\frac{-490}{9}\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4

1. **Determine which of the following polynomials has (x + 1) a factor:**

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1$
$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$
$=1-1+1-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii)$x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$
$=1-3+3-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv)$x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+ \sqrt{2}$
$=-1-1+2+\sqrt{2}+\sqrt{2}$
$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2-2x-1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3x2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii)$p(x)=x^3-4x^2+x+6$, $g(x) = x - 3$

Solution:

$p(x)= x^3-4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}\,(1)+1=0$

$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

$\Rightarrow k=\frac{3}{2}$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=1×12=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4=12]

$12x^2–7x+1=12x^2-4x-3x+1$
$=4x\,(3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=7 and product=$2\times 3=6$
We get 6 and 1 as the numbers [$6+1=7$ and $6\times 1=6$]

$2x^2+7x+3 =2x^2+6x+1x+3$
$=2x\,(x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=5 and product=$6\times -6= -36$
We get -4 and 9 as the numbers [$-4+9=5$ and $-4\times 9=-36$]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$
$=3x\,(2x + 3) – 2\,(2x + 3)$

$= (2x + 3)\,(3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-1 and product=$3\times -4= -12$
We get -4 and 3 as the numbers [$-4+3=-1$ and $-4\times 3=-12$]

$3x^2 – x – 4=3x^2–x–4$
$=3x^2–4x+3x–4$
$=x(3x–4)+1(3x–4)$
$=(3x–4)(x+1)$

5. Factorize:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
$p(1) = 0$
So, $(x+1)$ is factor of $p(x)$

Now,

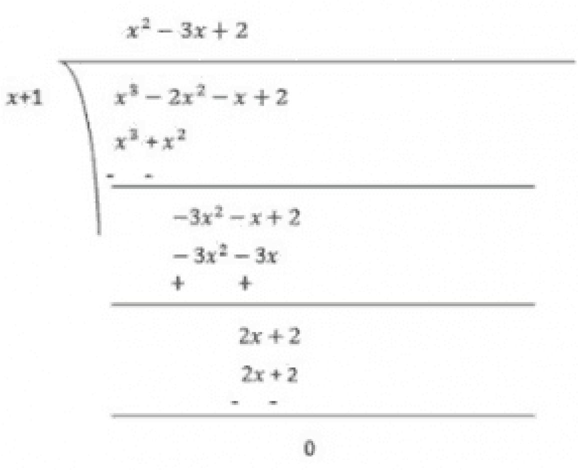
p(x)= $x^3–2x^2–x+2$

p(−1)=$(−1)^3–2(−1)^2–(−1)+2$

=−1−1+1+2

=0

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2)$ =$(x+1)(x^2–x–2x+2)$

=(x+1)(x(x−1)−2(x−1))

=(x+1)(x−1)(x-2)

(ii) $x^3–3x^2–9x–5$

Solution:

Let p(x) = $x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

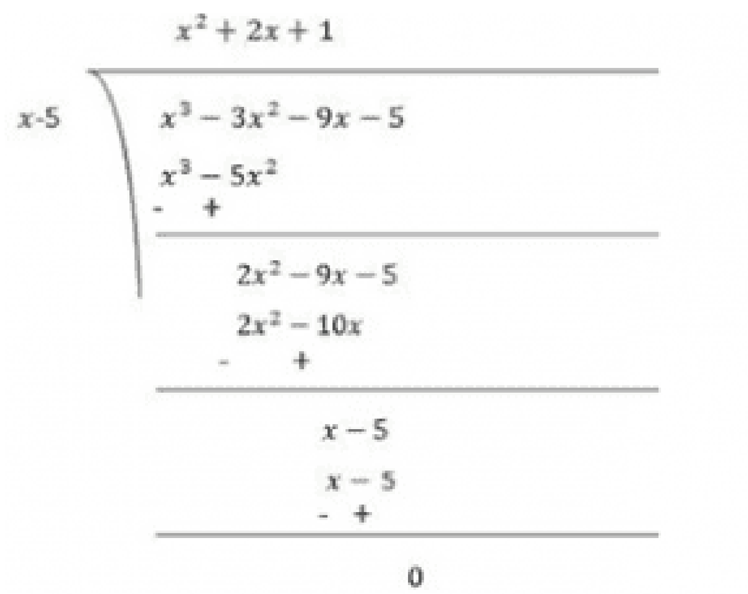
p(x) = $x^3–3x^2–9x–5$

p(5) = $(5)^3–3(5)^2–9(5)–5$

=125−75−45−5

=0

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$$
$$= (x-5)(x(x+1)+1(x+1))$$
$$= (x-5)(x+1)(x+1)$$

(iii) $\mathbf{x^3+13x^2+32x+20}$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$$p(x) = x^3+13x^2+32x+20$$
$$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$$
$$= -1+13-32+20$$
$$= 0$$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x + 20 \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(x+1)(x^2+12x+20) &= (x+1)(x^2+2x+10x+20) \\
&= (x+1)x(x+2)+10(x+2) \\
&= (x+1)(x+2)(x+10)
\end{aligned}
$$

(iv) $\mathbf{2y^3+y^2-2y-1}$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1) = -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, $(y-1)$ is the factor of $p(y)$

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & 3y^2 - 2y - 1 \\
 & 3y^2 - 3y \\
 & - \quad + \\
\hline
 & y - 1 \\
 & y - 1 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$

Exercise 2.5

1. Use suitable identities to find the following products:

(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$$
$$=x^2+14x+40$$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]

We get,

$$(x+8)(x-10)=x^2+(8+(-10))x+(8\times(-10))$$
$$=x^2+(8-10)x-80$$
$$=x^2-2x-80$$

(iii)(3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, x=3x, a=4 and b= −5]

We get,

$$(3x+4)(3x-5)=(3x)^2+4+(-5)3x+4\times(-5)$$
$$=9x^2+3x(4-5)-20$$
$$=9x^2-3x-20$$

(iv)$(y^2+\frac{3}{2})(y^2-\frac{3}{2})$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=\frac{3}{2}$]

We get,

$$(y^2+\tfrac{3}{2})(y^2-\tfrac{3}{2})= (y^2)^2-(\tfrac{3}{2})^2$$
$$=y^4-\frac{9}{4}$$

2. Evaluate the following products without multiplying directly:

(i) 103 × 107

Solution:

$103\times107=(100+3)\times(100+7)$

Using identity, [(x+a)(x+b)=x2+(a+b)x+ab

Here, x=100

a=3

b=7

We get, $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)100+(3\times7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95\times96=(100-5)\times(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, x=100

a=-5

b=-4

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii)104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, a=100

b=4

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, x=3x

y=y

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$
$=(3x+y)^2$
$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution: $4y^2-4y+1=(2y)^2-$

$(2\times2y\times1)+12$
Using identity, $x^2 - 2xy + y^2= (x - y)^2$
Here, $x=2y$
$y=1$
$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$
$=(2y-1)^2$
$=(2y-1)(2y-1)$

(iii) $x^2-\frac{y^2}{100}$

Solution:

$x^2-\frac{y^2}{100} = x^2-(\frac{y}{10})^2$
Using identity, $x^2 - y^2= (x - y) (x y)$
Here, $x=x$
$y=\frac{y}{10}$

$x^2 - \frac{y^2}{100} = x^2-(\frac{y}{10})^2$
$=(x-\frac{y}{10})(x+\frac{y}{10})$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii)$(-2x+3y+2z)^2$
(iv)$(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi)$(\frac{1}{4}a-\frac{1}{2}b+1)^2$

Solutions:

(i) **$(x+2y+4z)^2$**

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$$
$$= x^2+4y^2+16z^2+4xy+16yz+8xz$$

(ii) **$(2x-y+z)^2$**

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$$
$$= 4x^2+y^2+z^2-4xy-2yz+4xz$$

(iii) **$(-2x+3y+2z)^2$**

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$$
$$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$$

(iv) **$(3a - 7b - c)^2$**

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$$(3a - 7b - c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a\times -7b)+(2\times -7b\times -c)+(2\times -c\times 3a)$$

$$= 9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$y = 5y$

$z = -3z$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+(2\times -2x \times 5y)+(2\times 5y \times -3z)+(2\times -3z \times -2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $(\frac{1}{4}a - \frac{1}{2}b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = \frac{1}{4}a$

$y = -\frac{1}{2}b$

$z = 1$

$(\frac{1}{4}a - \frac{1}{2}b + 1)^2 = (\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+(1)^2+(2\times\frac{1}{4}a \times -\frac{1}{2}b)+(2\times -\frac{1}{2}b\times 1)+(2\times 1\times\frac{1}{4}a)$

$= \frac{1}{16}a^2+\frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b+\frac{2}{4}a$

$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab - b + \frac{1}{2}a$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x)^2+(3y)^2+(-4z)^2+(2\times 2x\times 3y)+(2\times 3y\times -4z)+(2\times -4z\times 2x)$

$= (2x+3y-4z)^2$

$= (2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$=(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{2}z\times-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $(\frac{3}{2}x+1)^3$

(iv) $(x-\frac{2}{3}y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times 2x\times 1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(2a-3b)^3=(2a)^3-(3b)^3-(3\times 2a\times 3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $(\frac{3}{2}x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(\frac{3}{2}x+1)^3=(\frac{3}{2}x)^3+1^3+(3\times\frac{3}{2}x\times 1)(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$

$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Solution:

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$\left(x-\frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - \left(3\times x\times\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= (x)^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= (x)^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

7. Evaluate the following using suitable identities:

(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(99)^3 = (100-1)^3$

$= (100)^3 - 1^3 - (3\times 100\times 1)(100-1)$

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + 2^3 + (3\times 100\times 2)(100+2)$

$= 1000000 + 8 + 600(100+2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(998)^3 = (1000-2)^3$

$= (1000)^3 - 2^3 - (3\times 1000\times 2)(1000-2)$

$= 1000000000 - 8 - 6000(1000-2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3-b^3-12a^2b+6ab^2$
(iii) $27 - 125a^3 - 135a + 225a^2$
(iv) $64a^3-27b^3-144a^2b+108ab^2$
(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 \quad =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$
$=(2a+b)^3$
$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ is used.

(ii) $8a^3-b^3-12a^2b+6ab^2$

Solution:

The expression, $8a^3-b^3-12a^2b+6ab^2$ can be written as $(2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$

$8a^3-b^3-12a^2b+6ab^2 \quad =(2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$
$=(2a-b)^3$
$=(2a-b)(2a-b)(2a-b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iii) $27 - 125a^3 - 135a + 225a^2$

Solution:

The expression, $27 - 125a^3 - 135a + 225a^2$ can be written as $3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$ $27-125a^3-135a+225a^2=3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$
$=(3-5a)^3$
$=(3-5a)(3-5a)(3-5a)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iv)64a3–27b3–144a^{2}b+108ab^2

Solution:

The expression, $64a^3-27b^3-144a^2b+108ab^2$ can be written as $(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$ $64a^3-27b^3-144a^2b+108ab^2=(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$
$=(4a-3b)^3$
$=(4a-3b)(4a-3b)(4a-3b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solution:

The expression, $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can be written as $(3p)^3 - (\frac{1}{6})^3 - 3(3p)^2(\frac{1}{6}) + 3(3p)(\frac{1}{6})^2$

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - (\tfrac{1}{6})^3 - 3(3p)^2(\tfrac{1}{6}) + 3(3p)(\tfrac{1}{6})^2$$
$$= (3p - \tfrac{1}{6})^3$$
$$= (3p - \tfrac{1}{6})(3p - \tfrac{1}{6})(3p - \tfrac{1}{6})$$

9. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$
(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2-3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)-3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2-xy)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

We know that,$(x-y)^3 = x^3-y^3-3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)^3+3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)[(x-y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3-y^3=(x-y)[(x^2+y^2-2xy)+3xy]$

$\Rightarrow x^3+y^3=(x-y)(x^2+y^2+xy)$

10. Factorize each of the following:

(i) $27y^3+125z^3$
(ii) $64m^3-343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 \quad =(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2-xy+y^2)$

$\therefore 27y^3+125z^3 =(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$ can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 \quad =(4m)^3-(7n)^3$

We know that, $x^3-y^3=(x-y)(x^2+xy+y^2)$

$\therefore 64m^3-343n^3 = (4m)^3-(7n)^3$
$= (4m+7n)[(4m)^2+(4m)(7n)+(7n)^2]$
$= (4m+7n)(16m^2+28mn+49n^2)$

11. Factorise : $27x^3+y^3+z^3-9xyz$

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz \quad =(3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\therefore 27x^3+y^3+z^3-9xyz \quad =(3x)^3+y^3+z^3-3(3x)(y)(z)$
$=(3x+y+z)(3x)^2+y^2+z^2-3xy-yz-3xz$
$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify that: $x^3+y^3+z^3-3xyz=^1(x+y+z)[(x-$

$y)^2+(y-z)^2+(z-x)^2_2]$

Solution:

We know that,
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz \quad =\frac{1}{2}\times(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$
$=\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$
$=\frac{1}{2}(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$
$=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,
$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$
Now, according to the question, let $(x + y + z) = 0$,
then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz =0$
$\Rightarrow \quad x^3+y^3+z^3 =3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $\mathbf{(28)^3+(-15)^3+(-13)^3}$

(i) $\mathbf{(-12)^3+(7)^3+(5)^3}$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let a= −12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, −12+7+5=0

$\therefore (-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3 \times -12 \times 7 \times 5$

$= -1260$

(ii) $\mathbf{(28)^3+(-15)^3+(-13)^3}$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$\therefore (28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

$=16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $\mathbf{25a^2-35a+12}$

(ii) Area : $\mathbf{35y^2+13y-12}$

Solution:

(i) Area : $25a^2-35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=25×12=300
We get -15 and -20 as the numbers [-15+-20=-35 and -3×-4=300]

$25a^2–35a+12 \;=25a^2–15a–20a+12$
$=5a(5a–3)–4(5a–3)$
$=(5a–4)(5a–3)$
Possible expression for length = 5a – 4
Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$
Using the splitting the middle term method,
We have to find a number whose sum= 13 and product=35× −12=420
We get -15 and 28 as the numbers [-15+28=-35 and -15× 28=420]

$35y^2+13y–12 \;=35y^2–15y+28y–12$
$=5y(7y–3)+4(7y–3)$
$=(5y+4)(7y–3)$
Possible expression for length = (5y + 4)
Possible expression for breadth = (7y – 3)

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:
(i) Volume : $3x^2–12x$
$3x^2–12x$ can be written as 3x(x – 4) by taking 3x out of both the terms.
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = (x – 4)

(ii) Volume : $12ky^2+8ky–20k$
$12ky^2+8ky–20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.
$12ky^2+8ky–20k =4k(3y^2+2y–5)$
[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$
$=4k[y(3y+5)–1(3y+5)]$
$=4k(3y+5)(y–1)$

Possible expression for length = 4k
Possible expression for breadth = (3y +5)
Possible expression for height = (y -1)

CHAPTER 2
POLYNOMIALS

Introduction

In this chapter, we shall study a particular type of algebraic expression, called polynomial, the Remainder Theorem and Factor Theorem and their use in the factorisation of polynomials.

ALGEBRAIC EXPRESSION

- The combination of constants and variables are called algebraic expression.

  For Example:-
  $2x+5$
  $-3xy + 4$
  $2a+5b +50$
  $-6x+5y+3z + 10$

Polynomials in One Variable

- $x^3 - x^2 + 4x + 7$ is a polynomial in one variable x.
- $3y^2 + 5y$ is a polynomial in one variable y.
- $t^2 + t + 4$ is a polynomial in one variable t.
- $6x^{3/4}+5$ is not a polynomial . WHY?

IMPORTANT TERMS

- **POLYNOMIAL:**

  AN ALGEBRAIC EXPRESSION IN WHICH THE VARIABLE INVOLVED HAVE ONLY NON-NEGATIVE INTEGRAL POWERS IS CALLED A POLYNOMIAL.

  $3y^2 + 5y$ IS A POLYNOMIAL.

  $6x^{3/4}+5$ IS NOT A POLYNOMIAL .

- **CONSTANT:**

  A SYMBOL HAVING A FIXED NUMERICAL VALUE IS CALLED A CONSTANT.

  FOR EXAMPLE IN POLYNOMIAL $-3xy + 4$, THE CONSTANTS ARE -3 AND 4 .

  IN POLYNOMIAL $2x+5$, THE CONSTANTS ARE 2 AND 5 .

- **VARIABLE:**

  A SYMBOL WHICH MAY BE ASSIGNED DIFFERENT NUMERICAL VALUES IS CALLED A VARIABLE.
  FOR EXAMPLE IN POLYNOMIAL -3XY + 4,
  X AND Y ARE VARIBLES.

- **COEFFICIENT:**

  THE NUMERICAVALUE (NUMBER/CONSTANT) THAT IS MULTIPLIED TO THE VARIABLE IN A TERM OF AN ALGEBRAIC EXPRESSION IS CALLED NUMERICAL COEFFICIENT.
  FOR EXAMPLE IN POLYNOMIAL 2X+5,
  THE NUMERICAL COEFFICIENT OF X IS 2.

DEGREE OF A POLYNOMIAL

- The highest power of the variable in a polynomial is called the degree of the polynomial.

  For example the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7. The degree of the polynomial $5y^6 - 4y^2 - 6$ is 6.

CONSTANT POLYNOMIALS

- A polynomial containing one term only, consisting of a constant is called constant polynomial.

Note: The degree of a non-zero constant polynomial is zero.
For example the degree of the polynomial 51 is 0.

ZERO POLYNOMIAL

- 0 IS A ZERO POLYNOMIAL.
- The degree of the zero polynomial is not defined.

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TYPES OF ALGEBRAIC EXPRESSION (ON THE BASIS OF TERMS)

- MONOMIALS
- BINOMIALS
- TRINOMIALS
- POLYNOMIALS

MONOMIALS

- Polynomials having only one term are called monomials (‘mono’ means ‘one’).
- For example the polynomials

  $2x$

  $5x^3$

  y

  u^4

BINOMIALS

- Polynomials having only two terms are called binomials ('bi' means 'two').

Observe each of the following polynomials:

$$p(z) = z + 1$$
$$q(x) = x^2 - x$$
$$r(y) = y + 1$$
$$t(u) = u^{43} - u$$

- How many terms are there in each of these?

TRINOMIALS

- Polynomials having only three terms are called trinomials ('tri' means 'three').

- Some examples of trinomials are

$$p(x) = x + x^2 + \pi,$$
$$q(x) = 2 + x - x^2,$$
$$r(u) = u + u^2 - 2,$$
$$t(y) = y^4 + y + 5.$$

POLYNOMIAL

- Polynomials having many terms are called polynomials.

  For example $p(x) = 3x^7 – 4x^6 + x + 9$ has more than three terms is called a polynomial.

- A polynomial in one variable x of degree n is an expression of the form:

  $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
  where $a_0, a_1, a_2, \ldots, a_n$ are constants and $a_n \neq 0$.

TYPES OF POLYNOMIALS (ON THE BASIS OF DEGREE)

- A polynomial of degree one is called a linear polynomial.

General form: $ax + b, where\ a \neq 0$

Eg. x + 2

- A polynomial of degree two is called a quadratic polynomial.

General form: $ax^2 + bx + c, where\ a \neq 0$

Eg. q^2 + 21

- A polynomial of degree three a cubic polynomial.

General form: $ax^3 + bx^2 + cx + d, where\ a \neq 0$

Eg. y^3

- A polynomial of degree four is called a biquadratic polynomial.

Eg. $p^4 - p^3 + p^2 - p + 33$

EXERCISE 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

(ii) $y^2 + 2^{1/2}$

(iii) $3X-7X^{8/9}$

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 + x^3$

(iii) $2x + \pi + 7$

(iv) 21

- Give one example of a binomial of degree 35.
- Give one example of a monomial of degree 100.

Write the degree of each of the following polynomials:

POLYNOMIALS	DEGREE
$5x^3 + 4x^2 + 7x$	
$4 - y^2$	
3	

Classify the following as linear, quadratic and cubic polynomials:

POLYNOMIALS	DEGREE	KIND OF POLYNOMIAL
$x^3 + x$	3	
$y + y^2 + 4$	2	
$1 + x$	1	

SOLVE EX. 2.1 IN YOUR CW NB

ZERO OF A POLYNOMIAL

- A zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.
- Consider the polynomial $p(x) = x - 1$.

  What is $p(1)$?

  $p(1) = 1 - 1 = 0$.

  As $p(1) = 0$, we say that 1 is a zero of the polynomial $p(x)$.

Example 3 :

- Check whether –2 and 2 are zeroes of the polynomial $x + 2$.

- Solution : Let $p(x) = x + 2$.

  Then $p(2) = 2 + 2 = 4$,

  $p(-2) = -2 + 2 = 0$

  Therefore, –2 is a zero of the polynomial $x + 2$, but 2 is not.

Example 4 :

- Find a zero of the polynomial $p(x) = 2x + 1$

- Solution : Finding a zero of $p(x)$, is the same as solving the equation $p(x) = 0$

  Now, $2x + 1 = 0$ gives us $x = -\frac{1}{2}$
  So, $-\frac{1}{2}$ is a zero of the polynomial $2x + 1$.

Solve Ex. 2.2 in your CW NB

Zero of a Polynomial

In a polynomial with one variable, the value of the variable where the value of the polynomial becomes zero, is the **zero of a polynomial**.

HOME WORK

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$

(ii) $y^2 + 23$

(iii) $4x - 31y^3 + z^{30}$

Q2. Write the degree of the polynomial: $7x^3 + 4x^2 - 3x + 12$

Q3. Classify the following polynomials on the basis of their degree:

(i) $x + x^2 + 4$

(ii) $3y$

(iii) 7

Q4. Write a trinomial of degree 25.

Q5. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

Remainder Theorem

$$
\begin{array}{r|l}
 & 3x - 2 \\
\hline
x + 1 & 3x^2 + x - 1 \\
 & 3x^2 + 3x \\
 & - \quad - \\
\hline
 & -2x - 1 \\
 & -2x - 2 \\
 & + \quad + \\
\hline
 & 1
\end{array}
$$

Find the remainder when $p(x) = 3x^2 + x - 1$ is divided by $g(x) = x + 1$.

Soln.

Let $g(x) = x + 1 = 0$

$\Rightarrow \quad x = -1$

$p(-1) = 3(-1)^2 + (-1) - 1$

$= 3 - 1 - 1$

$= 1$

Remainder Theorem

- Remainder Theorem :

  Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

Example 7 :

- Find the remainder when the polynomial $3x^4 - 4x^3 - 3x - 1$ is divided by $x - 1$.
- Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$.

- Solution :

The root of $x + 1 = 0$ is $x = -1$.

We see that $p(-1) = (-1)^3 + 1$

$= -1 + 1 = 0$,

which is equal to the remainder.

Example 10 :

- Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$

Solution :

As you know, q(t) will be a multiple of 2t + 1 only, if 2t + 1 divides q(t) leaving remainder zero.

Let, $2t + 1 = 0$, $\Rightarrow t = -\frac{1}{2}$

Now, $q(t) = 4t^3 + 4t^2 - t - 1$

$q(-\frac{1}{2}) = 4(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 - (-\frac{1}{2}) - 1 = 0$

As the remainder obtained on dividing q(t) by 2t + 1 is 0. So, 2t + 1 is a factor of the given polynomial q(t), That is q(t) is a multiple of 2t + 1.

HW: Ex. 2.3 Q1, Q2 and Q3

Factorisation of Polynomials

- Factor Theorem :

    If p(x) is a polynomial of degree $n \geq 1$ and a is any real number, then

    (i) $x - a$ is a factor of p(x)

    $\Rightarrow$ p(a) = 0, and

    (ii) p(a) = 0

    $\Rightarrow$ x – a is a factor of p(x).

Example 11 : Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.

- Solution : The zero of $x + 2$ is $–2$.

  Let $p(x) = x^3 + 3x^2 + 5x + 6$ and $s(x) = 2x + 4$

  Then, $p(–2) = (-2)^3 + 3(-2)^2 + 5(–2) + 6$

  $= –8 + 12 – 10 + 6 = 0$

  So, by the Factor Theorem, $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$.

  Again, $s(–2) = 2(–2) + 4 = 0$

  So, $x + 2$ is a factor of $2x + 4$.

Example 12 :

- Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.

- Solution : As $x - 1$ is a factor of

  $p(x) = 4x^3 + 3x^2 - 4x + k$

  $\Rightarrow p(1) = 0$

  $\Rightarrow p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k = 0$

  $\Rightarrow 4 + 3 - 4 + k = 0$

  $\Rightarrow k = -3$

Factorisation by splitting the middle term

- Example 13 : Factorise $6x^2 + 17x + 5$

- Solution:

  Let us look for the pairs of factors of 30 (6×5)

  Some are 1 and 30, 2 and 15, 3 and 10, 5 and 6.

  Of these pairs, 2 and 15 will give us

$$
\begin{aligned}
6x^2 + 17x + 5 &= 6x^2 + 2x + 15x + 5 \\
&= 2x(3x + 1) + 5(3x + 1) \\
&= (3x + 1)(2x + 5)
\end{aligned}
$$

To determine the factors in case of cubic polynomials

(i) If sum of coefficients is zero then (x - 1) is the factor.

(ii) If sum of coefficients of even powers of x = sum of coefficients of odd powers of x, then (x + 1) is the factor.

(iii) If both are not the factors, then to check for other factors we have to apply trial and error method

Example 15 : Factorise $x^3 - 23x^2 + 142x - 120$.

- Solution :

  Let $p(x) = x^3 - 23x^2 + 142x - 120$

  Sum of coefficients = 1-23+142-120 = 0

  $\therefore$ $(x - 1)$ is the factor

  $$x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$$
  $$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$$
  $$= (x - 1)(x^2 - 22x + 120)$$

  Now by splitting the middle term,

  $x^2 - 22x + 120 = (x - 12)(x - 10)$

  So, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$

Note: We can also divide the given polynomial by $(x - 1)$ to get the quotient $(x^2 - 22x + 120)$ and proceed further.

FACTORISE $x^3+13x^2+32x+20$

Given polynomial is $x^3+13x^2+32x+20$

Sum of coefficients of even powers of x
=sum of coefficients of odd power of x.
$\therefore (x+1)$ is the factor.
The remaining factors can be found by long division method

Quotient $=x^2+12x+20$
$=x^2+10x+2x+20$
$=x(x+10)+2(x+10)$
$=(x+2)(x+10)$

Hence, $x^3+13x^2+32x+20 = (x+1)(x+2)(x+10)$

Algebraic Identities

1. $(x + y)^2 = x^2 + 2xy + y^2$
2. $(x - y)^2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

6. $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
$$= x^3 + y^3 + 3x^2y + 3xy^2$$

---

7. $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
$$= x^3 - y^3 - 3x^2y + 3xy^2$$

8. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)$
$$(x^2 + y^2 + z^2 - xy - yz - zx)$$

9. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

10. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

1. Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

$= x^2 + (4 + 10)x + (4)(10)$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

$= x^2 + (8 - 10)x + (8)(-10)$

$= x^2 - 2x - 80$

2. Evaluate the following products without multiplying directly:

(i) 103×107

$(100 + 3)\ (100 + 7)$

$= (100)^2 + (3 + 7).100 + (3)(7)$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

$(100 - 5)\ (100 - 4)$

$= (100)^2 + (-5-4).100 + (-5)(-4)$

$= 10000 - 900 - 20$

$= 9079$

3. Factorise the following using appropriate identity

(i) $4y^2 - 4y + 1$

$=(2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(ii) $x^2 - \frac{y2}{100}$

$=(x)^2 - \left(\frac{y}{10}\right)^2$

$=\left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $(2x - y + z)^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$=(2x)^2 + (3y)^2 + (-4z)^2 +$

$2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + y - z)^2$

6. Write the following cubes in expanded form:

(i) $(2a - 3b)^3$

$= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

7. Evaluate the following using suitable identities:

(i) $(998)^3$

$= (1000 - 2)^3$

$=(1000)^3-(2)^3-3(1000)(2)(1000-2)$

$= 1000000000-8-6000(998)$

$= 999999992- 5988000$

$= 994011992$

8. Factorise each of the following:

- (i) $27 - 125a^3 - 135a + 225a^2$

Identity : $x^3 - y^3 - 3xy(x - y) = (x - y)^3$

$=3^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

9. Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3$

$= x^3 + y^3$

10. Factorise each of the following:

(i) $64m^3 – 343n^3$

Identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 – 343n^3 = (4m)^3 – (7n)^3$

$= (4m – 7n)(16m^2 + 28mn + 49n^2)$

Factorise :

- $8x^3 + y^3 + 27z^3 - 18xyz$

- Solution : $8x^3 + y^3 + 27z^3 - 18xyz$

$$= (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$$

$$= (2x + y + 3z)\left[(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)\right]$$

$$= (2x + y + 3z)\left(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz\right)$$

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

- We know that:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Putting $x+y+z=0$,

$$\therefore x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = 0$$

$$\therefore x^3 + y^3 + z^3 = 3xyz.$$

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

- Solution:

  Here, $x + y + z = (-12) + (7) + (5) = 0$

  So, $x^3 + y^3 + z^3 = 3xyz$

  $(-12)^3 + (7)^3 + (5)^3 = 3\,(-12)(7)(5)$

  $= -1260$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

- Area : $25a^2 - 35a + 12$

$$= 25a^2 - 15a - 20a + 12$$
$$= 5a(5a - 3) - 4(5a - 3)$$
$$= (5a - 4)(5a - 3)$$

LENGTH	BREADTH
5a - 4	5a – 3
5a – 3	5a - 4

- Solve Ex. 2.5 in your note book.

Summary

. A polynomial of one term is called a monomial.
. A polynomial of two terms is called a binomial.
. A polynomial of three terms is called a trinomial.
. A polynomial of degree one is called a linear polynomial.
. A polynomial of degree two is called a quadratic polynomial.
. A polynomial of degree three is called a cubic polynomial.
. A real number 'a' is a zero of a polynomial $p(x)$ if $p(a) = 0$. In this case, a is also called a root of the equation $p(x) = 0$.
. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial.
. Remainder Theorem : If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the linear polynomial $x – a$, then the remainder is $p(a)$.
. Factor Theorem : $x – a$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also, if $x – a$ is a factor of $p(x)$, then $p(a) = 0$.

ALGEBRAIC IDENTITIES

1. $(x + y)2 = x^2 + 2xy + y^2$
2. $(x - y)2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)(x - y)$
4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
   $= x^3 + y^3 + 3x^2y + 3xy^2$
7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
   $= x^3 - y^3 - 3x^2y + 3xy^2$
8. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)$
   $(x^2 + y^2 + z^2 - xy - yz - zx)$
9. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
10. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH. 3 COORDINATE GEOMETRY

MIND MAP

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 COORDINATES OF A POINT

Q.1 Write the perpendicular distance of point P (3, 4) from the y –axis.

Q.2 Write the point which lies on y-axis at a distance of 5 units in the negative direction of y-axis

Q.3 Write the point whose ordinate is 4 and which lies on y-axis.

Q.4 The points whose abscissa and ordinate have different signs will lie in which quadrant?

Q.5 Write the abscissa of all the points on the x-axis.

Q.6 If the coordinates of the two points are P (–2, 3) and Q (–3, 5), then find (abscissa of P) – (abscissa of Q).

Q.7 What is the name to the horizontal and vertical line in a coordinate system? The origin is indicated by what coordinates?

Q.8 How many quadrants are there in the Cartesian Plane?

Q.9 In which quadrant will the coordinates (-2,3) lie?

Q.10 In which quadrant will the coordinates (-3, -4) lie?

Q.11 What is the abscissa and the ordinate in the coordinates (3, -5)

Q.12 Write the abscissa and the ordinate of the coordinates of the points (0,3) (3,0) (0, 0).

TYPE: 2 PLOTTING OF POINTS IN THE CARTESIAN PLANE

Q.1 Plot the following point on the number line using a graph and join the points.

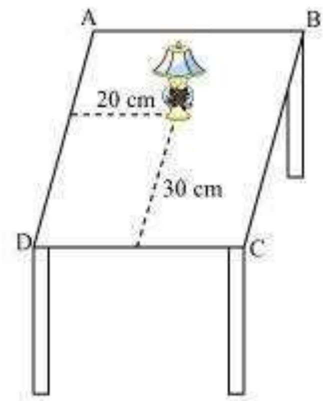
(a) (3, -4) (b) (-3, 2)

Q.2 Plot the points P (1, 0), Q (4, 0) and S (1, 3). Find the coordinates of the point R such that PQRS is a square.

Question 1:
Exercise 3.1

How will you describe the position of a table lamp on your study table to another person?

Answer:



Consider that the lamp is placed on the table. Choose two adjacent edges, DC and AD. Then, draw perpendiculars on the edges DC and AD from the position of lamp and measure the lengths of these perpendiculars. Let the length of these perpendiculars be 30 cm and 20 cm respectively. Now, the position of the lamp from the left edge (AD) is 20 cm and from the lower edge (DC) is 30 cm. This can also be written as (20, 30), where 20 represents the perpendicular distance of the lamp from edge AD and 30 represents the perpendicular distance of the lamp from edge DC.

Question 2:

(Street Plan): A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

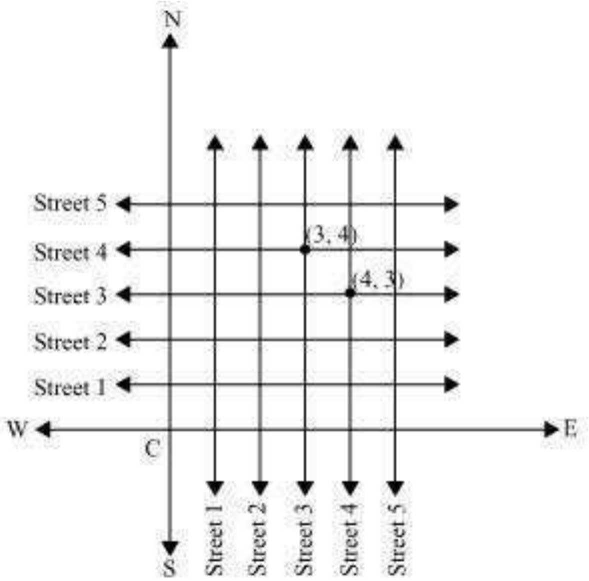
All the other streets of the city run parallel to these roads and are 200 m apart. There are about 5 streets in each direction. Using 1 cm = 100 m, draw a model of the city on your notebook Represent the roads/streets by single lines.

There are many cross-streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner: If the 2^{nd} street running in the North-South direction and 5^{th} in the East-West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) How many cross - streets can be referred to as (4, 3).

(ii) How many cross - streets can be referred to as (3, 4).

Answer:



Both the cross-streets are marked in the above figure. It can be observed that there is only one cross-street which can be referred as (4, 3), and again, only one which can be referred as (3, 4).

Question 1:

Exercise 3.2

Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?

(iii) Write the name of the point where these two lines intersect.

Answer:

(i) The name of horizontal lines and vertical lines drawn to determine the position of any point in the Cartesian plane is x-axis and y-axis respectively.

(ii) The name of each part of the plane formed by these two lines, x-axis and y-axis, is quadrant (one-fourth part).

(iii) The name of the point where these two lines intersect is the origin.

Question 2:

See the given figure, and write the following:

(i) The coordinates of B.

(ii) The coordinates of C.

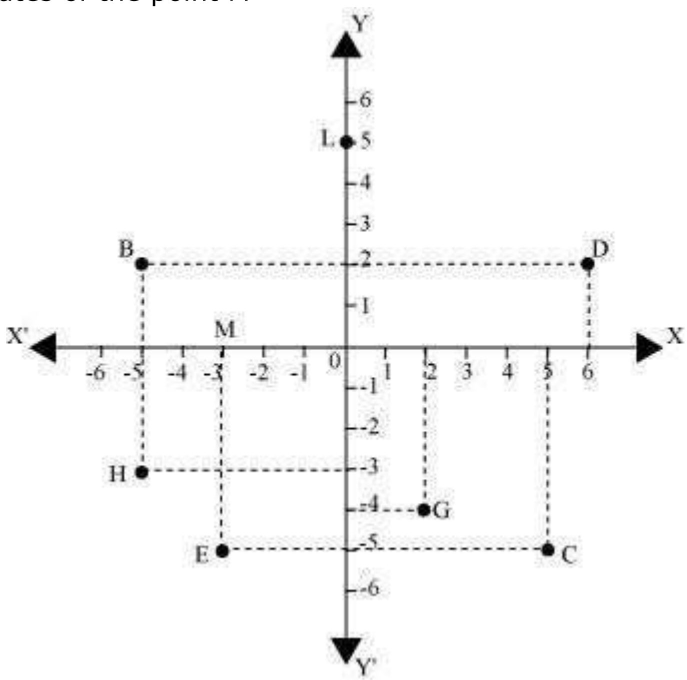
(iii) The point identified by the $(-3,-5)$ coordinates.

(iv) The point identified by the $(2,-4)$ coordinates (v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M



Answer:

(i) The x-coordinate and the y-coordinate of point B are −5 and 2 respectively. Therefore, the coordinates of point B are (−5, 2).

(ii) The x-coordinate and the y-coordinate of point C are 5 and −5 respectively. Therefore, the coordinates of point C are (5, −5).

(iii) The point whose x-coordinate and y-coordinate are −3 and −5 respectively is point E.

Question 1:

(iv) The point whose x-coordinate and y-coordinate are 2 and −4 respectively is point G.

(v) The x-coordinate of point D is 6. Therefore, the abscissa of point D is 6.

(vi) The y-coordinate of point H is −3. Therefore, the ordinate of point H is −3.

(vii) The x-coordinate and the y-coordinate of point L are 0 and 5 respectively. Therefore, the coordinates of point L are (0, 5).

(viii) The x-coordinate and the y-coordinate of point M are −3 and 0 respectively. Therefore, the coordinates of point M is (−3, 0).

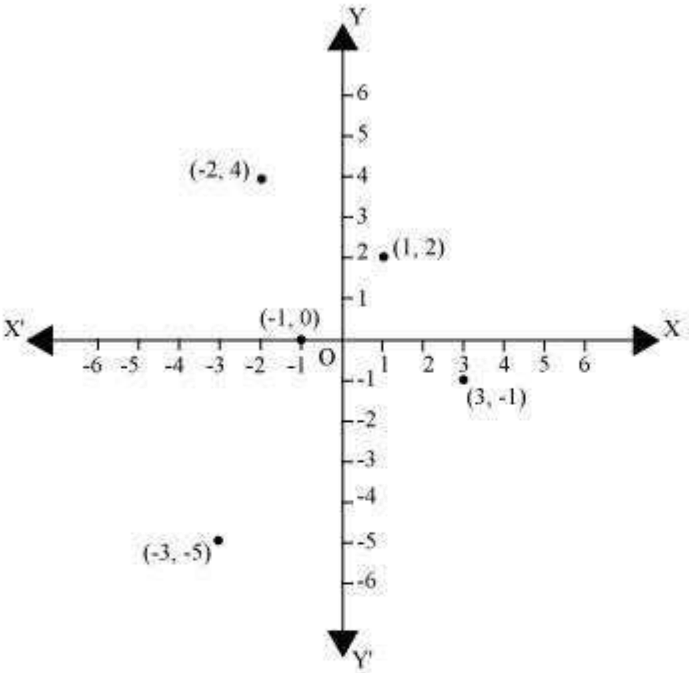
Exercise 3.3

$$(-2,4),(3,-1),(-1,0),(1,2)$$

In which quadrant or on which axis do each of the points

and $(-3,-5)$ lie? Verify your answer by locating them on the Cartesian plane.

Answer:



The point $(-2,4)$ lies in the IInd quadrant in the Cartesian plane because for point $(-2,4)$, x-coordinate is negative and y-coordinate is positive.

Again, the point $(3,-1)$ lies in the IV th quadrant in the Cartesian plane because for $(3,-1)$ point , x-coordinate is positive and y-coordinate is negative.

$(-1,0)$ The point lies on negative x-axis because for point $(-1,0)$, the value of ycoordinate is zero and the value of x-coordinate is negative.

Question 1:

The point $(1,2)$ lies in the I[st] quadrant as for point $(1,2)$, both x and y are positive.

The point $(-3,-5)$ lies in the III[rd] quadrant in the Cartesian plane because for point $(-3,-5)$, both x and y are negative.

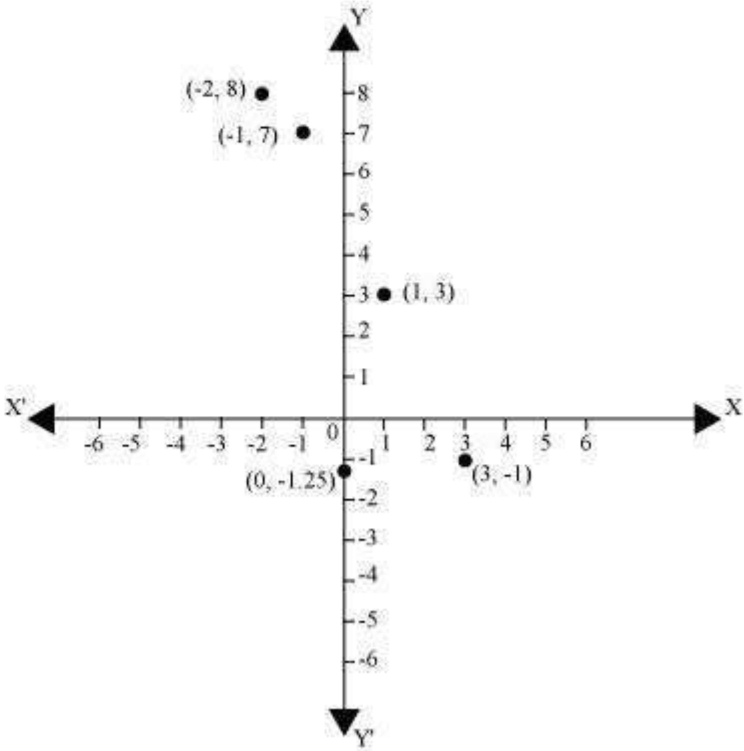
Question 2:

Plot the point (x, y) given in the following table on the plane, choosing suitable units of distance on the axis.

x	− 2	− 1	0	1	3
y	8	7	1.25	3	− 1

Answer:

The given points can be plotted on the Cartesian plane as follows.


Y
X'
X
Y'
(-2, 8)
(-1, 7)
(1, 3)
(0, -1.25)
(3, -1)
0
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER4 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

MIND MAP

This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 FORMATION OF LINEAR EQUATIONS IN TWO VARIABLES

Q.1 Write the equation $12x + 3y = 20$ in the form of $ax + by + c = 0$ and find out the values of a, b and c.

Q.2 Write $2x = 3y + 5$ in standard form of equation in two variables.

Q.3 Write the equation of the line parallel to the y-axis.

Q.4 Write the equation of the line parallel to x-axis.

TYPE: 2 GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

Q.1 The taxi fare in a city is as follows: For the first kilometer, the fare is ₹20 and for the subsequent distance it is ₹6 per km. Taking x km as the distance covered and ₹y as the total fare, write a linear equation for this information and draw its graph.

Q.2 Draw the graph of equation $3x + 4y = 24$.

Q.3 Draw the graph of equation $2y + 5 = 0$.

Class IX Chapter 4– Linear Equations in Two Variables Maths

Exercise 4.1 Question 1:

The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be Rs x and that of a pen to be Rs y.) Answer:

Let the cost of a notebook and a pen be x and y respectively.

Cost of notebook = 2 × Cost of pen $x = 2y$ $x - 2y = 0$

Question 2:

Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b, c in each case:

(i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$

(iv) $x = 3y$ (v) $2x = -5y$ (vi) $3x + 2 = 0$

(vii) y − 2 = 0 (viii) 5 = 2x

Answer:

(i) $2x + 3y = 9.3\bar{5}$

$2x + 3y - 9.3\bar{5}$ = 0

a = 2, b = 3, $c = -9.3\bar{5}$

(ii) $x - \frac{y}{5} - 10 = 0$

a = 1, b = $-\frac{1}{5}$, c = −10

(iii) − 2x + 3 y = 6

Comparing this equation with ax + by + c = 0,

Comparing this equation with ax + by + c = 0,

− 2x + 3 y − 6 = 0

Comparing this equation with ax + by + c = 0, a = −2, b = 3, c = −6 (iv) x = 3y

1x − 3y + 0 = 0

Comparing this equation with ax + by + c = 0, a = 1, b = −3, c = 0 (v) 2x = −5y

2x + 5y + 0 = 0

Comparing this equation with ax + by + c = 0, a = 2, b = 5, c = 0

(vi) 3x + 2 = 0

3x + 0.y + 2 = 0

Comparing this equation with ax + by + c = 0, a = 3, b = 0, c = 2 (vii) y − 2 = 0

0.x + 1.y − 2 = 0

Comparing this equation with $ax + by + c = 0$, $a = 0$, $b = 1$, $c = -2$ (vii) $5 = 2x$

$-2x + 0.y + 5 = 0$

Comparing this equation with $ax + by + c = 0$, a

$= -2$, $b = 0$, $c = 5$

Exercise 4.2 Question 1:

Which one of the following options is true, and why? y

$= 3x + 5$ has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions Answer:

$y = 3x + 5$ is a linear equation in two variables and it has infinite possible solutions. As for every value of x, there will be a value of y satisfying the above equation and vice-versa.

Hence, the correct answer is (iii).

Question 2:

Write four solutions for each of the following equations:

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$ Answer:
(i) $2x + y = 7$

For $x = 0$, $2(0) + y = 7 \Rightarrow$

$y = 7$

Therefore, (0, 7) is a solution of this equation. For $x = 1$, $2(1) + y = 7 \Rightarrow y = 5$

Therefore, (1, 5) is a solution of this equation.

For $x = -1$,

$2(-1) + y = 7 \Rightarrow$

$y = 9$

Therefore, (−1, 9) is a solution of this equation.

For $x = 2$,

$2(2) + y = 7$

$\Rightarrow y = 3$

Therefore, (2, 3) is a solution of this equation.

(ii) $\pi x + y = 9$

For $x = 0$, $\pi(0)$

$+ y = 9$

$\Rightarrow y = 9$
Therefore, $(0, 9)$ is a solution of this equation.

For $x = 1$, $\pi(1) + y = 9 \Rightarrow y = 9 - \pi$

Therefore, $(1, 9 - \pi)$ is a solution of this equation.

For $x = 2$, $\pi(2) + y = 9 \Rightarrow y = 9 - 2\pi$

Therefore, $(2, 9 - 2\pi)$ is a solution of this equation.

For $x = -1$, $\pi(-1) + y = 9 \Rightarrow y = 9 + \pi$

$\Rightarrow (-1, 9 + \pi)$ is a solution of this equation.

(iii) $x = 4y$

For $x = 0$,

$0 = 4y \Rightarrow$

$y = 0$

Therefore, $(0, 0)$ is a solution of this equation.

For $y = 1$, $x = 4(1) = 4$

Therefore, (4, 1) is a solution of this equation.

For y = −1, x = 4(−1) $\Rightarrow$ x = −4

Therefore, (−4, −1) is a solution of this equation.

For x = 2,

2 = 4y

$\Rightarrow y = \frac{2}{4} = \frac{1}{2}$

Therefore, $\left(2, \frac{1}{2}\right)$ is a solution of this equation.

Question 3:

Check which of the following are solutions of the equation x − 2y = 4 and which are not:

(i) (0, 2 (ii) (2, 0) (iii) (4, 0)

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$ (v) (1, 1)

Answer:

(i) (0, 2)

Putting x = 0 and y = 2 in the L.H.S of the given equation, x

− 2y = 0 − 2× − 4 ≠ 4 2 =
L.H.S ≠ R.H.S

Therefore, (0, 2) is not a solution of this equation.

(ii) (2, 0)

Putting x = 2 and y = 0 in the L.H.S of the given equation, x

− 2y 2 − 2 × 0 = 2 ≠ 4 =

≠

L.H.S R.H.S

Therefore, (2, 0) is not a solution of this equation.

(iii) (4, 0)

Putting x = 4 and y = 0 in the L.H.S of the given equation, x

− 2y = 4 − 2(0)

= 4 = R.H.S

Therefore, (4, 0) is a solution of this equation.

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$

Putting $x=\sqrt{2}$ and $y=4\sqrt{2}$ in the L.H.S of the given equation,

$$x-2y=\sqrt{2}-2\left(4\sqrt{2}\right)$$
$$=\sqrt{2}-8\sqrt{2}=-7\sqrt{2}\neq 4$$

L.H.S ≠ R.H.S

Therefore, $\left(\sqrt{2}, 4\sqrt{2}\right)$ is not a solution of this equation.

(v) (1, 1)

Putting x = 1 and y = 1 in the L.H.S of the given equation, x

− 2y 1 − 2(1) = 1 − 2 = − 1 ≠ 4 =

L.H.S ≠ R.H.S

Therefore, (1, 1) is not a solution of this equation.

Question 4:

Find the value of k, if x = 2, y = 1 is a solution of the equation 2x + 3y = k.

Answer:

Putting x = 2 and y = 1 in the given equation,
2x + 3y = k
⇒

⇒ 2(2) + 3(1) = k
4 + 3 = k ⇒ k

= 7

Therefore, the value of k is 7.

Exercise 4.3 Question 1:

Draw the graph of each of the following linear equations in two variables:

$x + y = 4$ (ii) $x - y = 2$

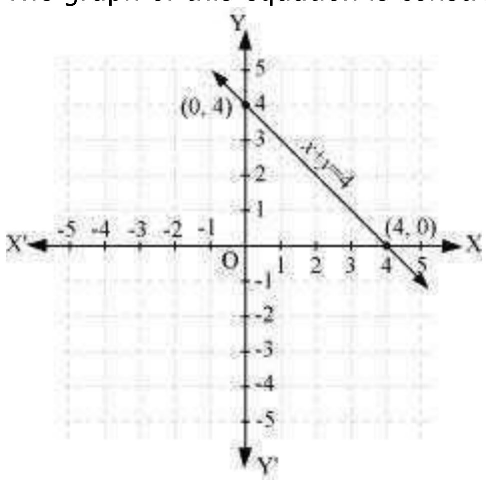
Answer:

(i) $x + y = 4$ (i) (iii) y = 3x (iv) 3 = 2x + y

It can be observed that x = 0, y = 4 and x = 4, y = 0 are solutions of the above equation. Therefore, the solution table is as follows.

x	0	4
y	4	0

The graph of this equation is constructed as follows.

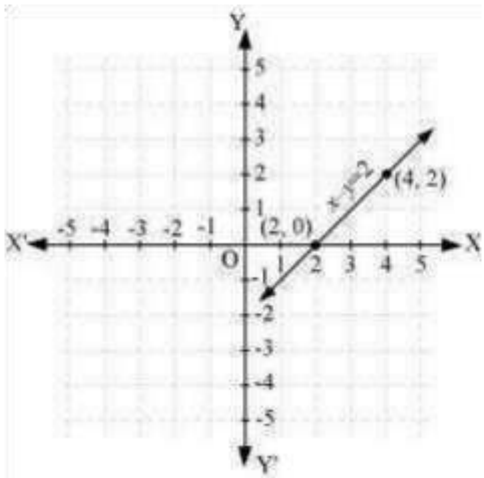


(ii) $x - y = 2$

It can be observed that x = 4, y = 2 and x = 2, y = 0 are solutions of the above equation. Therefore, the solution table is as follows.

x	4	2
y	2	0

The graph of the above equation is constructed as follows.

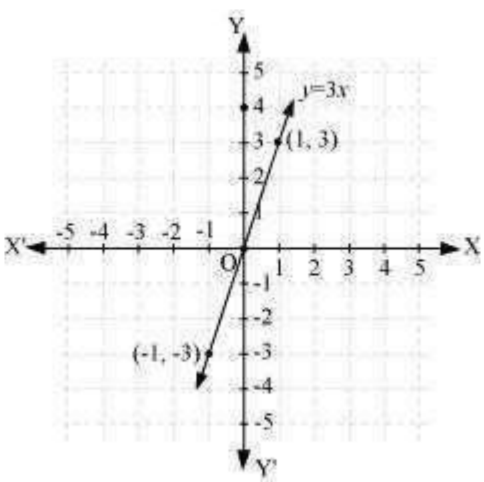


(iii) y = 3x

It can be observed that x = −1, y = −3 and x = 1, y = 3 are solutions of the above equation. Therefore, the solution table is as follows.

x	− 1	1
y	− 3	3

The graph of the above equation is constructed as follows.

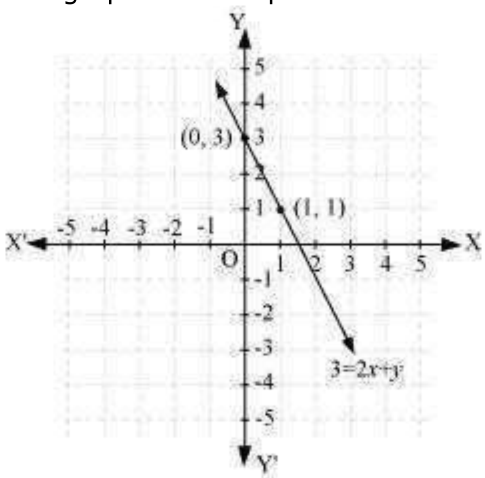


(iv) 3 = 2x + y

It can be observed that x = 0, y = 3 and x = 1, y = 1 are solutions of the above equation. Therefore, the solution table is as follows.

x	0	1
y	3	1

The graph of this equation is constructed as follows.



Question 2:

Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Answer:

It can be observed that point (2, 14) satisfies the equation 7x − y = 0 and x − y + 12 = 0.

Therefore, 7x − y = 0 and x − y + 12 = 0 are two lines passing through point (2, 14).

As it is known that through one point, infinite number of lines can pass through, therefore, there are infinite lines of such type passing through the given point.

Question 3:

If the point (3, 4) lies on the graph of the equation 3y = ax + 7, find the value of a.

Answer:

Putting x = 3 and y = 4 in the given equation, 3y = ax + 7

3 (4) = a (3) + 7

5 = 3a

$a = \frac{5}{3}$

Question 4:

The taxi fare in a city is as follows: For the first kilometre, the fares is Rs 8 and for the subsequent distance it is Rs 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.

Answer:

Total distance covered = x km

Fare for 1st kilometre = Rs 8

Fare for the rest of the distance = Rs (x − 1) 5

Total fare = Rs [8 + (x − 1) 5] y = 8 + 5x − 5 y

= 5x + 3

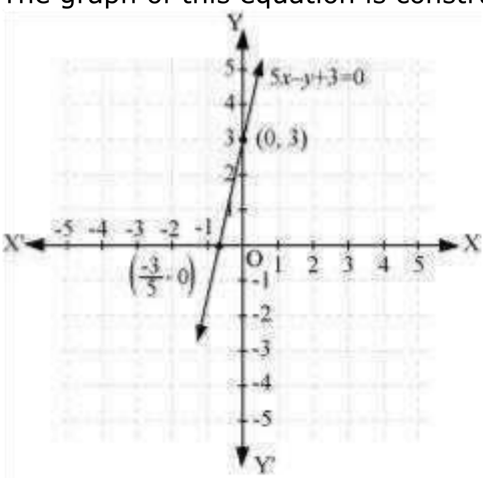
5x − y + 3 = 0

It can be observed that point (0, 3) and $\left(-\frac{3}{5}, 0\right)$ satisfies the above equation.

Therefore, these are the solutions of this equation.

x	0	$\frac{-3}{5}$
y	3	0

The graph of this equation is constructed as follows.



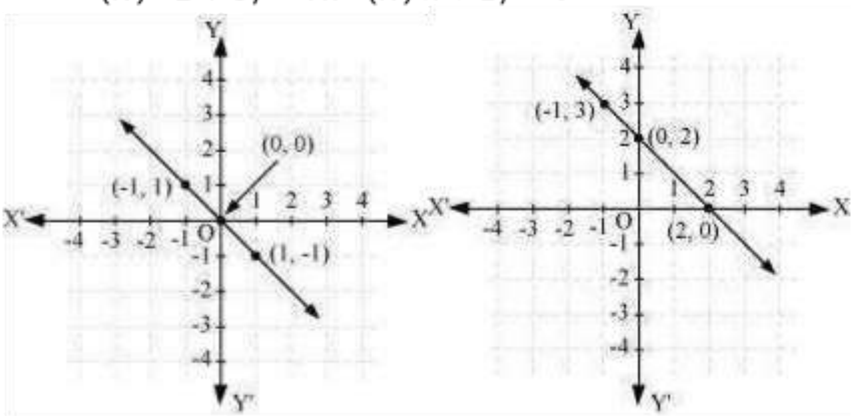
Here, it can be seen that variable x and y are representing the distance covered and the fare paid for that distance respectively and these quantities may not be negative. Hence, only those values of x and y which are lying in the 1st quadrant will be considered.

Question 5:

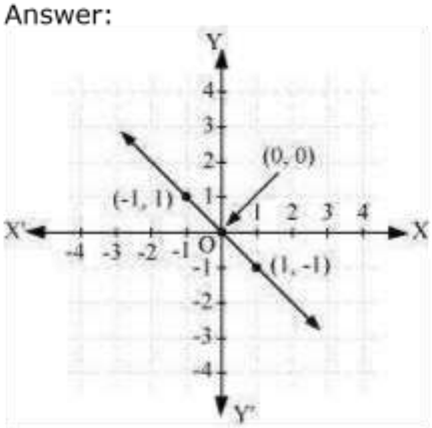
From the choices given below, choose the equation whose graphs are given in the given figures.

	For the first figure	For the second figure
(i)	$y = x$	(i) $y = x + 2$
(ii)	$x + y = 0$	(ii) $y = x - 2$
(iii)	$y = 2x$	(iii) $y = -x + 2$

(iv) 2 + 3y = 7x (iv) x + 2y = 6



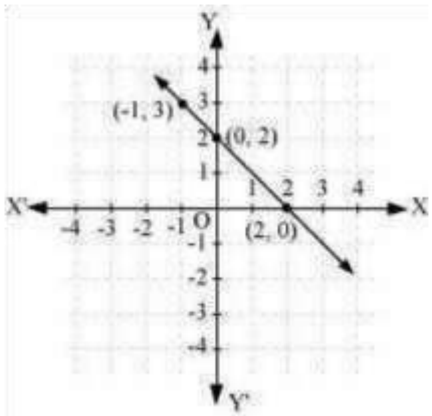
Answer:



Points on the given line are (−1, 1), (0, 0), and (1, −1).

It can be observed that the coordinates of the points of the graph satisfy the equation x + y = 0. Therefore, x + y = 0 is the equation corresponding to the graph as shown in the first figure.

Hence, (ii) is the correct answer.



Points on the given line are (−1, 3), (0, 2), and (2, 0). It can be observed that the coordinates of the points of the graph satisfy the equation $y = -x + 2$.

Therefore, $y = -x + 2$ is the equation corresponding to the graph shown in the second figure.

Hence, (iii) is the correct answer.

Question 6:

If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units.

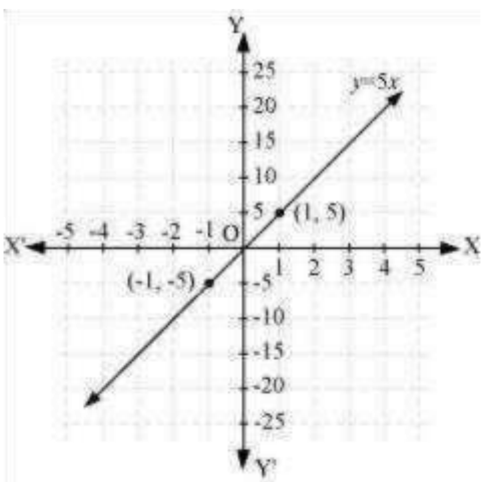
Also read from the graph the work done when the distance travelled by the body is (i) 2 units (ii) 0 units Answer:

Let the distance travelled and the work done by the body be x and y respectively.

Work done $\propto$ distance travelled $y \propto x$ $y = kx$

Where, k is a constant

If constant force is 5 units, then work done $y = 5x$
It can be observed that point (1, 5) and (−1, −5) satisfy the above equation. Therefore, these are the solutions of this equation. The graph of this equation is constructed as follows.



(i)From the graphs, it can be observed that the value of y corresponding to x = 2 is 10. This implies that the work done by the body is 10 units when the distance travelled by it is 2 units.

(ii) From the graphs, it can be observed that the value of y corresponding to x = 0 is 0. This implies that the work done by the body is 0 units when the distance travelled by it is 0 unit.

Question 7:

Yamini and Fatima, two students of Class IX of a school, together contributed Rs 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as Rs x and Rs y.) Draw the graph of the same.

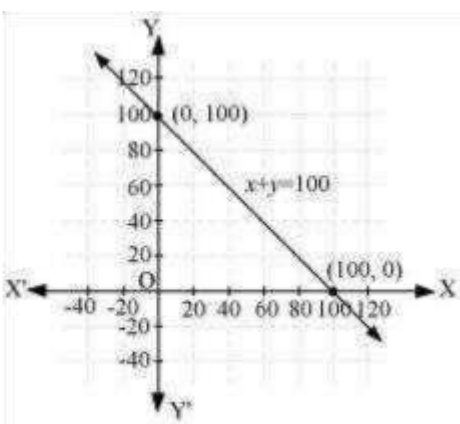
Answer:

Let the amount that Yamini and Fatima contributed be x and y respectively towards the Prime Minister's Relief fund.

Amount contributed by Yamini + Amount contributed by Fatima = 100 x

+ y = 100

It can be observed that (100, 0) and (0, 100) satisfy the above equation. Therefore, these are the solutions of the above equation. The graph is constructed as follows.



Here, it can be seen that variable x and y are representing the amount contributed by Yamini and Fatima respectively and these quantities cannot be negative. Hence, only those values of x and y which are lying in the 1st quadrant will be considered.

Question 8:

In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

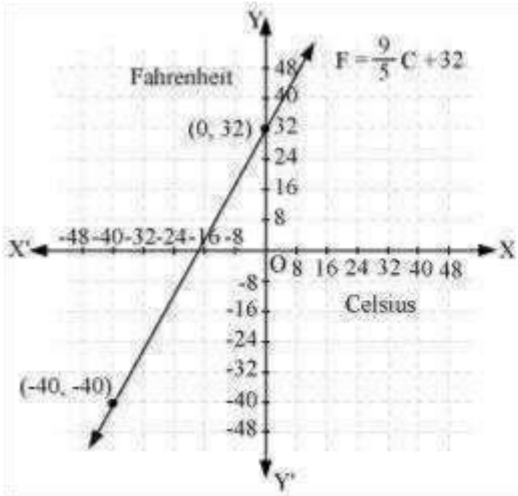
Answer:

(i) $F = \left(\frac{9}{5}\right)C + 32$

It can be observed that points (0, 32) and (−40, −40) satisfy the given equation.

Therefore, these points are the solutions of this equation.

The graph of the above equation is constructed as follows.



(ii) Temperature = 30°C

$$F = \left(\frac{9}{5}\right)C + 32$$

$$F = \left(\frac{9}{5}\right)30 + 32 = 54 + 32 = 86$$

Therefore, the temperature in Fahrenheit is 86°F.

(iii) Temperature = 95°F

$$F = \left(\frac{9}{5}\right)C + 32$$

$$95 = \left(\frac{9}{5}\right)C + 32$$

$$63 = \left(\frac{9}{5}\right)C$$

$$C = 35$$

Therefore, the temperature in Celsius is 35°C.

(iv)

$$F = \left(\frac{9}{5}\right)C + 32$$

If C = 0°C, then

$$F = \left(\frac{9}{5}\right)0 + 32 = 32$$

Therefore, if C = 0°C, then F = 32°F

If F = 0°F, then

$$0=\left(\frac{9}{5}\right)C+32$$

$$\left(\frac{9}{5}\right)C=-32$$

$$C=\frac{-160}{9}=-17.77$$

Therefore, if F = 0°F, then C = −17.8°C

$$F=\left(\frac{9}{5}\right)C+32$$

(v)

Here, F = C

$$F=\left(\frac{9}{5}\right)F+32$$

$$\left(\frac{9}{5}-1\right)F+32=0$$

$$\left(\frac{4}{5}\right)F=-32$$

$$F=-40$$

Yes, there is a temperature, −40°, which is numerically the same in both Fahrenheit and Celsius.

Exercise 4.4 Question

1:

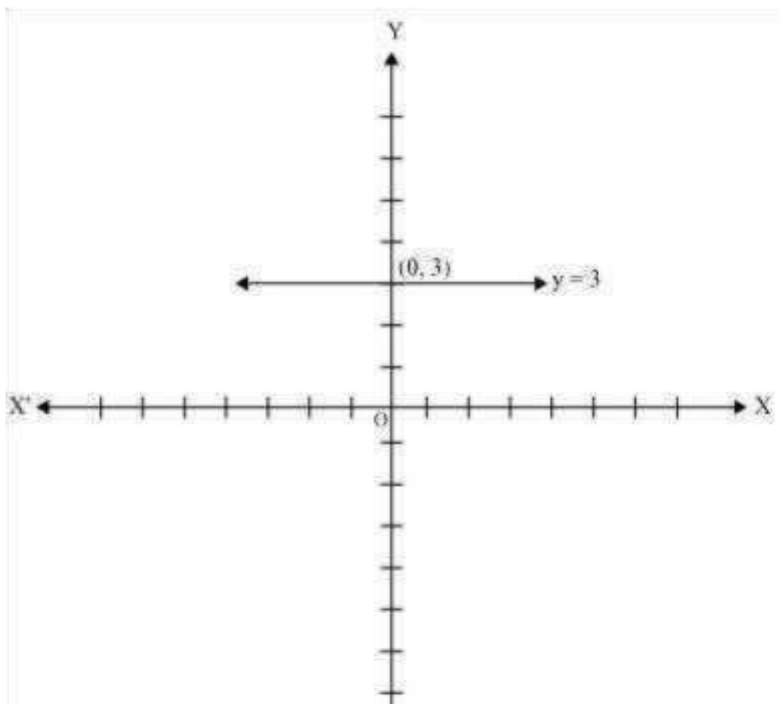
Give the geometric representation of y = 3 as an equation

(I) in one variable (II) in two variables

Answer:

In one variable, y = 3 represents a point as shown in following figure.

In two variables, y = 3 represents a straight line passing through point (0, 3) and parallel to x-axis. It is a collection of all points of the plane, having their y-coordinate as 3.



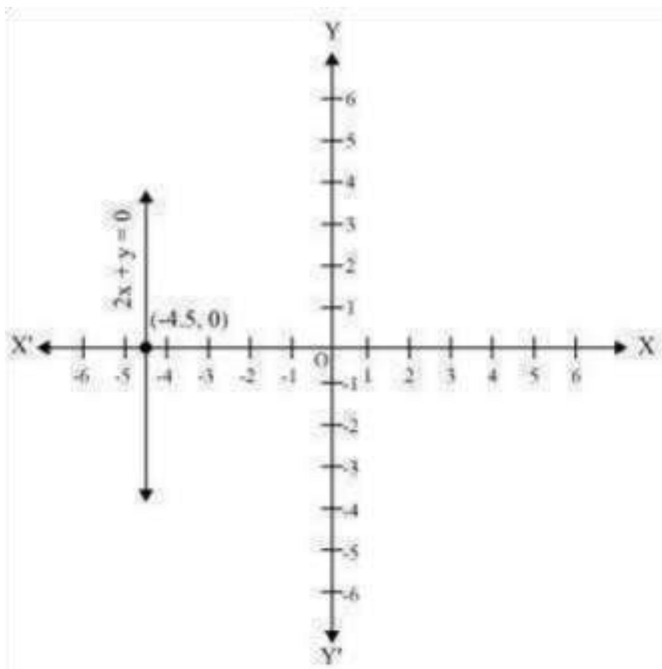
Question 2:

Give the geometric representations of 2x + 9 = 0 as an equation

Answer:

(1) In one variable, 2 x + 9 = 0 represents a point $\boxed{x = \frac{-9}{2} = -4.5}$ as shown in the following figure.

(2) In two variables,2x + 9 = 0 represents a straight line passing through point (−4.5,
(2) in two variables
(1) in one variable
0) and parallel to y-axis. It is a collection of all points of the plane, having their xcoordinate as 4.5.



<u>**DELHI PUBLIC SCHOOL, GANDHINAGAR**</u>

<u>**CHAPTER 6: LINES AND ANGLES**</u>

<u>**MIND MAP**</u>

This chapter consists of three different topics. The most probable questions from examination point of view are given below.

<u>TYPE: 1</u> TYPES OF ANGLES

Q.1 If an angle is 28° less than its complement, find its measure.

Q.2 If an angle is 30° more than one half of its complement, find the measure of the angle.

Q.3 Two supplementary angles differ by 48°. Find the angles.

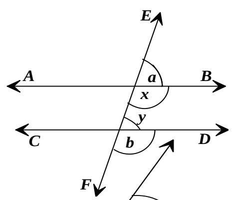
Q.4 If the angles $(2x - 10)°$ and $(x - 5)°$ are complementary angles, find x.

<u>TYPE: 2</u> PROPERTY OF PARALLEL LINES

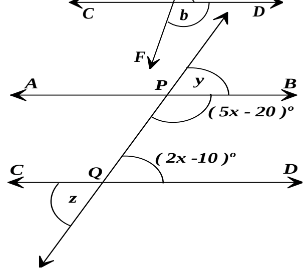
Q.1 A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Q.2 If two lines intersect each other, then vertically opposite angles are equal.

Q.3 In the adjoining figure, if $AB \parallel CD$ and $x : y = 3 : 2$, find $a : b$.



Q.4 In the adjoining figure, if $AB \parallel$ CD, $\angle BPQ = (5x - 20)^\circ$ and $\angle PQD = (2x - 10)^\circ$, find the value of y and z.

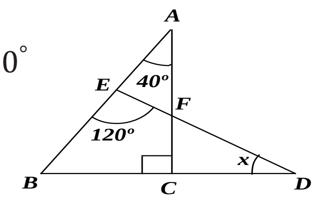


<u>TYPE: 3</u> PROPERTIES OF TRIANGLE

Q.1 Theorem: The sum of three angles of a triangle is 180°.

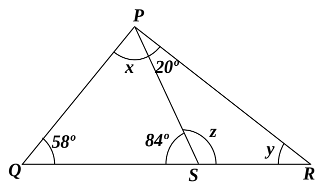
Q.2 Theorem: The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Q.3 Find the value of x in the given figure, where $\angle A = 40^\circ$ and $\angle BED = 120^\circ$

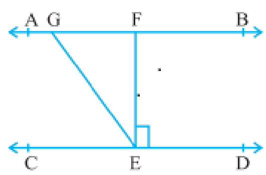


Q.4 An exterior angle of a triangle is 110° and the two interior opposite angles are equal. Find the measure of each of these opposite angles

Q.5 From the adjoining figure, find the values of x, y and z.



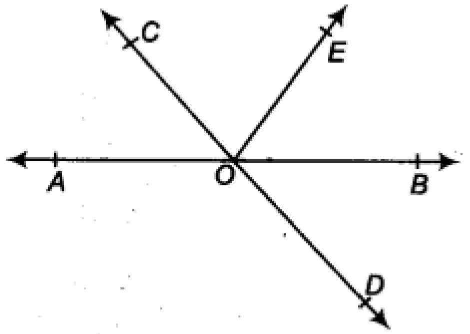
Q.6 In the adjacent figure if AB∥CD, EF⊥CD and ∠GED = 130^0, find ∠AGE, ∠GEF and ∠FGE.



CHAPTER 6: LINES AND ANGLES

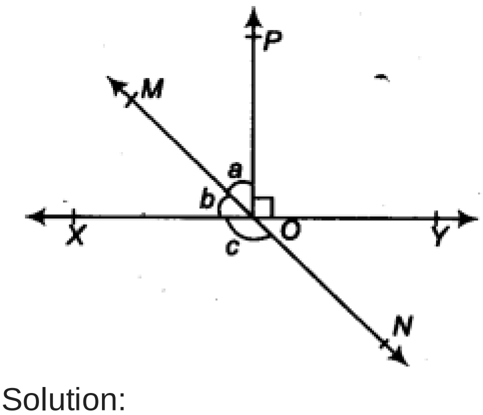
EXERCISE 6.1

Q.1. In figure, lines AB and CD intersect at 0. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Solution:
Since AB is a straight line,
∴ ∠AOC + ∠COE + ∠EOB = 180°
or (∠AOC + ∠BOE) + ∠COE = 180° or 70° + ∠COE = 180° [∵∠AOC + ∠BOE = 70° (Given)]
or ∠COE = 180° – 70° = 110°
∴ Reflex ∠COE = 360° – 110° = 250°
Also, AB and CD intersect at O.
∴∠COA = ∠BOD [Vertically opposite angles]
But ∠BOD = 40° [Given]
∴ ∠COA = 40°
Also, ∠AOC + ∠BOE = 70°
∴ 40° + ∠BOE = 70° or ∠BOE = 70° -40° = 30°
Thus, ∠BOE = 30° and reflex ∠COE = 250°.

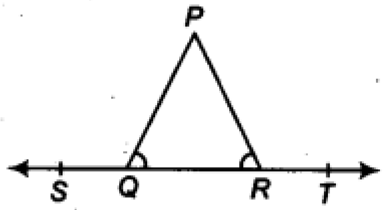
.
Q.2. In figure, lines XY and MN intersect at 0. If ∠POY = 90° , and a : b = 2 : 3. find c.



Solution:
Since XOY is a straight line.
∴ b+a+∠POY= 180°
But ∠POY = 90° [Given]
∴ b + a = 180° – 90° = 90° …(i)
Also a : b = 2 : 3 ⇒ b = $\frac{3a}{2}$ …(ii)
Now from (i) and (ii), we get
$\frac{3a}{2}$ + A = 90°
⇒ $\frac{5a}{2}$ = 90°
⇒ a = $\frac{90°}{5} \times 2 \quad = \quad 36°$
From (ii), we get

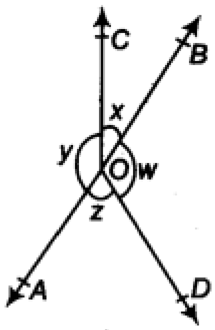
b = $\frac{3}{2}$ x 36° = 54°
Since XY and MN intersect at O,
∴ c = [a + ∠POY] [Vertically opposite angles]
or c = 36° + 90° = 126°
Thus, the required measure of c = 126°.

Q.3. In figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Solution:
ST is a straight line.
∴ ∠PQR + ∠PQS = 180° …(1) [Linear pair]
Similarly, ∠PRT + ∠PRQ = 180° …(2) [Linear Pair]
From (1) and (2), we have
∠PQS + ∠PQR = ∠PRT + ∠PRQ
But ∠PQR = ∠PRQ [Given]
∴ ∠PQS = ∠PRT

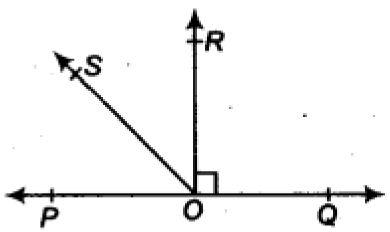
Q.4. In figure, if x + y = w + ⇒, then prove that AOB is a line.



Solution:
Sum of all the angles at a point = 360°
∴ x + y + ⇒ + w = 360° or, (x + y) + (⇒ + w) = 360°
But (x + y) = (⇒ + w) [Given]
∴ (x + y) + (x + y) = 360° or,
2(x + y) = 360°
or, (x + y) = $\frac{360°}{2}$ = 180°
∴ AOB is a straight line.

Q.5. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

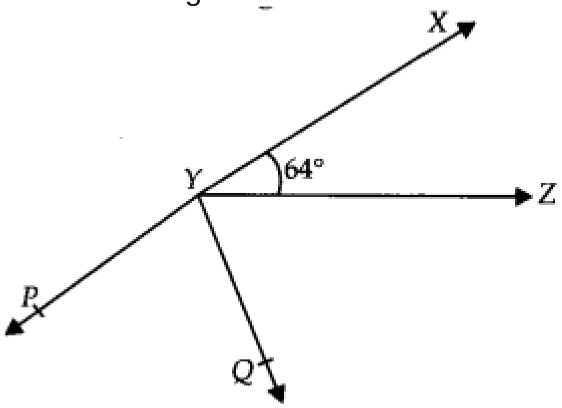
$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Solution:
POQ is a straight line. [Given]
∴ ∠POS + ∠ROS + ∠ROQ = 180°

But OR ⊥ PQ
∴ ∠ROQ = 90°
⇒ ∠POS + ∠ROS + 90° = 180°
⇒ ∠POS + ∠ROS = 90°
⇒ ∠ROS = 90° – ∠POS … (1)
Now, we have ∠ROS + ∠ROQ = ∠QOS
⇒ ∠ROS + 90° = ∠QOS
⇒ ∠ROS = ∠QOS – 90° ……(2)
Adding (1) and (2), we have
2 ∠ROS = (∠QOS – ∠POS)
∴ ∠ROS = $\frac{1}{2}(\angle QOS - \angle POS)$

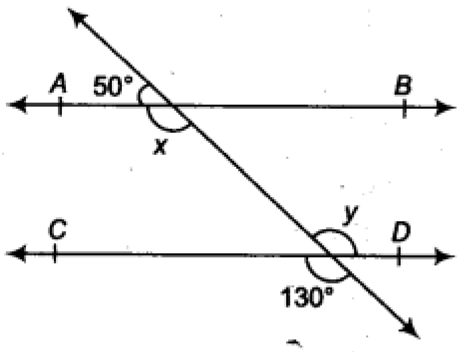
Q.6. It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.
Solution:
XYP is a straight line.



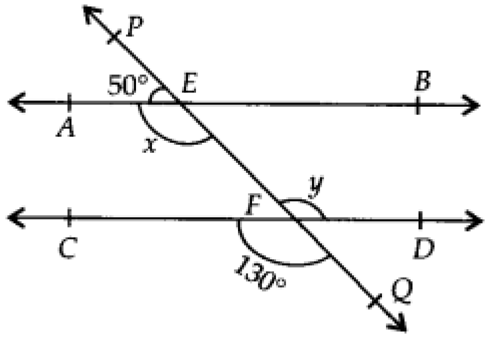
∴ ∠XYZ + ∠ZYQ + ∠QYP = 180°
⇒ 64° + ∠ZYQ + ∠QYP = 180°
[∵ ∠XYZ = 64° (given)]
⇒ 64° + 2∠QYP = 180°
[YQ bisects ∠ZYP so, ∠QYP = ∠ZYQ]
⇒ 2∠QYP = 180° – 64° = 116°
⇒ ∠QYP = $\frac{116°}{2}$ = 58°
∴ Reflex ∠QYP = 360° – 58° = 302°
Since ∠XYQ = ∠XYZ + ∠ZYQ
⇒ ∠XYQ = 64° + ∠QYP [∵∠XYZ = 64°(Given) and ∠ZYQ = ∠QYP]
⇒ ∠XYQ = 64° + 58° = 122° [∠QYP = 58°]
Thus, ∠XYQ = 122° and reflex ∠QYP = 302°.

EXERCISE 6.2

Q.1. In figure, find the values of x and y and then show that AB || CD.

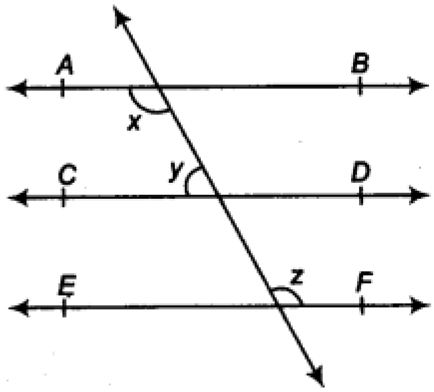


Solution:
In the figure, we have CD and PQ intersect at F.



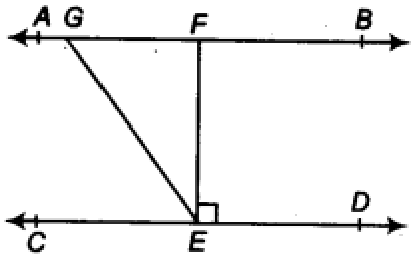
∴ y = 130° …(1)
[Vertically opposite angles]
Again, PQ is a straight line and EA stands on it.
∠AEP + ∠AEQ = 180° [Linear pair]
or 50° + x = 180°
⇒ x = 180° – 50° = 130° …(2)
From (1) and (2), x = y
As they are pair of alternate interior angles.
∴ AB || CD

Q.2. In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.



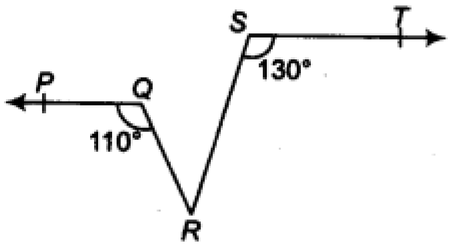
Solution:
AB || CD, and CD || EF [Given]
∴ AB || EF
∴ x = z [Alternate interior angles] ….(1)
Again, AB || CD
⇒ x + y = 180° [Co-interior angles]
⇒ z + y = 180° … (2) [By (1)]
But y : z = 3 : 7
z = $\frac{7}{3}$ y = $\frac{7}{3}$(180°- z) [By (2)]
⇒ 10z = 7 x 180°
⇒ z = 7 x 180° /10 = 126°
From (1) and (3), we have
x = 126°.

Q.3. In figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.

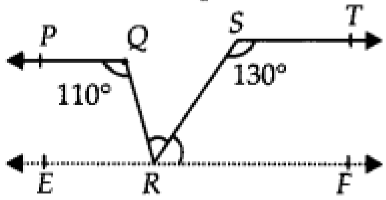


Solution:
AB || CD and GE is a transversal.
∴ ∠AGE = ∠GED [Alternate interior angles]
But ∠GED = 126° [Given]
∴∠AGE = 126°
Also, ∠GEF + ∠FED = ∠GED
or ∠GEF + 90° = 126° [∵ EF ⊥ CD (given)]
x = z [Alternate interior angles]… (1) Again, AB || CD
⇒ x + y = 180° [Co-interior angles]
∠GEF = 126° -90° = 36°
Now, AB || CD and GE is a transversal.
∴ ∠FGE + ∠GED = 180° [Co-interior angles]
or ∠FGE + 126° = 180°
or ∠FGE = 180° – 126° = 54°
Thus, ∠AGE = 126°, ∠GEF=36° and ∠FGE = 54°.

Q.4. In figure, if PQ || ST, ∠ PQR = 110° and ∠ RST = 130°, find ∠QRS.

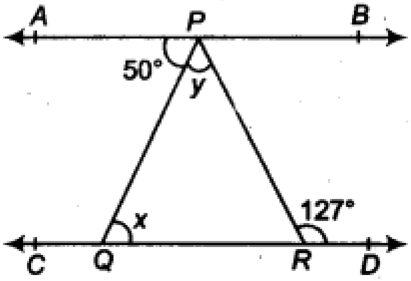


Solution:
Draw a line EF parallel to ST through R.



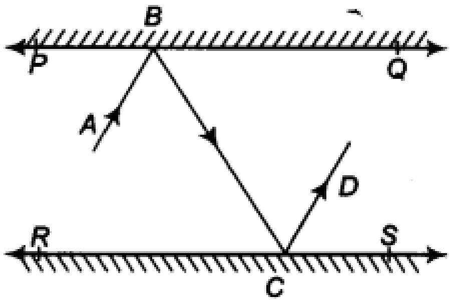
Since PQ || ST [Given]
and EF || ST [Construction]
∴ PQ || EF and QR is a transversal
⇒ ∠PQR = ∠QRF [Alternate interior angles] But ∠PQR = 110° [Given]
∴∠QRF = ∠QRS + ∠SRF = 110° …(1)
Again ST || EF and RS is a transversal
∴ ∠RST + ∠SRF = 180° [Co-interior angles] or 130° + ∠SRF = 180°
⇒ ∠SRF = 180° – 130° = 50°
Now, from (1), we have ∠QRS + 50° = 110°
⇒ ∠QRS = 110° – 50° = 60°
Thus, ∠QRS = 60°.

Q.5. In figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

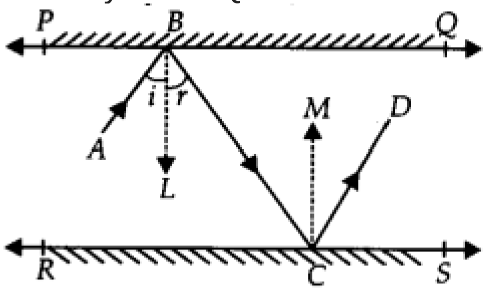


Solution:
We have AB || CD and PQ is a transversal.
∴ ∠APQ = ∠PQR
[Alternate interior angles]
⇒ 50° = x [∵ ∠APQ = 50° (given)]
Again, AB || CD and PR is a transversal.
∴ ∠APR = ∠PRD [Alternate interior angles]
⇒ ∠APR = 127° [∵ ∠PRD = 127° (given)]
⇒ ∠APQ + ∠QPR = 127°
⇒ 50° + y = 127° [∵ ∠APQ = 50° (given)]
⇒ y = 127°- 50° = 77°
Thus, x = 50° and y = 77°.

Q.6. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Solution:
Draw ray BL ⊥PQ and CM ⊥ RS

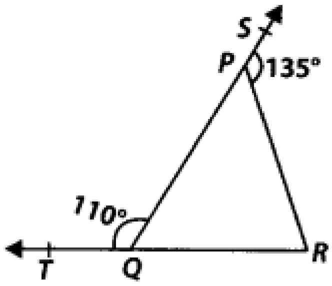


∵ PQ || RS ⇒ BL || CM
[∵ BL || PQ and CM || RS]
Now, BL || CM and BC is a transversal.
∴ ∠LBC = ∠MCB …(1) [Alternate interior angles]
Since, angle of incidence = Angle of reflection
∠ABL = ∠LBC and ∠MCB = ∠MCD
⇒ ∠ABL = ∠MCD …(2) [By (1)]
Adding (1) and (2), we get
∠LBC + ∠ABL = ∠MCB + ∠MCD
⇒ ∠ABC = ∠BCD

i. e., a pair of alternate interior angles are equal.
∴ AB || CD.

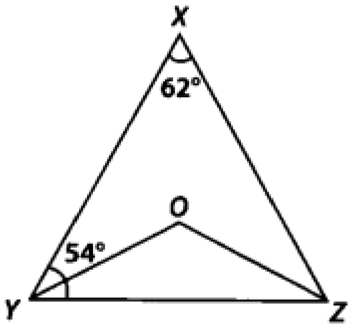
EXERCISE 6.3

Q.1. In figure, sides QP and RQ of ΔPQR are produced to points S and T, respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.



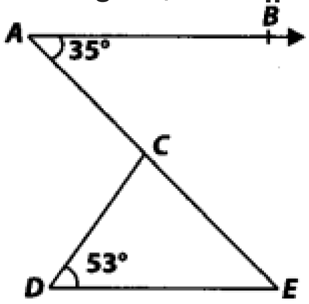
Solution:
We have, ∠TQP + ∠PQR = 180°
[Linear pair]
⇒ 110° + ∠PQR = 180°
⇒ ∠PQR = 180° – 110° = 70°
Since, the side QP of ΔPQR is produced to S.
⇒ ∠PQR + ∠PRQ = 135°
[Exterior angle property of a triangle]
⇒ 70° + ∠PRQ = 135° [∠PQR = 70°]
⇒ ∠PRQ = 135° – 70° ⇒ ∠PRQ = 65°

Q.2.In figure, ∠X = 62°, ∠XYZ = 54°, if YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OZY and ∠YOZ.



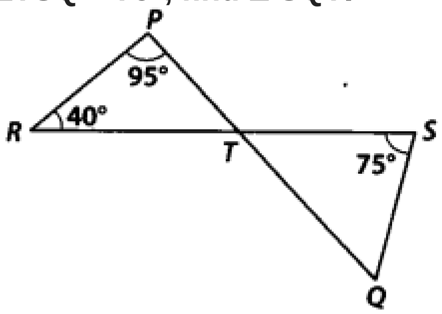
Solution:
In ΔXYZ, we have ∠XYZ + ∠YZX + ∠ZXY = 180°
[Angle sum property of a triangle]
But ∠XYZ = 54° and ∠ZXY = 62°
∴ 54° + ∠YZX + 62° = 180°
⇒ ∠YZX = 180° – 54° – 62° = 64°
YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively.
∴ ∠OYZ = $\frac{1}{2}\angle XYZ$ = $\frac{1}{2}$(54°) = 27°
and ∠OZY = $\frac{1}{2}\angle YZX$ = $\frac{1}{2}$(64°) = 32°
Now, in ΔOYZ, we have
∠YOZ + ∠OYZ + ∠OZY = 180°
[Angle sum property of a triangle]
⇒ ∠YOZ + 27° + 32° = 180°
⇒ ∠YOZ = 180° -27° – 32° = 121°
Thus, ∠OZY = 32° and ∠YOZ = 121°

Q.3. In figure, if AB || DE, ∠BAC = 35° and ∠CDE = 539 , find ∠DCE.



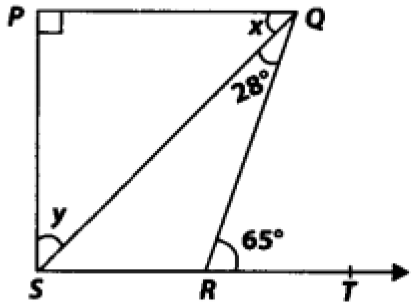
Solution:
AB || DE and AE is a transversal.
So, ∠BAC = ∠AED
[Alternate interior angles]
and ∠BAC = 35° [Given]
∴ ∠AED = 35°
Now, in ΔCDE, we have ∠CDE + ∠DEC + ∠DCE = 180°
{Angle sum property of a triangle]
∴ 53° + 35° + ∠DCE =180°
[∵ ∠DEC = ∠AED = 35° and∠CDE = 53° (Given)]
⇒ ∠DCE = 180° – 53° – 35° = 92°
Thus, ∠DCE = 92°

Q.4. In figure, if lines PQ and RS intersect at point T, such that ∠ PRT = 40°, ∠ RPT = 95° and ∠TSQ = 75°, find ∠ SQT.



Solution:
In ΔPRT, we have ∠P + ∠R + ∠PTR = 180°
[Angle sum property of a triangle]
⇒ 95° + 40° + ∠PTR =180°
[∵ ∠P = 95°, ∠R = 40° (given)]
⇒ ∠PTR = 180° – 95° – 40° = 45°
But PQ and RS intersect at T.
∴ ∠PTR = ∠QTS
[Vertically opposite angles]
∴ ∠QTS = 45° [∵ ∠PTR = 45°]
Now, in Δ TQS, we have ∠TSQ + ∠STQ + ∠SQT = 180°
[Angle sum property of a triangle]
∴ 75° + 45° + ∠SQT = 180° [∵ ∠TSQ = 75° and ∠STQ = 45°]
⇒ ∠SQT = 180° – 75° – 45° = 60°
Thus, ∠SQT = 60°

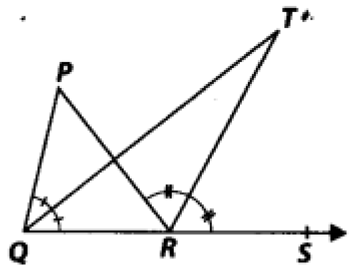
Q.5. In figure, if PQ ⊥ PS, PQ||SR, ∠SQR = 2S° and ∠QRT = 65°, then find the values of x and y.



Solution:
In ∆ QRS, the side SR is produced to T.
∴ ∠QRT = ∠RQS + ∠RSQ
[Exterior angle property of a triangle]
But ∠RQS = 28° and ∠QRT = 65°
So, 28° + ∠RSQ = 65°
⇒ ∠RSQ = 65° – 28° = 37°
Since, PQ || SR and QS is a transversal.
∴ ∠PQS = ∠RSQ = 37°
[Alternate interior angles]
⇒ x = 37°
Again, PQ ⊥ PS ⇒ AP = 90°
Now, in ∆PQS,
we have ∠P + ∠PQS + ∠PSQ = 180°
[Angle sum property of a triangle]
⇒ 90° + 37° + y = 180°
⇒ y = 180° – 90° – 37° = 53°
Thus, x = 37° and y = 53°

Q.6. In figure, the side QR of ∆PQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that

$$\angle QTR = \frac{1}{2}\angle QPR.$$



Solution:
In ∆PQR, side QR is produced to S, so by exterior angle property,
∠PRS = ∠P + ∠PQR
⇒ $\frac{1}{2}$∠PRS = $\frac{1}{2}$∠P + $\frac{1}{2}$∠PQR
⇒ ∠TRS = $\frac{1}{2}$∠P + ∠TQR …(1)
[∵ QT and RT are bisectors of ∠PQR and ∠PRS respectively.]
Now, in ∆QRT, we have
∠TRS = ∠TQR + ∠T …(2)
[Exterior angle property of a triangle]
From (1) and (2),
we have ∠TQR + $\frac{1}{2}$∠P = ∠TQR + ∠T

$\Rightarrow \frac{1}{2}\angle P = \angle T$

$\Rightarrow \frac{1}{2}\angle QPR = \angle QTR$ or $\angle QTR = \frac{1}{2}\angle QPR$

DELHI PUBLIC SCHOOL – GANDHINAGAR

CHAPTER 7 TRIANGLES

MIND MAP

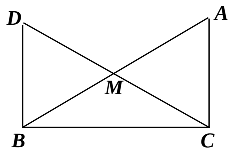
This chapter consists of two different topics. The most probable questions from examination point of view are given below.

TYPE: 1 CONGRUENCE OF TRIANGLES

Q.1 In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect at O. Join A and O. Show that:

(i) $OB = OC$. (ii) AO bisects $\angle A$.

Q.2 In a right angled triangle ABC, right angled at C, M is the mid- point of hypotenuse $AB. C$ is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that

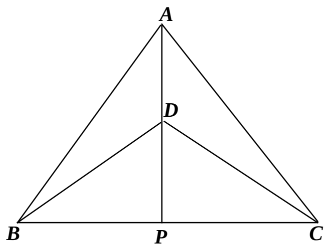


(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

Q.3 ΔABC and ΔDBC are two isosceles triangles on the same BC and vertices A and D are on the same side of BC as shown in the figure. If AD is extended to intersect BC at P, Show that:



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$.

Q.4 Show that the angles of an equilateral are $60°$ each.

Q.5 ABC is a right angled triangle in which $\angle A$=$90°$ and $AB = AC$. Find $\angle B$ and $\angle C$.

TYPE: 2 INEQUALITIES IN A TRIANGLE

Q.1 Show that in a right angled triangle, the hypotenuse is the longest side.

Q.2 Sides AB *and* AC of ΔABC are extended to points P *and* Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

Q.3 AB and CD are respectively the smallest and longest sides of a quadrilateral $ABCD$. Show that $\angle A > \angle C$ and $\angle B > \angle D$.

Q.4 Prove that angles opposite to equal sides of an isosceles triangle are equal.

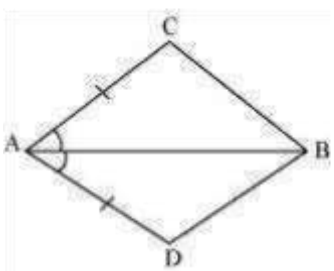
Class IX Chapter 7 – Triangles Maths

Exercise 7.1 Question

1:

In quadrilateral ACBD, AC = AD and AB bisects ∠A (See the given figure). Show that

≅



Answer: ΔABC ΔABD. What can you say about BC and BD?

In ΔABC and ΔABD,

AC = AD (Given)

∠CAB = ∠DAB (AB bisects ∠A)

AB = AB (Common)

∴ ΔABC ≅ ΔABD (By SAS congruence rule)

∴ BC = BD (By CPCT)

Therefore, BC and BD are of equal lengths.

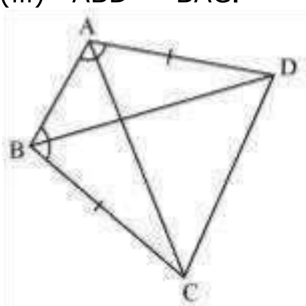
Question 2:

ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (See the given figure). Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.



Answer:

In ΔABD and ΔBAC,

AD = BC (Given)

∠DAB = ∠CBA (Given)

AB = BA (Common)

∴ ΔABD ≅ ΔBAC (By SAS congruence rule)

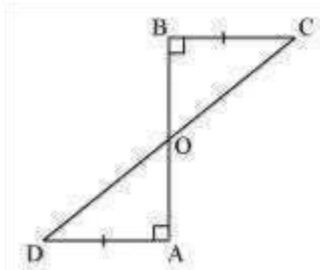
∴ BD = AC (By CPCT) And, ∠ABD

= ∠BAC (By CPCT)

Question 3:

AD and BC are equal perpendiculars to a line segment AB (See the given figure).

Show that CD bisects AB.



Answer:
In ΔBOC and ΔAOD,

∠BOC = ∠AOD (Vertically opposite angles)

∠CBO = ∠DAO (Each 90º)

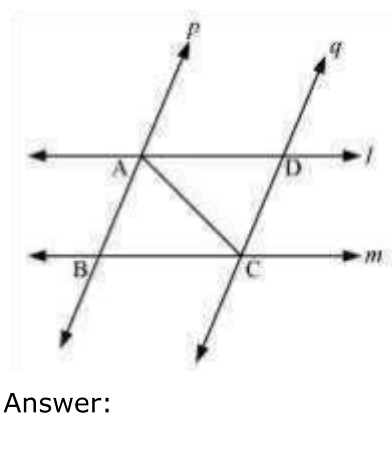
BC = AD (Given)

∴ ΔBOC ≅ ΔAOD (AAS congruence rule)
∴ BO = AO (By CPCT)

⇒ CD bisects AB.

Question 4: l and m are two parallel lines intersected by another pair of parallel lines p and q (see

the given figure). Show that ΔABC ≅ ΔCDA.



Answer:

In ΔABC and ΔCDA,

∠BAC = ∠DCA (Alternate interior angles, as p || q)

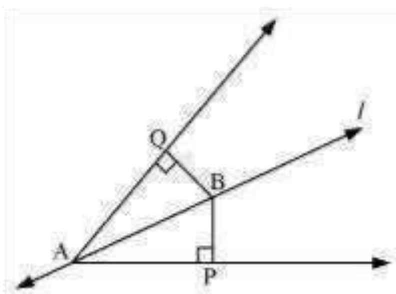
AC = CA (Common)

∠ ∠ BCA = DAC (Alternate interior angles, as l || m)

∴ ∴ ΔABC ΔCDA (By ASA congruence rule)

Question 5:

Line l ∴A is the bisector of an angle and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∴A (see the given figure). Show that: i) ΔAPB ∴ ΔAQB (ii) BP = BQ or B is equidistant from the arms of ∴(A.



Answer:

In ΔAPB and ΔAQB,

∴ ∴ APB = AQB (Each 90º)

∴ ∴ PAB = QAB (l is the angle bisector of ∴A)

AB = AB (Common)

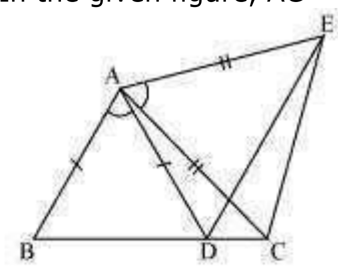
∴ ΔAPB ∴ ΔAQB (By AAS congruence rule) ∴ BP = BQ (By CPCT)

rms of ∴A. Or,

it can be said that B is equidistant from the a

Question 6:

In the given figure, AC = AE, AB = AD and ∴BAD = ∴EAC. Show that BC = DE.



Answer:

It is given that ∴BAD = ∴EAC

∴BAD + ∴DAC = ∴EAC + ∴DAC

∴BAC = ∴DAE

In ΔBAC and ΔDAE, AB = AD

(Given) ∴BAC =

∴DAE (Proved above)

AC = AE (Given)

∴ ΔBAC ∴ ΔDAE (By SAS congruence rule)

∴ BC = DE (By CPCT)

Question 7:

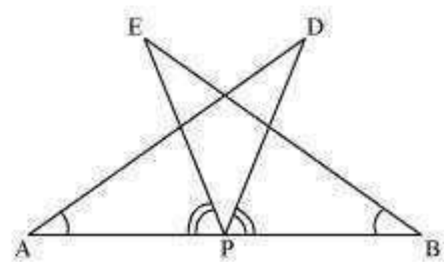
AB is a line segment and P is its mid-point. D and E are points on the same side of AB

such that BAD = ∴ABE and ∴EPA = ∴DPB (See the given figure). Show that i)

∴

ΔDAP ΔEBP (

(ii) AD = BE



Answer:

It is given that EPA = DPB

∴ ∴ EPA + DPE = DPB + DPE

∴ ∴ DPA = EPB

In Δ DAP and Δ EBP,

∴ DAP = ∴ EBP (Given)

AP = BP (P is mid-point of AB)

∴ ∴ DPA = EPB (From above)

∴ ∴ ΔDAP ΔEBP (ASA congruence rule)

∴ AD = BE (By CPCT)

Question 8:

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point

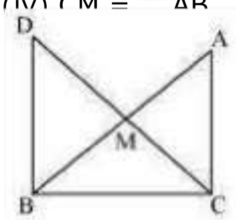
B (see the given figure). Show that: i)

ΔAMC ∴ ΔBMD (

ii) ∴DBC is a right angle. (iii)

ΔDBC ∴ ΔACB (

(iv) CM = $\frac{1}{2}$ AB



Answer:

(i) In ΔAMC and ΔBMD,
AM = BM (M is the mid-point of AB)

∴AMC = ∴BMD (Vertically opposite angles)

CM = DM (Given)

∴ ΔAMC ∴ ΔBMD (By SAS congruence rule)

∴ AC = BD (By CPCT) And,

∴ ACM = ∴BDM (By CPCT) ii)

∴ ACM = ∴BDM (

However, ∴ACM and ∴BDM are alternate interior angles.

Since alternate angles are equal,

It can be said that DB || AC

∴ ∴ DBC + ∴ACB = 180º (Co-interior angles)

∴ ∴ DBC + 90º = 180º

∴ ∴ DBC = 90º

(iii) In ΔDBC and ΔACB,
DB = AC (Already proved)

∠DBC = ∠ACB (Each 90°)

BC = CB (Common)

∴ ΔDBC ≅ ΔACB (SAS congruence rule) iv)

ΔDBC ≅ ΔACB (

∴ AB = DC (By CPCT)

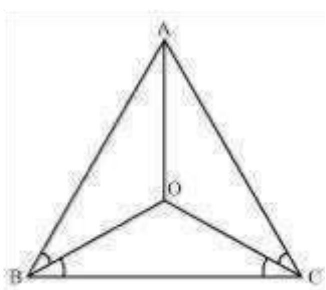
∴ AB = 2 CM

∴ CM = $\frac{1}{2}$ AB

Exercise 7.2 Question

1:
In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

i) OB = OC (ii) AO bisects ∠A (Answer:



(i) It is given that in triangle ABC, AB = AC

∴ ∴ ∴ ACB = ABC (Angles opposite to equal sides of a triangle are equal)

∴ $\frac{1}{2}$∴ ACB = $\frac{1}{2}$∴ ABC

∴ ∴OCB = ∴OBC

∴

OB = OC (Sides opposite to equal angles of a triangle are also equal)

(ii) In ΔOAB and ΔOAC, AO =AO (Common)

AB = AC (Given)

OB = OC (Proved above)

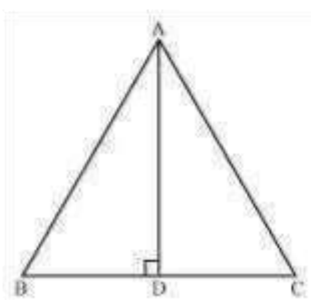
Therefore, ΔOAB ∴ΔOAC (By SSS congruence rule)

∴ ∴ BAO ≟ CAO (CPCT)

∴ AO bisects A.

Question 2:

In ΔABC, AD is the perpendicular bisector of BC (see the given figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Answer:

In ΔADC and ΔADB,

AD = AD (Common)

∴ADC = ∴ADB (Each 90º)

CD = BD (AD is the perpendicular bisector of BC)

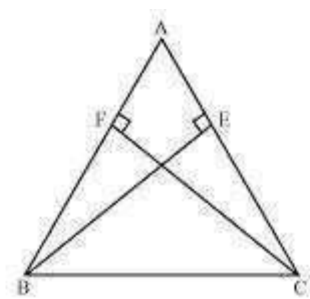
∴ ΔADC ≅ ΔADB (By SAS congruence rule)

∴ AB = AC (By CPCT)

Therefore, ABC is an isosceles triangle in which AB = AC.

Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.



Answer:

In ΔAEB and ΔAFC,

∠AEB and ∠AFC (Each 90º) ∠A =

∠A (Common angle)

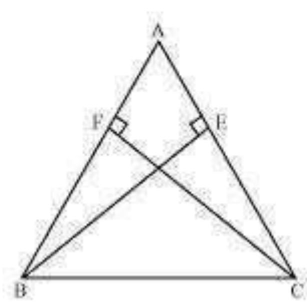
AB = AC (Given)

∴ ΔAEB ∴ ΔAFC (By AAS congruence rule) ∴ BE = CF (By CPCT)

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the given figure). Show that

(i) ΔABE ∴ ΔACF



Answer:

(ii) AB = AC, i.e., ABC is an isosceles triangle.

(i) In ΔABE and ΔACF,

∴ABE and ∴ACF (Each 90º)

∴A = ∴A (Common angle)

BE = CF (Given)

∴ ΔABE ∴ ΔACF (By AAS congruence rule)

(ii) It has already been proved that

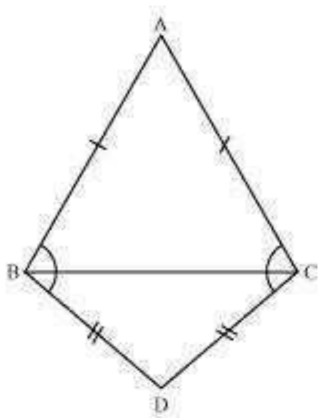
ΔABE ∴ΔACF

∴ AB = AC (By CPCT)

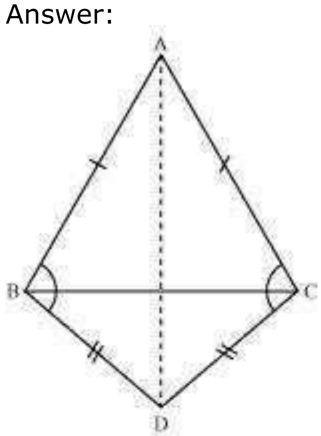
Question 5:

ABC and DBC are two isosceles triangles on the same base BC (see the given figure).

Show that ∴ABD = ∴ACD.



Answer:



Let us join AD.

In ΔABD and ΔACD,

AB = AC (Given)

BD = CD (Given)

AD = AD (Common side)

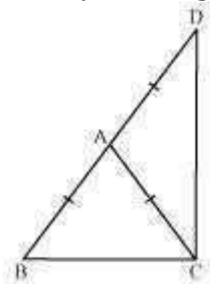
∴ ΔABD ≅ ΔACD (By SSS congruence rule)

∴ ∠ABD = ∠ACD (By CPCT)

Question 6:

ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD

= AB (see the given figure). Show that ∠BCD is a right angle.



Answer:

In ΔABC,

AB = AC (Given)

∴ ∠ACB = ∠ABC (Angles opposite to equal sides of a triangle are also equal)

In ΔACD,

AC = AD

∴ ∠ADC = ∠ACD (Angles opposite to equal sides of a triangle are also equal)

In ΔBCD,

∠ABC + ∠BCD + ∠ADC = 180º (Angle sum property of a triangle)

∴ ∠ACB + ∠ACB + ∠ACD + ∠ACD = 180º

∴ 2(∠ACB + ∠ACD) = 180º

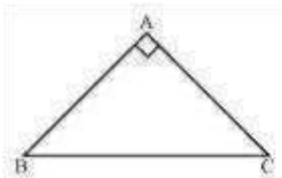
∴ 2(∠BCD) = 180º

∴ ∠BCD = 90º

Question 7:

ABC is a right angled triangle in which ∠A = 90º and AB = AC. Find ∠B and ∠C.

Answer:



It is given that

AB = AC

∴ C = B (Angles opposite to equal sides are also equal)

In ΔABC,

∴ A + B + C = 180º (Angle sum property of a triangle)

∴ 90º + B + C = 180º

∴ 90º + B + B = 180º

∴ 2 B = 90º

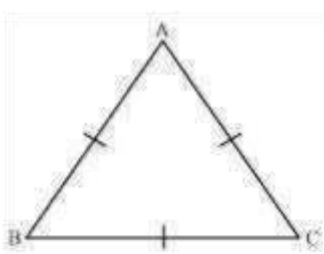
∴ B = 45º

∴ B = C = 45º

Question 8:

Show that the angles of an equilateral triangle are 60º each.

Answer:



Let us consider that ABC is an equilateral triangle.

Therefore, AB = BC = AC

AB = AC

∴C = B (Angles opposite to equal sides of a triangle are equal)

Also,

AC = BC

∴ $\angle B = \angle A$ (Angles opposite to equal sides of a triangle are equal)

Therefore, we obtain $\angle A$

$= \angle B = \angle C$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle A + \angle A = 180°$

$3\angle A = 180°$

$\angle A = 60°$

$\angle A = \angle B = \angle C = 60°$ Hence, in an equilateral triangle, all interior angles are of measure 60º.

Exercise 7.3

Question 1:

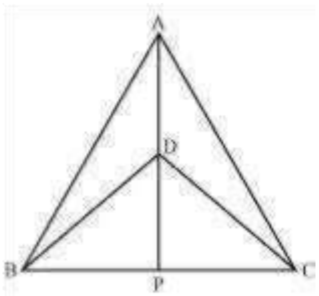
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect

BC at P, show that

i) ΔABD $\cong$ ΔACD (ii) ΔABP $\cong$ ΔACP

(iii) AP bisects $\angle A$ as well as $\angle D$. (

(iv) AP is the perpendicular bisector of BC.



Answer:

(i) In ΔABD and ΔACD,

AB = AC (Given)
BD = CD (Given)

AD = AD (Common)

∴ ΔABD ≅ ΔACD (By SSS congruence rule)

∴ ∠BAD = ∠CAD (By CPCT)

∴ ∠BAP = ∠CAP …. (1)

(ii) In ΔABP and ΔACP,
AB = AC (Given)

∠BAP = ∠CAP [From equation (1)]

AP = AP (Common)

∴ ΔABP ≅ ΔACP (By SAS congruence rule)

∴ BP = CP (By CPCT) … (2)

(iii) From equation (1),

∠BAP = ∠CAP

Hence, AP bisects ∠A.

In ΔBDP and ΔCDP,

BD = CD (Given)

DP = DP (Common)

BP = CP [From equation (2)]

∴ ΔBDP ≅ ΔCDP (By S.S.S. Congruence rule)

∴ ∠BDP = ∠CDP (By CPCT) … (3) Hence,

AP bisects ∠D. iv) ΔBDP ≅

ΔCDP (
∴ ∠BPD = ∠CPD (By CPCT) …. (4)

∴ ∠BPD + ∠CPD = 180° (Linear pair angles)
∠BPD + ∠BPD = 180°

∴ 2∠BPD = 180° [From equation (4)]

∴ ∠BPD = 90° … (5)

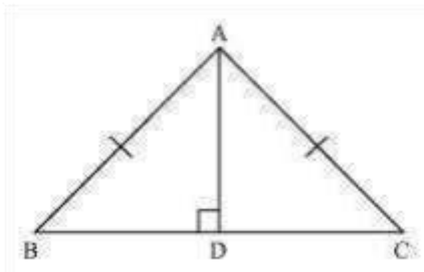
From equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.

Question 2:

AD is an altitude of an isosceles triangles ABC in which AB = AC. Show that

i) AD bisects BC (ii) AD bisects ∠A. (

Answer:



(i) In ΔBAD and ΔCAD,

∴ADB = ∴ADC (Each 90º as AD is an altitude)

AB = AC (Given)

AD = AD (Common)

∴ ΔBAD ∴ ΔCAD (By RHS Congruence rule)

∴ BD = CD (By CPCT)

Hence, AD bisects BC.
(ii) Also, by CPCT,

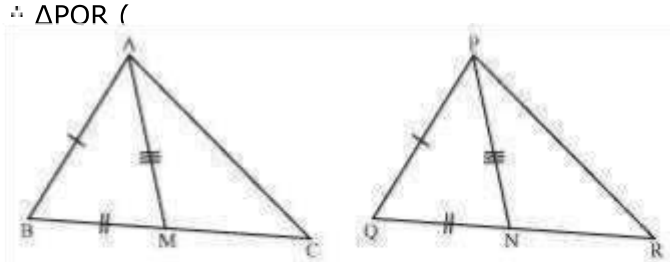
∴BAD = ∴CAD Hence, AD

bisects A. ∴

Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the given figure). Show that: i) ΔABM

ΔPQN (ii) ΔABC ∴ ΔPQR (



∴

Answer:

(i) In ΔABC, AM is the median to BC.

∴ BM = $\frac{1}{2}$ BC

∴ QN = $\frac{1}{2}$ QR

However, BC = QR

∴ $\frac{1}{2}$ BC = $\frac{1}{2}$ QR

∴ BM = QN ... (1)

In ΔABM and ΔPQN,In ΔPQR, PN is the median to QR.

AB = PQ (Given)

BM = QN [From equation (1)]

AM = PN (Given)

∴ ∴ ΔABM ΔPQN (SSS congruence rule)

∴ ∴ ABM = PQN (By CPCT)

∴ ∴ ABC = PQR ... (2)

(ii) In ΔABC and ΔPQR,

AB = PQ (Given)

∴ABC = ∴PQR [From equation (2)]

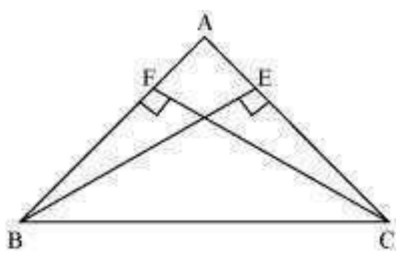
BC = QR (Given)

∴ ΔABC ∴ ΔPQR (By SAS congruence rule)

Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer:



In ΔBEC and ΔCFB,

∴BEC = ∴CFB (Each 90°)

BC = CB (Common)
BE = CF (Given)

∴ ΔBEC ∴ΔCFB (By RHS congruency)

∴ BCE = CBF (By CPCT)

∴ AB = AC (Sides opposite to equal angles of a triangle are equal)

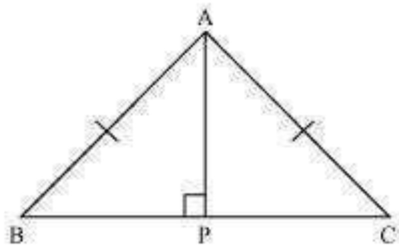
Hence, ΔABC is isosceles.

Question 5:

∴ ∴

Answer:

ABC is an isosceles triangle with AB = AC. Drawn AP ⊥ BC to show that ∠B = ∠C.



In ΔAPB and ΔAPC,

∠APB = ∠APC (Each 90º)

AB =AC (Given)

AP = AP (Common)

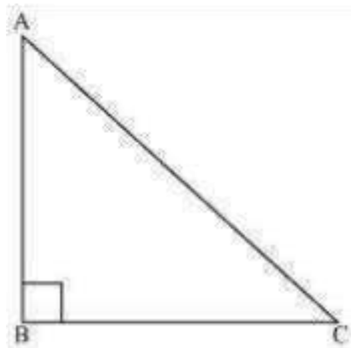
∴ ΔAPB ≅ ΔAPC (Using RHS congruence rule)

∴ ∠B = ∠C (By using CPCT)

Exercise 7.4 Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.

Answer:



Let us consider a right-angled triangle ABC, right-angled at B.

In ΔABC,

∴A + B + C = 180° (Angle sum property of a triangle)

∴A + 90º + C = 180°

∴A + C = 90°

Hence, the other two angles have to be acute (i.e., less than 90º).

∴ ∴ B is the largest angle in ΔABC.

∴ ∴ B > A and B > C ∴

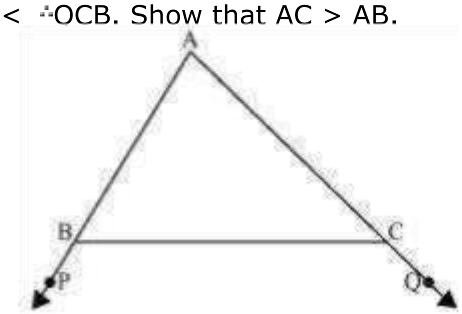
∴ AC > BC and AC > AB

[In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, AC is the largest side in ΔABC.

However, AC is the hypotenuse of ΔABC. Therefore, hypotenuse is the longest side in a right-angled triangle.

Question 2:

In the given figure sides AB and AC of ΔABC are extended to points P and Q respectively. Also, ∴PBC < ∴QCB. Show that AC > AB.



Answer:

In the given figure,

∴ABC + PBC = 180° (Linear pair)

∴ ABC = 180° − ∴PBC ... (1)

Also,

∴ACB + ∴QCB = 180°

∴

∴

ACB = 180° − QCB ... (2)

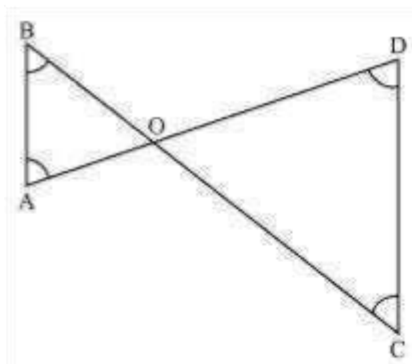
As PBC < QCB,

∴ 180° − PBC > 180° − ∴QCB

∴ ∴ ABC > ACB [From equations (1) and (2)] ∴ AC >

AB (Side opposite to the larger angle is larger.) Question 3:

In the given figure, ∠B < ∠A and ∠C < ∠D. Show that AD < BC.



Answer:

In ΔAOB,

∴ B < A AO < BO (Side opposite to smaller angle is smaller) ... (1)

In ΔCOD,

∴ C < D

∴ OD < OC (Side opposite to smaller angle is smaller) ... (2)

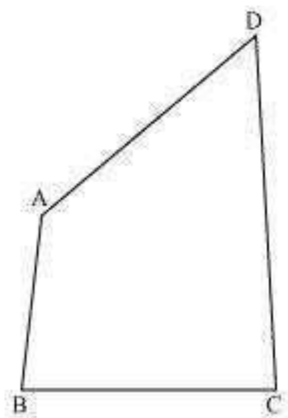
On adding equations (1) and (2), we obtain

AO + OD < BO + OC

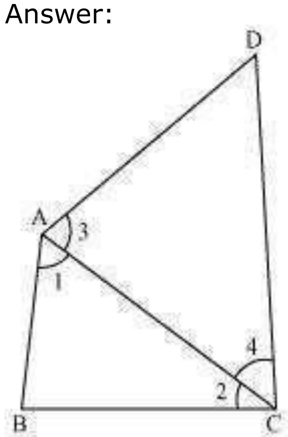
AD < BC

Question 4:

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD see the given figure). Show that A > C and ∴B > (∴D.



Answer:



Let us join AC.
In ΔABC,

AB < BC (AB is the smallest side of quadrilateral ABCD)

∴ ∴ 2∴< 1 (Angle opposite to the smaller side is smaller) ... (1)

In ΔADC,

AD < CD (CD is the largest side of quadrilateral ABCD)

∴ ∴ 4∴< 3 (Angle opposite to the smaller side is smaller) ... (2)

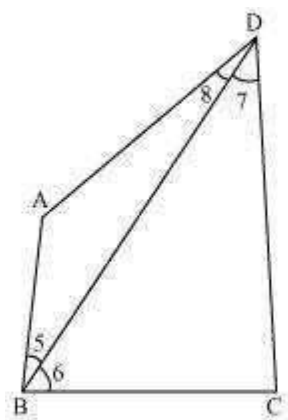
On adding equations (1) and (2), we obtain

$\angle 2 + \angle 4 < \angle 1 + \angle 3$ ∴

$\angle C < \angle A$

∴ $\angle A > \angle C$

Let us join BD.



In ΔABD,

AB < AD (AB is the smallest side of quadrilateral ABCD)

∴ $\angle 8 < \angle 5$ (Angle opposite to the smaller side is smaller) ... (3)

In ΔBDC,

BC < CD (CD is the largest side of quadrilateral ABCD)

∴ $\angle 7 < \angle 6$ (Angle opposite to the smaller side is smaller) ... (4)

On adding equations (3) and (4), we obtain

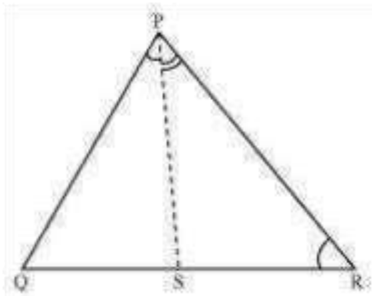
$\angle 8 + \angle 7 < \angle 5 + \angle 6$

∴ $\angle D < \angle B$

∴ $\angle B > \angle D$

Question 5:

In the given figure, PR > PQ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.



Answer:
As PR > PQ,

$\therefore \angle PQR > \angle PRQ$ (Angle opposite to larger side is larger) ... (1) PS is the bisector of $\angle QPR$.

$\therefore \angle QPS = \angle RPS$ ... (2)

$\angle PSR$ is the exterior angle of ΔPQS.

$\therefore \angle PSR = \angle PQR + \angle QPS$ ... (3)

$\angle PSQ$ is the exterior angle of ΔPRS.

$\therefore \angle PSQ = \angle PRQ + \angle RPS$ ... (4)

Adding equations (1) and (2), we obtain

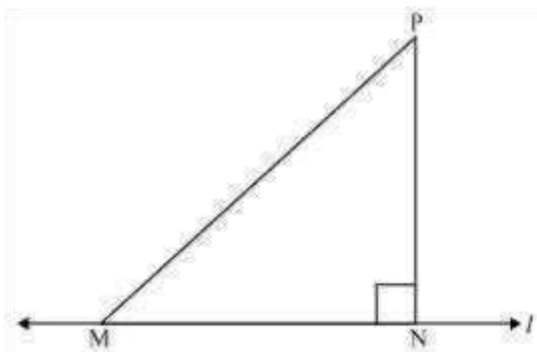
$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$

$\therefore \angle PSR > \angle PSQ$ [Using the values of equations (3) and (4)]

Question 6:

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Answer:



Let us take a line l and from point P (i.e., not on line l), draw two line segments PN and PM. Let PN be perpendicular to line l and PM is drawn at some other angle.

In ΔPNM,

$\angle N = 90^\circ$

$\angle P + \angle N + \angle M = 180^\circ$ (Angle sum property of a triangle)

$\angle P + \angle M = 90^\circ$

Clearly, $\angle M$ is an acute angle.

∴ $\angle M < \angle N$

∴ PN < PM (Side opposite to the smaller angle is smaller)

Similarly, by drawing different line segments from P to l, it can be proved that PN is smaller in comparison to them.

Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Exercise 7.5 Question

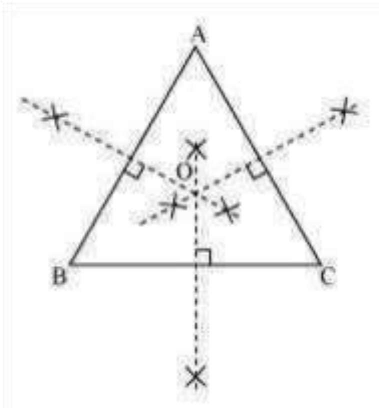
1:

ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Answer:

Circumcentre of a triangle is always equidistant from all the vertices of that triangle.

Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.



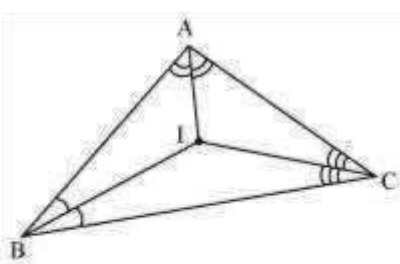
In ΔABC, we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of ΔABC.

Question 2:

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Answer:

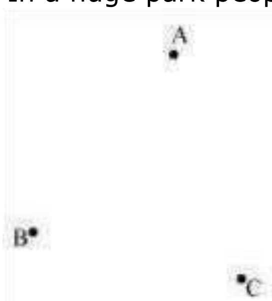
The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



Here, in ΔABC, we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of ΔABC.

Question 3:

In a huge park people are concentrated at three points (see the given figure)



A: where there are different slides and swings for children,

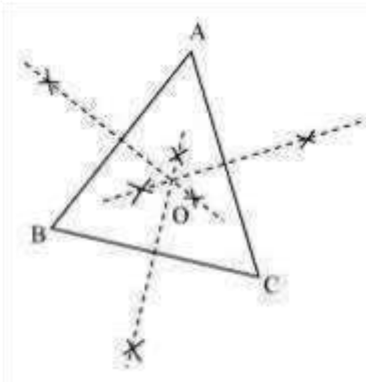
B: near which a man-made lake is situated,

C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

(Hint: The parlor should be equidistant from A, B and C) Answer:

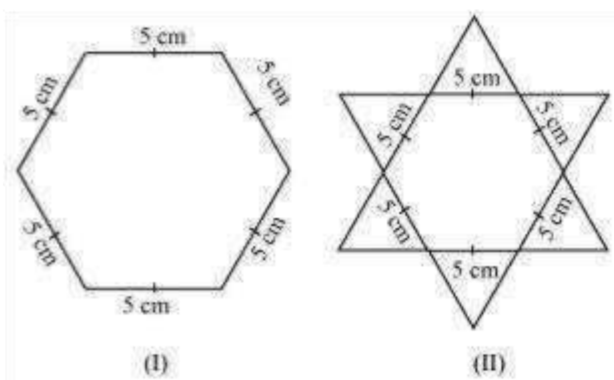
Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre O of ΔABC.



In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.

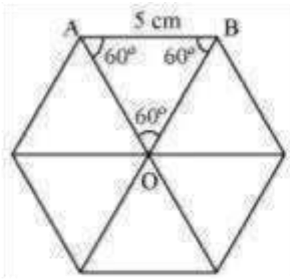
Question 4:

Complete the hexagonal and star shaped rangolies (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



Answer:

It can be observed that hexagonal-shaped rangoli has 6 equilateral triangles in it.



Area of ΔOAB $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2$

$= \frac{\sqrt{3}}{4}(25) = \frac{25\sqrt{3}}{4}\ \text{cm}^2$

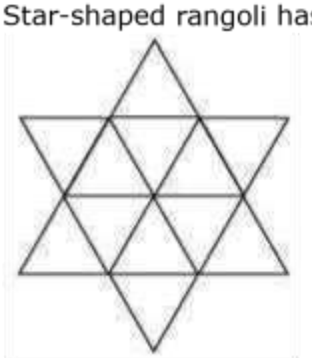
Area of hexagonal-shaped rangoli $= 6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}\ \text{cm}^2$

Area of equilateral triangle having its side as 1 cm $= \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}\ \text{cm}^2$

Number of equilateral triangles of 1 cm side that can be filled

in this hexagonal-shaped *rangoli* $= \dfrac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}} = 150$

Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.



Area of star-shaped rangoli = $12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$

Number of equilateral triangles of 1 cm side that can be filled

in this star-shaped *rangoli* $= \dfrac{75\sqrt{3}}{\dfrac{\sqrt{3}}{4}} = 300$

Therefore, star-shaped rangoli has more equilateral triangles in it.